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Chaired by
S.P. Shah and A. Aguado

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Concrete Technology

New Trends, Industrial Applications

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13 CONCRETE CREEP AND SHRINKAGE PREDICTION MODELS FOR DESIGN CODES

Recent results and future directions

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Abstract
An improved and simplified prediction model for creep and shrinkage of concrete is presented. New simplified expressions for empirical parameters representing the influence of the composition and strength of concrete on its long term deformations are given. This simplification is achieved by using sensitivity analysis which helped to identify which empirical parameters are important and which are not. A new expression for additional creep due to drying is derived based on the hypothesis of stress-induced shrinkage. Expressions for the effect of constant elevated temperature on basic creep are developed by generalizing the basic creep expressions using the concept of activation energy. Comparative statistics of errors of the proposed model and the current ACI and CEB models are given. The statistics are formulated so as to minimize the bias due to selection of specimen ages, test durations and reading times made by the experimenter. The problem of improving the predictions by extrapolating short-time test data on creep and on shrinkage is discussed.
Keywords : Creep, Concrete, Design codes, Errors, Prediction, RILEM, Shrinkage, Statistics.

1 Introduction

A realistic and accurate prediction of creep and shrinkage of concrete is a formidably difficult problem of great complexity, in which progress has been hard to achieve and has been coming only gradually. Because of the importance of the problem for durability and serviceability of concrete structures, effective transfer of the latest improvements to structural engineering practice is of paramount importance. The present paper, in keeping with the goals of the present symposium, is intended to help this transfer by summarizing the basic guidelines for creep and shrinkage prediction, recently formulated by RILEM Technical Committee 107, and by presenting the latest improvement of the creep and shrinkage prediction model, which has been achieved at Northwestern University as a result of improved understanding of the physical mechanism of creep and drying effects.

The model presented here represents an improvement, based on the aforementioned guidelines, of the previously presented BP-KX model [5]. Although the

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basic form of the time curve for shrinkage has been retained from a previous publication [5], the influence of composition and aging has been simplified. The basic creep compliance function is taken from solidification theory [3] and the composition formulas for the material parameters involved are also simplified [5]. The expression for drying creep is derived starting from the assumption of stress-induced-shrinkage [8] and is better theoretically justified than that in the existing models [2,3,9,10,13]. The formulas for the temperature effect on basic creep are based on the concept of activation energy. Two activation energies, those of hydration and of creep, which have opposing influences on the deformation are used.

The experimental results accumulated during the last several decades, as well as theoretical and numerical studies of the physical phenomena involved in creep and shrinkage of concrete, such as moisture and temperature diffusion, have led the RILEM committee to formulate the guidelines for creep and shrinkage prediction models to be used in design codes (or design recommendations). These guidelines, presented for the reader's convenience in appendix I, evolved from RILEM TC67 conclusions [1] and were produced by RILEM Committee TC107(chaired by Z.P. Bažant) in collaboration with RILEM Committee TC114 (chaired by I. Carol), include a number of requirements and consistency conditions which should be satisfied by any creep or shrinkage prediction model in order to avoid conflict with experimental evidence, agree with solidly established theoretical concepts, and achieve mathematical consistency. It is highly desirable that the models incorporated in design codes or design recommendations would satisfy the foregoing basic guidelines. Unfortunately, a close examination of the existing models shows that this is not the case. It is important that revisions that might be adopted in the future would adhere to these guidelines.

2 A new prediction model

In view of the aforementioned guidelines the following model for practical prediction for creep and shrinkage of concrete has been developed. It represents a simplification as well as an improvement of the existing BP and the BP-KX models. Simplification in the sense that the empirical formulas for the influence of composition and strength of concrete on its long term deformations have been simplified considerably without much loss of accuracy. And improvement in the sense that the present version of the model is more consistent with the underlying theoretical concepts.

The following symbols are introduced : t = time, representing the age of concrete, t_0 = age when drying begins, t' = age at loading, $\hat{t} = t - t_0$ = duration of drying, t'_e = effective age at loading, t_T = effective creep duration, (all the times must be given in days), $\epsilon_{sh\infty}$ = ultimate shrinkage strain, h = relative humidity of the environment ($0 \le h \le 1$), H = spatial average pore humidity, τ_{sh} = shrinkage half-time, $\bar{D}$ = effective cross-section thickness in millimeters, v/s = volume-to-surface ratio in millimeters, c = cement content in lb/ft^3 (lb/ft^3 = 16.03kg/m^3), w/c = water/cement ratio by weight, $w = (w/c)c$ = water content in lb/ft^3 f'_c = 28-day standard cylinder strength in psi, a/c = aggregate-cement ratio by weight.

