Bažant, Z.P., and Kazemi, M. T. (1994). "Localization of softening damage in frames and implications for earth-quake resistance." *Proc., 5th U.S. National Conference on Earthquake Engineering* (held in Chicago, July 1994), Earth-quake Engineering Research Institute, Oakland, California, Vol. 1, 313–322.

FIFTH U.S. NATIONAL CONFERENCE ON EARTHQUAKE ENGINEERING

July 10 - 14, 1994
Chicago, Illinois

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P R O C E E D I N G S

VOLUME I

Earthquake Engineering Research Institute

LOCALIZATION OF SOFTENING DAMAGE IN FRAMES AND IMPLICATIONS FOR EARTHQUAKE RESISTANCE

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Abstract

The paper demonstrates that, in statically indeterminate building frames, the symmetric plastic hinge mechanisms typically considered as a mode of inelastic deformation in an earthquake can be severely altered by localization of flexural softening behavior. Due to softening, the symmetry of response normally assumed in calculations may break down. This may cause the strain energy of the structure released by softening to be fed into a lesser number of softening hinges, thus provoking a faster collapse that predicted on the basis of symmetric response. Analysis of static monotonic loading of a portal frame shows that more than one solution exists when the inelastic hinges exhibit flexural softening during collapse. The response path bifurcates in a way that breaks the symmetry of response. Comparison of the tangential stiffnesses for the symmetric and nonsymmetric response path reveals that the nonsymmetric response path is that which actually occurs. For the nonsymmetric path, the damage localizes into fewer softening hinges. By implication, for cyclic or dynamic loading, or both, one must expect a similar localization phenomenon and bifurcation of the solution in time. Aside form the symmetric solution of the response path, there must exist a nonsymmetric solution in which the damage is localized into fewer softening hinges. To detect such behavior, the frame cannot be simplified as a single-degree-of-freedom oscillator.

Introduction

Although it would be preferable to design building frames so that their load-deflection diagram exhibit a horizontal yield plateau, in practice this turns out to be usually impossible to achieve. This is true not only for reinforced concrete frames but also for steel frames.

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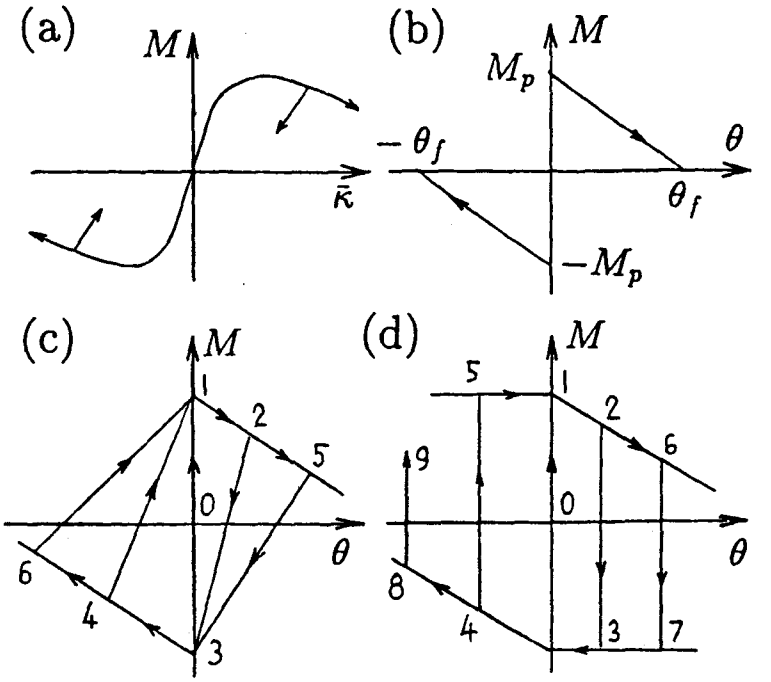

Figure 1: (a) Diagram of bending moment vs. average curvature in softening segment of beam, (b–d) Simplified moment rotation diagrams for softening hinges.

