COMPUTER MODELLING OF CONCRETE STRUCTURES

Edited by

H. MANG
Technical University, Vienna, Austria

N. BIĆANIĆ
University of Wales, Swansea, U. K.

R. DE BORST
Delft University of Technology, Netherlands

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Microplane Model with Stress-Strain Boundaries and Its Identification from Tests with Localized Damage

By ZDENĚK P. BAŽANT[1], MILAN JIRÁSEK[2], YUYIN XIANG[3], and PERE C. PRAT[4]

ABSTRACT. — The paper deals with the microplane model for nonlinear triaxial behavior of quasibrittle material such as concrete and its method of identification from test data. In the microplane model, the constitutive behavior is characterized by relations between the stress and strain components on planes of various orientation in the material, called the microplanes, which are constrained to the macroscopic strain tensor. Certain limitations of the recently developed microplane model are pointed out and an improvement avoiding these limitations is proposed. The idea is to introduce stress-strain boundary curves on the microplane level. Further, a simple method to take into account the localization of strain softening damage within test specimens the identification of the stress-strain relation from test data is proposed.

1 Introduction

Despite significant progress in the nonlinear triaxial modeling of concrete and interpretation of failure test data, fully realistic models are not yet available. Among various approaches, such as plasticity, continuum damage mechanics, fracturing or plastic-fracturing theory, etc., a very powerful, versatile and simple in concept is perhaps the microplane model. This model represents a generalization of the idea of G.I. Taylor [1] which was developed for hardening

[1]Walter P. Murphy Professor of Civil Engineering, Northwestern University, Evanston, IL 60208 USA

[2]Post Doctoral Research Associate presently Assistant Professor, Czech Technical University (ČVUT), Prague, Czech Republic

[3]Graduate Research Assistant, Northwestern University, Evanston, IL 60208 USA

[4]Visiting Scholar, Associate Professor on leave from Polytechnic University of Catalunya, Barcelona, Spain

plasticity of polycrystalline metals by Batdorf and Budianski [2] and others, and for soils by Pande et al. [4]. The idea was further extended to materials undergoing softening damage by Bažant and Oh [5] for tensile failure, and by Bažant and Prat [6] for compressive and shear failures (also [3]). A further generalization to a nonlocal form which can properly handle localization of damage was presented by Bažant and Ožbolt [7]. The approach was originally called the slip theory of plasticity [2], but this term became inappropriate for adaptations of Taylor's idea to materials with damage which exhibit no plastic slip. For this reason, the neutral term "microplane model" was coined [5]. It reflects the basic feature that in microplane model the material properties are characterized by relations between the stress and strain components independently for planes of various orientation within the material. The behavior for various orientations is then combined to a macroscopic tensorially invariant form by means of a variational principle. In contrast to the original applications to hardening metals, the modeling of damage requires these planes, called the microplanes, to be constrained to the macroscopic material behavior on the basis of strain rather than stress, or else a model that is unstable for post-peak softening would result.

Recently, in connection with the development of a computer program for the simulation of impact, certain limitations of the previously proposed microplane model [6] have been detected. The present paper proposes in general terms an improvement of the microplane model from [6] which overcomes these limitations and also combines certain other improvements made by Carol et al. Aside from that a new but rather simple approach will be proposed for the method of identification of the material model from test data obtained from specimens that must be assumed to have undergone a certain degree of localization of softening damage. A detailed journal article on these improvements is in preparation.

The limitations of the microplane model from [6] are encountered for complex triaxial tensile behavior at large post-peak strains. Among the volumetric and deviatoric strains on the microplanes, the tensile softening tends to localize into the volumetric components and the deviatoric components exhibit unloading. This is unreasonable and leads to grossly unrealistic lateral strains in tensile loading at large post-peak axial strains. A remedy to this problem will be shown.

2 The Microplane Concept

To remedy the aforementioned problem, the existing microplane concept must be changed to some extent. One remedy would be to return to the original microplane model from [5] in which the normal strain on the microplanes was not separated into its volumetric and deviatoric parts, and the post-peak strain softening was characterized in terms of the relations between the normal stresses and strains on the microplanes. This model worked well for tensile cracking, however, as it turned out, generalization to damage in compression turned out to be impossible. If the relation between the normal stress and strain on a

microplane is made to exhibit post-peak tensile softening in compression, one can model well the uniaxial and biaxial compression tests but not the response to high hydrostatic pressure or high uniaxial compressive strain. For such loadings there is no peak and no softening. However any model in which the relationship between the normal stress and strain on the microplane exhibits post-peak softening would also exhibit softening in the hydrostatic test and the uniaxial strain test. This conflict was overcome in the latest microplane model [6-9] by splitting the normal microplane strains into their volumetric and deviatoric parts, and compression softening was considered only for the deviatoric part. At the same time, since the volumetric part exhibits no post-peak softening, no problems with localization of post-peak softening into one of the volumetric and deviatoric components could develop for compression, unlike in tension.