2.1 Shrinkage

Mean shrinkage strain in the cross section:

$$\epsilon_{sh}(t, t_0) = \epsilon_{sh\infty}\, k_h\, S \tag{1}$$

Time curve:

$$S = \tanh\sqrt{\hat{t}/\tau_{sh}}\ , \qquad \hat{t} = t - t_0 \tag{2}$$

Humidity dependence:

$$k_h = \begin{cases} 1 - h^3 & \text{for} \quad h \le 0.98 \\ -0.2 & \text{for} \quad h = 1 \\ \text{linear interpolation} & \text{for} \quad 0.98 \le h \le 1 \end{cases} \tag{3}$$

Size dependence:

$$\tau_{sh} = \frac{0.45\, D^2}{C1}, \qquad D = \frac{2v}{s}, \qquad C1 = 10\left(\frac{t_0}{7}\right)^{0.08}\left(\frac{f'_c}{1000}\right)^{1/4} \tag{4}$$

The final shrinkage strain including the effect of aging and microcracking can be approximately predicted from the following empirical formulas

$$\epsilon_{s\infty} = \alpha_1\alpha_2\left[0.026w^{2.1}(f'_c)^{-0.28} + 0.27\right] \qquad (\text{in } 10^{-3}) \tag{5}$$

where

$$\alpha_1 = \begin{cases} 1.0 & \text{for type I cement;} \\ 0.85 & \text{for type II cement;} \\ 1.1 & \text{for type III cement.} \end{cases} \tag{6}$$

and

$$\alpha_2 = \begin{cases} 0.75 & \text{for steam} - \text{cured specimens;} \\ 1.0 & \text{for specimens cured in water or 100\% R.H.;} \\ 1.2 & \text{for specimens sealed during curing.} \end{cases} \tag{7}$$

Age and microcracking effect:

$$\epsilon_{sh\infty} = \epsilon_{s\infty}\frac{E(7 + 600)}{E(t_0 + \tau_{sh})} \tag{8}$$

where

$$E(t) = E(28)\left(\frac{t}{4 + 0.85t}\right)^{0.5} \tag{9}$$

Fig. 1 shows the comparison of the predictions of these formulas with some important data taken from the literature [17,18,19].