In reinforced concrete frames, the large deformations that occur in earthquake cause material damage in the form of distributed cracking as well as localized fracture of concrete. Such damage is manifested by development of the so-called "plastic" hinges which, however, are not plastic. Rather, they exhibit gradual softening, that is, a flexural softening response in which the bending moment decreases at increasing rotation (Fig. 1a). Such behavior is typical of columns or prestressed beams, in which concrete typically undergoes softening damage in compression before the steel reinforcement reaches its limit capacity. Under seismic loading the cracking damage that causes degradation of the yield stress and reduction of stiffness is intensified by repeated loading.

Steel frames have been widely thought to be free of such softening because the material per se does not exhibit strain softening. However, as shown experimentally already by Maier and Zavelani (1970), post-peak softening in the moment-rotation diagram of the plastic hinges in steel beams of large or thin-wall cross sections can be caused plastic buckling of the flanges and webs.

It is well-known that softening damage in general leads to localization instabilities and

response path bifurcations which break the symmetry of response (Maier, 1971; Maier, Zavelani and Dotreppe, 1973; Bažant, 1976; Bažant, Pijaudier-Cabot and Pan, 1987; Bažant, Pan and Pijaudier-Cabot, 1987; Bažant and Cedolin, 1991; Hunt and Baker, 1993). These phenomena have recently been extensively studied with regard to the formation and propagation of damage bands and fracture. Some studies have also been devoted to the localization of flexural softening damage in various simple structures. However, these phenomena have apparently not been studied with regard to the behavior of building frames in an earthquake. Yet it is clear that, in principle, the softening damage that is produced in an earthquake must lead to localization and response path bifurcation.

The purpose of the present conference paper is to show that, in statically indeterminate frames, some symmetric plastic hinge mechanisms typically considered as a mode of inelastic deformation in an earthquake can be severely altered by localization of softening behavior. Due to softening, the symmetry of response normally assumed in calculations may break down. This may cause the strain energy of the structure released by softening to be fed into a lesser number of softening hinges, thus provoking a faster collapse that predicted on the basis of symmetric response.

Example of a Portal Frame with Softening Hinges

As an example, consider that a large horizontal displacement u is imposed in the portal frame in Fig. 2a. Let F be the corresponding horizontal load (or reaction). The frame has one statically indeterminate internal force. During plastic collapse, the frame becomes a single-degree-of-freedom mechanism with two plastic hinges, as shown in Fig. 2b. However, in the case that the diagram of the bending moment M versus hinge rotation θ exhibits post-peak softening (Fig. 1a,b), the symmetric deformation mode with equal deflection curves of both columns (legs) and equal rotations of both hinges (Fig. 2b) is only one possible post-peak response to imposed horizontal displacement. Another possible post-peak response is the deformation mode shown in Fig. 2c, in which the softening damage localizes from two softening hinges into one hinge while the other hinge gradually unloads, behaving elastically.

For the sake of simple illustration, consider the moment-rotation diagram to be linear, characterized by the peak bending moment M_p and by rotation θ_f at which the bending moment is reduced to zero; see Fig. 1b. By elastic analysis according to the principle of virtual work, the horizontal deflection and force at the limit of elastic response (peak of the load-deflection diagram) are:

$$u_p = \frac{M_p \gamma}{6EH^2}, \qquad F_p = \frac{2M_p}{H} \tag{1}$$

$$\gamma = H^3 \left(\frac{L}{I_b} + \frac{2H}{I_c} \right) \tag{2}$$

where H = height of the frame, L= its span, and I_b, I_c = centroidal moments of inertia of the cross sections of the beam and the columns (legs) in the frame. At the limit of elastic