Another reason for splitting the normal microplane strain ϵ_N into the volumetric strain ϵ_V and the deviatoric strain $\epsilon_D = \epsilon_N - \epsilon_V$ was to capture the Poisson effect on the microplane level and thus be able to control the value of the Poisson ratio on the macroscale. In analogy to the multiaxial elastic Hooke's law [which can be written as $\sigma_x = E'(\epsilon_x + \mu'\epsilon_y + \mu'\epsilon_z)$ where E' and μ' are constants, σ_x, ϵ_x = axial stress and strain, ϵ_y, ϵ_z = lateral strains] one may introduce for each microplane the elastic relation $\sigma_N = C(\epsilon_N + \mu'\epsilon_K + \mu'\epsilon_M)$ where C = constant. This may be rewritten as

$$\sigma_N = C_N^0 \epsilon_N + C_L^0 \epsilon_L \tag{1}$$

where C_N^0, C_L^0 = constants and

$$\epsilon_L = \frac{1}{2}(\epsilon_K + \epsilon_M) = \frac{1}{2}(3\epsilon_V - \epsilon_N) = \epsilon_V - \frac{1}{2}\epsilon_D \tag{2}$$

ϵ_K, ϵ_M = normal strains in the directions of two Cartesian coordinate axes within the microplane, and ϵ_L = mean transverse strain called lateral strain, which is invariant with respect to coordinate rotations about the axis normal to the microplane.

Substituting (3), we may rewrite Eq. (1) as:

$$\sigma_N = \sigma_V + \sigma_D, \quad \sigma_V = C_V^0 \epsilon_V, \quad C_D^0 = C_D^0 \epsilon_D \tag{3}$$

where C_V^0 and C_D^0 are constants. Eqs. (1) and Eq. (3) are equivalent, and the latter one was used in the latest microplane model from [6].

For reasons explained before [5, 6], the microplane strains are constrained to the macroscopic strain tensor ϵ_{ij} assuming a kinematic constraint, in which the strain components on each microplane are the resolved components of the macroscopic stress tensor, that is

$$\epsilon_N = n_i n_j \epsilon_{ij}, \quad \epsilon_V = \epsilon_{kk}/3, \quad \epsilon_D = \epsilon_N - \epsilon_V \tag{4}$$

$$\epsilon_{T_i} = \frac{1}{2}(\delta_{ik} n_m + \delta_{im} n_k - 2 n_i n_k n_m)\, \epsilon_{km} \tag{5}$$

Here n_i = direction cosines of the unit normal of each microplane in Cartesian coordinates x_i; $\sigma_{T_k}, \epsilon_{T_k}$ = component of the vectors of the shear strain and shear stress within each microplane, and Ω = surface of a unit hemisphere. The macroscopic stress tensor is obtained from the principle of virtual work as:

$$\sigma_{ij} = \int_\Omega \left[\sigma_N n_i n_j + \frac{1}{2}\sigma_{T_k}\left(n_i\delta_{kj} + n_j\delta_{ki} - 2n_i n_j n_k\right)\right] d\Omega \tag{6}$$

where Ω = surface of a unit hemisphere and σ_{T_k} = components of the shear stress vector on each microplane. In numerical applications, the foregoing integral is evaluated by an optimal Gaussian integration scheme on a hemispherical surface [5].

The constitutive properties were in the latest form of the microplane model defined by the functions F_V, F_D and F_T in the form: $\sigma_V = f_V(\epsilon_V)$, $\sigma_D = f_D(\epsilon_D)$, $\sigma_T = f_T(\epsilon_T, \epsilon_V)$, among which F_D and F_T exhibited post-peak softening for compression.