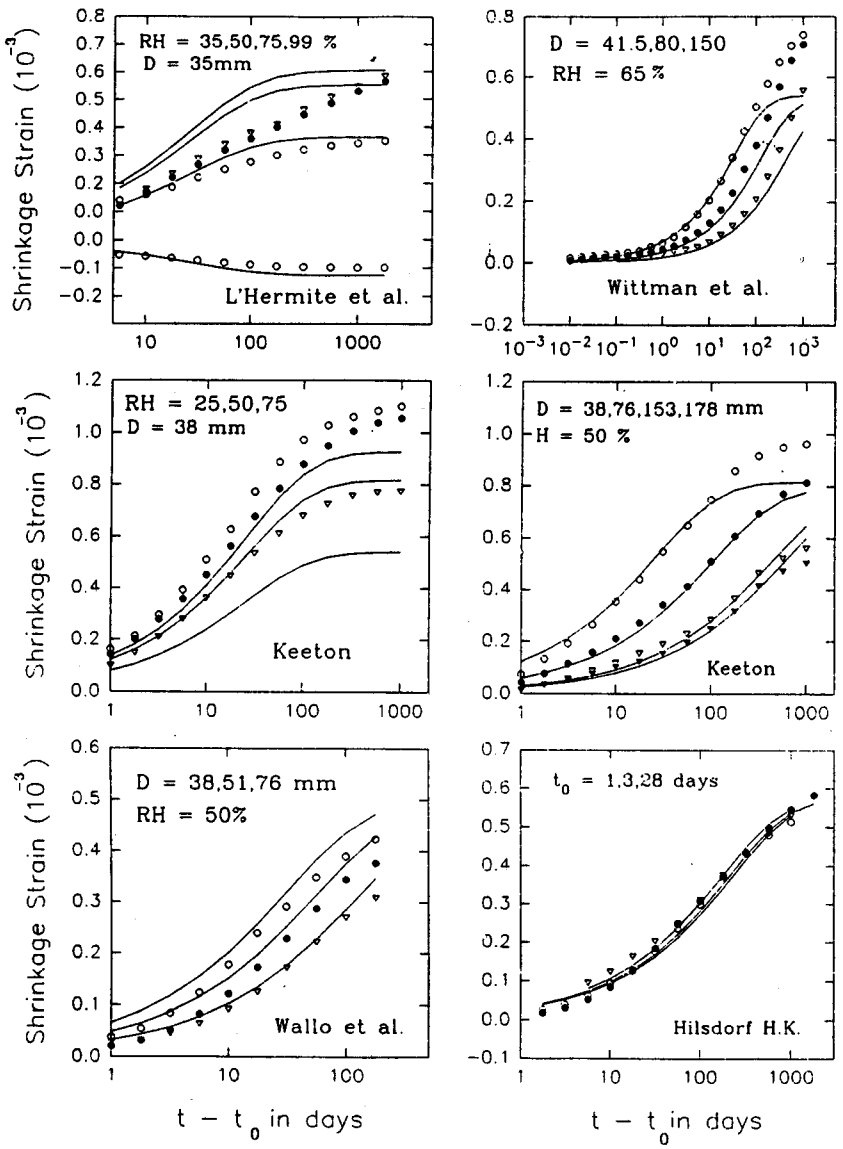

Figure 1: Predictions of the shrinkage formulas compared with some important test data from the literature

2.2 Basic Creep

The expression of the derivative of the compliance function $\dot{J}(t,t')$ results directly from the solidification theory[3]. It has the following simple form

$$\dot{J}(t,t') = \dot{\phi}(t,t') + \frac{q_4}{t}; \qquad \dot{\phi}(t,t') = \frac{n(q_2 t^{-m} + q_3)}{(t-t') + (t-t')^{1-n}} \tag{10}$$

where $m = 0.5$ and $n = 0.1$. For computer solutions this simple expression for $\dot{J}(t,t')$ is more convenient because the step-by-step algorithm for creep structural analysis can be based directly on the above expression for creep rate and the expansion into Dirichlet series and conversion of the stress-strain relation to a rate type form is best done directly on the basis of $\dot{J}(t,t')$. The terms containing q_2, q_3 and q_4 represent the rates of aging viscoelastic compliance, non-aging viscoelastic compliance and flow compliance respectively. To obtain the expression for the compliance function $J(t,t')$ the expression for $\dot{\phi}$ has to be integrated. Unfortunately this integration cannot be carried out in closed form because the function multiplying q_2 yields the binomial integral. Denoting the integral by $Q(t,t')$ the expression for the compliance function can be written as

$$J(t,t') = q_1 + C_0(t,t') \tag{11}$$

where

$$C_0(t,t') = q_2 Q(t,t') + q_3 \ln[1 + (t-t')^{0.1}] + q_4 \ln\left(\frac{t}{t'}\right) \tag{12}$$

The function $Q(t,t')$ can be obtained by integrating (10)(see ref [3]) For the evaluation of this integral either an approximate formula given in appendix II can be used. Alternatively Eq.(10) can be integrated numerically in time steps or a table given in [3] can be used.

The parameters q_1, q_2, q_3 and q_4 based on composition and strength of concrete are given by the following simplified empirical formulas. These formulas are much simpler than the corresponding formulas in [4]. This simplification has been achieved using sensitivity analysis[12].

$$q_1 = \frac{0.6 \times 10^6}{E_{28}}, \qquad E_{28} = 57000\sqrt{f'_c} \tag{13}$$

$$q_2 = 0.9c^{0.5}(0.001 f'_c)^{-0.9}, \qquad q_3 = q_2\left[0.29(w/c)^4\right], \qquad q_4 = 0.14(a/c)^{-0.7} \tag{14}$$

The compliance function is obtained in $10^{-6}/psi$ (1 psi = 6895 Pa). The ranges of composition and strength parameters for which these formulas are valid are wide enough to cover most normal concretes used in practice. These ranges are :

$$2500 \le f'_c \le 10,000 \quad f'_c \text{ in psi} \tag{15}$$

$$0.3 \le w/c \le 0.85 \tag{16}$$

$$10.0 \le c \le 45.0 \quad \text{cement content in lbs/ft}^3 \tag{17}$$

$$3.0 \le a/c \le 13.5 \tag{18}$$

Fig. 2 shows some important basic creep data in the literature compared with the predictions of these formulas.