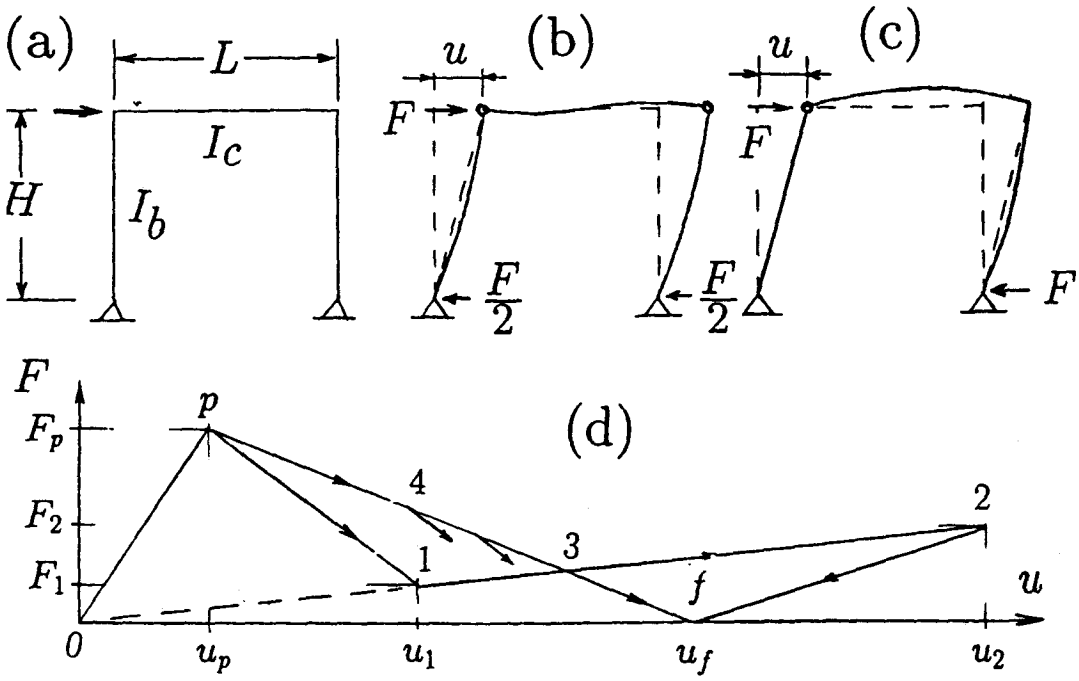

Figure 2: Portal frame, its symmetric and nonsymmetric deflections with softening hinges, and load-deflection diagrams.

response, moment M_p is reached simultaneously at both corners.

For the symmetric deformation mode with two softening hinges (Fig. 2b), the post-peak segment of the load-deflection diagram (Fig. 2d) must be linear (because the moment-rotation diagram is linear) and must terminate with a state at which the rotations of both hinges are θ_f, $F = 0$, and the bending moments are everywhere zero. At that state, the displacement is $u_f = \theta_f H$.

For the nonsymmetric post-peak deformation mode, in which the left corner of the frame undergoes softening and the moment at the right corner decreases elastically from its peak value M_p, the terminal state (point 1 in Fig. 2d) of the initial post-peak linear segment corresponds to the bending moment $M_a = 0$ and hinge rotation $\theta_1 = \theta_f$ at the left corner of the frame, while the rest of the frame is elastic. The magnitude of the (negative) bending moment at the right corner is $M_b = F_1 H$ which must not be larger than M_p. By elastic analysis according to the principle of virtual work, the deflection and the load at this terminal state (point 1 in Fig. 2d) are:

$$u_1 = \frac{2\alpha}{\beta}\theta_f H, \qquad F_1 = \frac{6}{\beta}EH^2\theta_f \tag{3}$$

$$\alpha = H^3\left(\frac{L}{I_b} + \frac{H}{I_c}\right), \qquad \beta = H^3\left(\frac{3L}{I_b} + \frac{2H}{I_c}\right) \tag{4}$$

At point 1, the horizontal reaction at the right support is equal to F_1 (since at the left support the reaction is zero). So the bending moment at the right corner of the frame is $M_b = -F_1 H = -6EH^3\theta_f/\beta$, the absolute value of which must not be larger than M_p. From this we conclude that the nonsymmetric response path exist only if

$$\theta_f \leq \frac{\beta}{6EH^3} M_p \tag{5}$$

Beyond point 1, the frame behaves elastically as if there were a real hinge (with zero moment) at the left corner. The load increases linearly up to point 2 (Fig. 2d) for which elastic analysis according to the principle of virtual work gives

$$u_2 = \frac{M_p \alpha}{3EH^2}, \qquad F_2 = \frac{M_p}{H} \tag{6}$$

Then a softening hinge forms at the right corner and, as this hinge softens, the load decreases linearly to point f in Fig. 2d. The slope of the path from point 2 to point f can be positive or negative (Fig. 2d shows the case of negative slope, representing snapback instability). The energy dissipated by each softening hinge up to total failure is $M_p\theta_f/2$, and so the energy dissipated by the whole frame during failure is $W_f = M_p\theta_f$. This energy is also represented by the area under the complete load-deflection diagram. Therefore, the areas 0pf0 and 0p12f0 in Fig. 2d must each equal W_f. Thus, the triangles p13p and 32f3 must have equal areas, which means that point 2 for nonsymmetric response must lie above the line pf for the symmetric response.