3 Stress-Strain Boundaries

To circumvent the aforementioned problem with localization of tensile softening into the volumetric component, as well as for other reasons, a new idea of stress-strain boundaries is proposed [10]. These boundaries are represented by the curves:

$$\sigma_N = F_N(\epsilon_N), \quad \sigma_V = F_V(\epsilon_V), \quad \sigma_D = F_D(\epsilon_D), \quad \sigma_T = F_T(\epsilon_T, \epsilon_N) \tag{7}$$

with functions F_N, F_V, F_D and F_T sketched in Fig. 1. The stress-strain response is not allowed to get beyond these boundary curves. Outside the boundary curves, for the sake of simplicity, the response is assumed to be elastic, given by straight lines and described by Eq. (3) or (1). Upon arriving to the boundary curve, there is a sudden change of slope since the response is forced to follow the boundary curve. When loading reverts to unloading, the response departs from the boundary curve and is again elastic, given by Eq. (3) or (1). Despite the sudden change of slope, the macroscopic response is an almost smooth curve since there are many microplanes (at least 21) and different microplanes enter the boundary curve at different moments.

Note that the boundary curve for tensile softening is given in terms of normal stress and strain, with no volumetric-deviatoric split. On the other hand, the boundary curve for compression softening is given only in terms of the deviatoric components. For the volumetric components in compression, there is a boundary curve but no softening. For the shear strains on the microplanes, the boundary curves are not fixed but depend on the normal strain as a parameter; for large normal compressive strain, the boundary a in Fig. 1 applies, and for large normal tensile strain, the boundary b applies, with a gradual transition in between. The dependence of the shear stress σ_T on the normal strain ϵ_N indirectly characterizes the friction on a microplane or the fact that a widely opened rough crack offers less resistance to shear than a narrow crack.

4 Identification of Strain Softening Constitutive Model from Tests with Localized Damage

The stress-strain relations with strain softening have so far been identified from test data on small specimens ignoring the possibility of nonuniform states of damage. However, beginning in the mid-1970's [14] it became clear that strain softening damage must localize even within the small laboratory test specimens. This was later clearly demonstrated for compression [15, 16, 21] as well as tension [17]. However, because of the tremendous complexity of the general problem of material identification in presence of strain softening localization [22], this aspect has been ignored in the evaluation of test data. But such an approach is no longer reasonable. The localization must be taken into account, although not necessarily with a high degree of accuracy and sophistication.

In the present study, the test data from small specimens are being analyzed taking into account localization in an approximate manner, based on two simple concepts: (1) localization in the series coupling model, and (2) the effect of energy release due to localization on the maximum load, as described by the size effect law proposed by Bažant [18]. According to the series coupling model, the mean strain observed in a test of a specimen [19] is

$$\bar{\epsilon} = \lambda\epsilon + (1-\lambda)\epsilon_u, \qquad \lambda = L/\ell \tag{8}$$

in which ϵ is the actual strain in the strain softening damage band, which we want to model, ϵ_u is the strain in the rest of the specimen which undergoes unloading from the peak stress point (Fig. 4), L = specimen length, and ℓ = length (width) of the strain softening band. The unloading strain is $\epsilon_u = \epsilon_p - (\sigma_p - \sigma)/E$ where E = Young's elastic modulus, ϵ_p and σ_p = strain and stress at the peak of the stress-strain curve for the given type of loading. What the microplane model or any constitutive model for damage predi is the strain ϵ, while the strain that is observed is $\bar{\epsilon}$. Obviously, to make a comparison with test data one must know the value of ℓ. It is next to impossible to determine this value for the test data reported in the literature. However, a reasonable estimate can be made by experience from some other test analyses, and it transpires that a reasonable estimate is perhaps $\ell = 3d_a$ where d_a = the maximum size of the aggregate in concrete, in the case of normal concretes (for high-strength concrete ℓ might perhaps be as small as d_a). The value of ℓ has also to do with the size of the finite elements which are to be described by the damage constitutive law derived from the test data. It is intended that the size of these elements should be roughly $3d_a$. Anyway, for a smaller element size the assumption of a continuum, implied in the use of the finite element method would hardly be justifiable.

Extending the length of a long specimen does not appear to influence its strength, however, changing all the specimen sizes in proportion does. This type of size effect is not describable by the series coupling model. The response is a complex mixture of series and parallel coupling and is better looked at a in a different way—through the energy release. A damage zone releases energy not only from the zone itself but also from the adjacent material that is getting

5 Concluding Remarks

Constitutive modeling of quasibrittle materials such as concrete is a difficult problem in which improvements have been only gradual. The current knowledge nevertheless represents a major advance compared to the situation 20 years ago. The present paper presents another gradual improvement, based on the concept of the microplane model—a powerful model which trades simplicity of concept for the penalty of larger computational demands. The new idea of stress-strain boundary curves makes it possible to obtain a realistic response at large tensile strain softening deformations over a broader range of triaxial behavior. Another improvement is proposed for the method of identification of the constitutive relation from the test data, in which the localization in the test specimens in taken in an approximate but tractable manner.

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