SIZE DEPENDENCE OF STRENGTH AND FRACTURE PROPERTIES OF BRICK MASONRY WALLS[a]

Discussion by Zdeněk P. Bažant,[4] Fellow, ASCE

The test data on the size effect in the failure of masonry walls and some parts of the discussion of size effect are valuable. However, the theoretical description of the size effect is in my opinion invalid. The use of the "multifractal scaling law" proposed earlier by the senior author is unjustified, for the same reasons as it is for concrete structures. The reasons are explained in detail in Bažant (1997a,b) and Bažant and Planas (1997).

Briefly, one fault, in the discusser's opinion, is that the size effect caused by the release of energy from the structure is ignored, yet it must be taking place if the crack formed before maximum load is large compared with the dimensions of the structure. The explanation is that the energy released due to fracture increases with the structure size faster than the energy is consumed and dissipated by the fracture.

The second fault, which is less obvious, is that the multifractal concept, in my opinion, could apply only if the fractures that cause failures of structures of different sizes were happening on different scales of the material—in other words, if the fracture of a large structure were decided by a crack longer than several brick sizes and the fracture of a small structure were decided by a crack in the layer of mortar in one brick joint, smaller than one brick size. In that case, the representative volumes of the relevant materials on the different scales would not be the same, and their different fractal dimensions could conceivably matter.

However, in the discusser's opinion, this is not the case if one deals with masonry walls (or concrete structures) per se. The authors consider the macroscopic cracks in the masonry as a whole. It has been shown mathematically (Bažant 1997a; Bažant and Chen 1997; Bažant and Planas 1997) that if there is only one and the same material, which must inevitably be characterized by one and the same representative volume, the hypothesis of multifractality reduces to monofractality. The reason is that any material volume relevant to the failure of a larger structure can be subdivided into the representative volumes of the material considered for the smaller structure (by definition, or else one would not speak of the same material, masonry in this case).

Consequently, in my opinion, the only conceivable (albeit still questionable) consequence of the hypothesis of fractality of fracture (whether lacunar or invasive) would be a power law scaling, which is represented by a straight line in the log-log plot rather than a curve of declining slope. Such scaling, however, implies the absence of any characteristic length (Bažant 1997b)—obviously untrue for masonry walls, in which the brick size imposes a finite characteristic length of the material.

The test data of the authors could be equally well fitted by the quasibrittle size effect law based on energy release, which was initially proposed in Bažant (1984) and was in 1987 extended to a form with a finite large-size asymptotic value of nominal strength, exhibiting at larger sizes a positive curvature in the log-log plot of nominal strength versus the size.

The type of the size effect law to use unfortunately cannot be decided solely by size effect experiments (of limited size range). The scatter of the test results is just too large [see Fig. 10(a) of the paper]. A correct theory, agreeing with all the other experimental evidence, is important.

APPENDIX. REFERENCES

Bažant, Z. P. (1984). "Size effect in blunt fracture: Concrete, rock, and metal." *J. Engrg. Mech.*, ASCE, 110, 518–535.

Bažant, Z. P. (1997a). "Scaling of quasibrittle fracture: Hypotheses of invasive and lacunar fractality, their critique and Weibull connection." *Int. J. Fracture*, 83(1), 41–65.

Bažant, Z. P., and Chen, E.-P. (1997). "Scaling of structural failure." *Appl. Mech. Rev.*, 50(10), 593–627.

Bažant, Z. P., and Planas, J. (1997). *Fracture and size effect in concrete and other quasibrittle materials.* CRC Press, Boca Raton, Fla.

Closure by Alberto Carpinteri,[5] Fellow, ASCE, Bernardino Chiaia,[6] and Pietro Bocca[7]

The writers would like to thank the discusser for his kind words at the beginning of the discussion and for considering as valuable at least some parts of the paper. They will try to convince him that even the other parts are quite reasonable and to remove some of his criticisms.

The discusser properly affirms that the energy released due to fracture increases with the structure size faster than the energy consumed and dissipated by the fracture. This is nothing but the very well-known Griffith's energy approach, which produces a strength decrease with slope equal to $-1/2$ in the log-log plot. The writers have taken into consideration this size effect caused by the release of energy from the structure exactly when as the discusser requests, the crack formed before maximum load is large compared with the dimensions of the structure, i.e., when the structure is sufficiently small.

When the structure is larger, the crack formed before maximum load could be very small, or even absent when a very brittle (catastrophic) failure occurs. On the other hand, a homogenization effect should prevail for very large specimen sizes, due to the limited size of the heterogeneities (aggregates, pores, cracks, etc.). The two limit situations of small scales (slow crack growth and geometric disorder) and large scales (fast crack propagation and geometric order) must be connected by an envelope curve with decreasing slope (from $-1/2$ to zero). The multifractal scaling law deriving from the above arguments presents a characteristic internal length, as the discusser expects.

The writers agree with the discusser that several of the test data reported in the literature could be equally well fitted by the size effect law proposed by him in 1984, but only when the scale range is below one order of magnitude. In the few other cases, a finite asymptotic value of nominal strength emerges for large-sized uncracked specimens, exhibiting a positive curvature in the log-log plot. This is nothing but the homogenization effect mentioned above.

The writers wish to take this opportunity to affirm that they never used the invasive fractals to explain the strength size effects, as the discusser erroneously reported (Bažant 1997a,b).

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[a]August 1997, Vol. 123, No. 8, by Alberto Carpinteri, Bernardino Chiaia, and Pietro Bocca (Paper 10729).

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The invasive fractals, on the other hand, are useful to explain the fracture energy size effects (Carpinteri and Chiaia 1995). In addition, the lacunar fractality assumption regards the material ligament (or the net cross section) and not the microcrack array, as the discusser probably misunderstood.

Fractals represent the geometric aspect of renormalization group theory (RGT) and RGT is an extension of dimensional analysis. When we apply fractal concepts to mechanics we create a multidisciplinary approach, and should not be accused of abandoning mechanics. In the same way, the Reynolds' number is a topic belonging to hydraulics as well as to dimensional analysis.

As a last remark, we willingly admit that the size effect law according to the discusser is very appropriate when initially cracked or notched structures are considered, with their cracks or notches scaling proportionally to the structure dimensions. On the contrary, when the structures are initially uncracked and unnotched, this law is not appropriate and must be generalized to consider the homogenization effects in the framework of self-affine geometry.