The initial post-peak tangential stiffnesses of the frame, which represent the initial post-peak slopes, are obtained as $K = \Delta F/\Delta u$ where Δ refers to the change from the peak state to point 1. The stiffnesses for the symmetric and nonsymmetric responses are thus calculated as:

$$K^{(1)} = \frac{-F_p}{u_f - u_p}, \qquad K^{(2)} = \frac{F_1 - F_p}{u_1 - u_p} \tag{7}$$

These stiffnesses can be either negative (as shown in Fig. 2d) or positive. For a positive post-peak stiffness, there is snapback instability, which means the structure is unstable even under displacement control. For a negative post-peak stiffness, the structure is stable under displacement control.

As we have shown, the equilibrium response path of the frame bifurcates at the peak load state, that is, at the start of softening. It can similarly be shown that there is bifurcation at every point (e.g. point 4 or 5) along the symmetric response path (the straight line connecting the peak to point f), such that the softening hinge at one corner starts to unload while the other hinge continues softening. But what matters is the first bifurcation, which occurs at the peak. Now, which of the two bifurcating paths will actually occur?

The path that will occur is that which descends more to the left in Fig. 2d, as has

been generally shown in Bažant and Cedolin (1991, chapter 10 and 13) on the basis of thermodynamic considerations. Thus, the actual post-bifurcation path is that for which the compliance $1/K$ is larger. So, the nonsymmetric path, labeled by superscript $^{(2)}$ will occur if and only if

$$\frac{1}{K^{(2)}} > \frac{1}{K^{(1)}} \tag{8}$$

If we substitute here the foregoing expressions and rearrange, we obtain the same inequality as (5), as might have been expected. Thus the hinge rotation at which the bending moment is reduced to zero (Fig. 1b) must be sufficiently small for the nonsymmetric response path to exist and to occur.

Example of a Column with Softening Hinges

As another even simpler example, consider the column in Fig. 3a, with one fixed end and one sliding end restrained against rotation. This column, which is loaded by prescribing the horizontal displacement u at the top and fails by softening hinges at the ends, represents a simplified model for the behavior of columns in multi-story frames in an earthquake. The peak value of the horizontal load F is $F_p = 2M_p/L$ and the corresponding displacement is $u_p = M_pL^2/6EI$.

One possible post-peak response is symmetric, with equal magnitudes of bending moments and hinge rotations at the ends. The load-deflection diagram is linear and again is given by line pf in Fig. 2d, in which now $u_f = \theta_f L$ where L is the height of the column.

Another possible response is nonsymmetric, such that the bottom hinge first rotates while the top cross section unloads elastically. The path is represented by line p1 in Fig. 2d, and the terminal point 1 of this path, corresponding to $M = 0$ and $\theta = \theta_f$ at the bottom is found by elastic deformation analysis to be $u_1 = 2L\theta_f/3$ and $F_1 = 2EI\theta_f/l^2$. The requirement that at point 1 the moment on top would not exceed M_p yields the condition $\theta_f \leq M_pL/2EI$. Otherwise, the nonsymmetric response does not exist.

After point 1, the column loads elastically along path 12 and at point 2 the moment on top reaches M_p. At point 2, $u_2 = M_pL^2/3EI$ and $F_2 = M_p/L$. The areas under the load-deflection diagrams must both be equal to the energy dissipated in the softening hinges, which is $W_f = M_p\theta_f$. For this reason point 2 must lie above the path for the symmetric response.

General Comments on Softening Behavior of Frames

Comparing the symmetric and nonsymmetric responses, we see that in our example the softening damage is more localized in the case of the nonsymmetric response (it is localized into one instead of two hinges). This is a general characteristic of softening

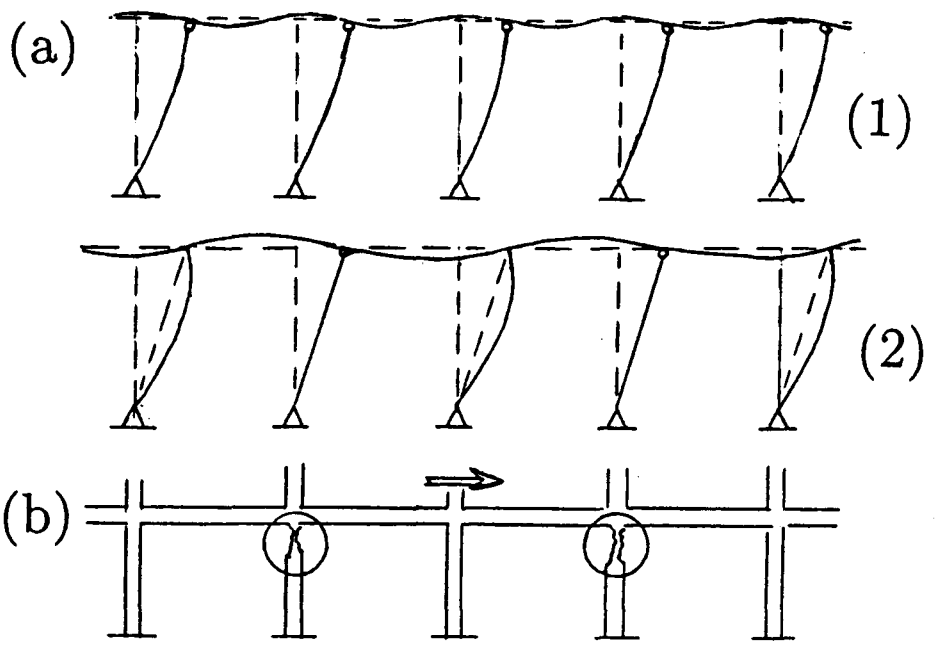

Figure 3: (a) Symmetric and nonsymmetric deflections of multi-bay frame with softening hinges, and (b) damage suffered.

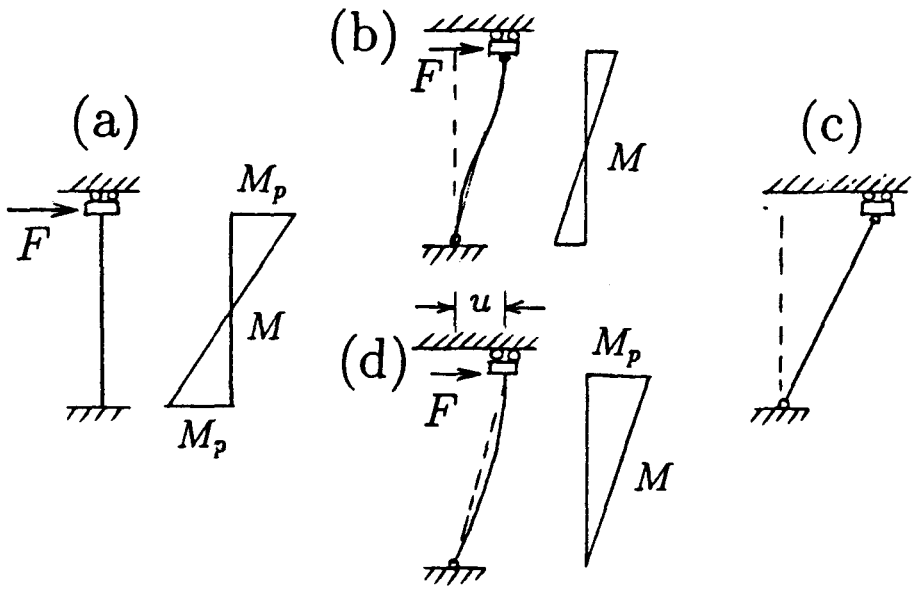

Figure 4: Column loaded by horizontal load, and its symmetric and nonsymmetric deflections with softening hinges.

behavior, contrasting with plastic (non-softening) behavior. This means that the elastic strain energy that is released due to the softening of inelastic hinges flows into fewer hinges—into one rather than two. The larger energy release associated with nonsymmetric response must obviously drive the collapse faster. This is intuitively understood by looking at the deflection shapes of the frames for the symmetric and nonsymmetric responses, as sketched in Fig. 2b,c. In Fig. 2c for the nonsymmetric response, the beam and the right column have at a given displacement smaller curvatures than in Fig. 3b for the symmetric response, and thus they have released more strain energy, which has nowhere else to go but into the softening hinges.

Note that the behavior is analogous to that deduced by fracture mechanics for bodies with multiple cracks (Bažant and Cedolin, 1991, chapter 12). There, too, propagation of a single crack releases more energy into the propagating crack than propagation of several cracks, and therefore fracture tends to localize. The same is true for the propagation of tensile cracking bands or shear bands with softening damage.

A similar behavior in the post-peak softening regime can be demonstrated for various more complex frames, for example multi-bay frames (Fig. 4) or multi-bay multi-story frames. Fig. 4a shows the deflections for symmetric and nonsymmetric post-peak softening responses. For the nonsymmetric deflection shape, the strain energy release for a given horizontal displacement can clearly by greater and it is fed into fewer softening hinges, which means that the damage in those hinges that soften must advance faster.

Implications for Behavior in an Earthquake

Although the foregoing analysis has covered only monotonic static loading, it has important implications for behavior under cyclic and dynamic loadings which occur in an earthquake. A detailed discussion of such behavior is planned for the conference presentation and a subsequent journal article, but is beyond the scope of this brief paper. Only a few comment will be made here.

Under static cyclic loading, the softening hinges follow paths such as 0123456 in Fig. 1c or 1d, in which two simple rules for unloading are described. The hysteretic loops described by such a cyclic path are important for dissipating the kinetic energy supplied by earthquake. During such loading, the softening happens in the hinges in small increments which accumulate from cycle to cycle. Again, there must obviously be bifurcations because the responses can be symmetric or nonsymmetric. It may naturally be expected that a cyclic response that is nonsymmetric would release more of the strain energy stored in the structure, and that more energy will thus flow into fewer softening hinges, which would lead to a faster collapse than for a symmetric response.

Under dynamic cyclic loading, the picture is complicated by inertial forces. But it is nevertheless evident that symmetric as well as nonsymmetric responses represent possible solutions of the dynamic problem when softening damage occurs. Whereas for static loading we know from thermodynamics that the nonsymmetric response is usually the correct one, we cannot obtain such a conclusion from thermodynamics in the case of dynamic behavior. One must analyze the stability of motion for the symmetric and nonsymmetric responses according to Lyapunov's concept of a stable solution in time and one must take small imperfections into account. From experience, small imperfections normally induce breakdown of symmetry and development of large nonsymmetric deformations.

For the purpose of dynamic analysis, structures subjected to earthquake are often simplified as a nonlinear single-degree-of-freedom oscillator. However, such a simplification becomes questionable when the response path bifurcates and localization takes place.

Observations of damage in building frames after an earthquake give hints that localization of damage is taking place. For example, if the deflections were symmetric one would expect in multi-bay frames a symmetric damage, such that every column suffers about the same damage. However, observations show that even if all the columns are identical, only some of the columns are damaged, and usually severely so; see Fig. 4b. Such observations confirm that the damage is nonsymmetric.

Conclusions

1. Analysis of static monotonic loading of a portal frame shows that more than one solution exists when the inelastic hinges exhibit softening during collapse. The response path bifurcates at the peak load in a way that breaks the symmetry of response. Comparison of the tangential stiffnesses for the symmetric and nonsymmetric response

path reveals that the nonsymmetric response path is that which actually occurs. For the nonsymmetric path, the damage localizes into fewer softening hinges.

2. By implication, for cyclic or dynamic loading, or both, one must expect a similar localization phenomenon and bifurcation of the solution in time. Aside from the symmetric solution of the response path, there must exist a nonsymmetric solution in which the damage is localized into fewer softening hinges. In that case, the path with maximum stiffness degradation will occur. The classical simplification as a single-degree-of-freedom oscillator is questionable in that case.

ACKNOWLEDGMENT.–Partial financial support under AFOSR Grant 91-0140 to Northwestern University (monitored by Dr. James Chang) is gratefully acknowledged. Further support for material models of softening cracking damage in concrete was received from Center for Advanced Cement-Based Materials at Northwestern University.

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