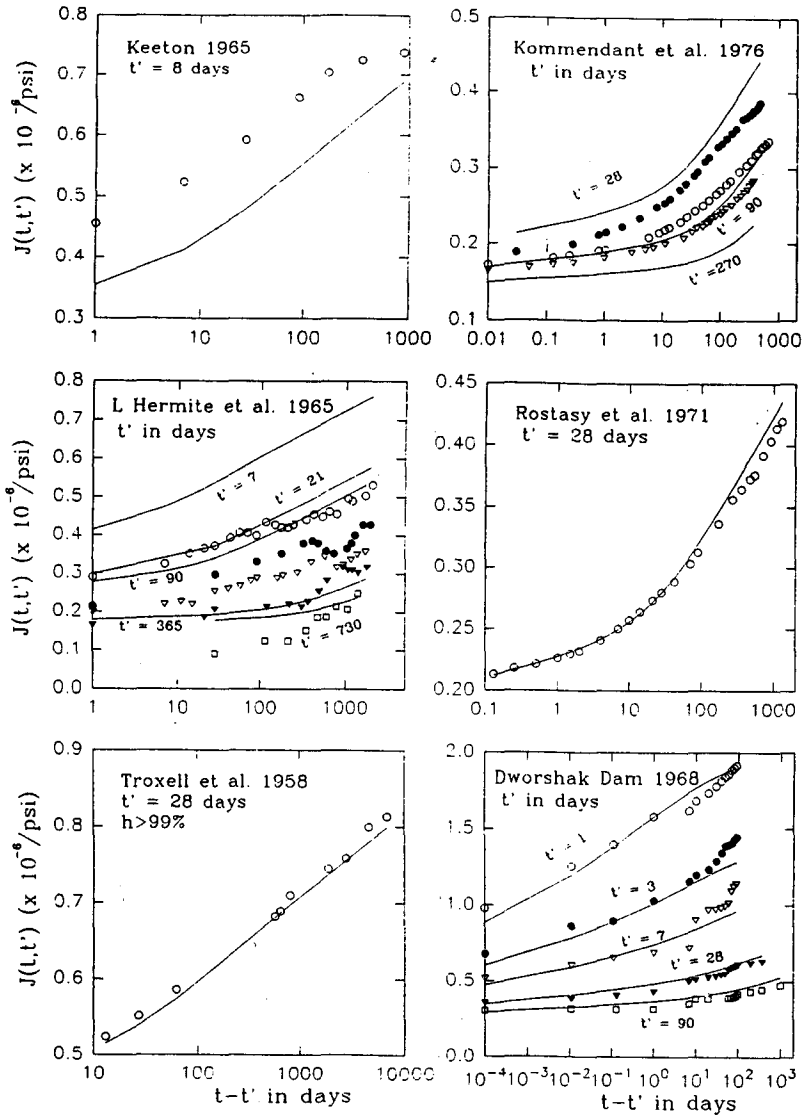

Figure 2: Predictions of the basic creep formulas compared with some important test data from the literature

2.3 Creep at Drying

The phenomenon of stress-induced-shrinkage[8] or the cross-effect between stress and drying shrinkage has been demonstrated to be the dominant effect in the additional creep due to drying by finite element studies of test data. Based on the hypothesis that the drying creep is adequately described by stress -induced-shrinkage alone, the additional creep due to drying is derived by averaging over the cross-section.

We may write the value of the compliance rate averaged over the cross-section as:

$$\dot{J}(t,t') = \dot{C}_0(t,t') + \dot{C}_d(t,t',t_0) \tag{19}$$

With the basic hypothesis similar to that used by Bažant and Chern[8]

$$\dot{C}_d(t,t',t_0) = \frac{\rho(H)k_{sh}\dot{H}}{C_d} \tag{20}$$

where H is spatial average of the pore humidity over the cross-section, ρ is a coefficient depending on H and k_{sh} is a constant. This can be rewritten as:

$$\frac{d}{dt}(C_d^2) = 2\rho(H)k_{sh}\dot{H} \tag{21}$$

Integrating from age at loading t' to the current time t

$$C_d^2 = 2k_{sh}\int_{t'}^{t}\rho(H)\dot{H}d\tau = 2k_{sh}\int_{H(t')}^{H(t)}\rho(H)dH \tag{22}$$

Since drying creep, similarly to shrinkage, is caused by water content changes governed by the diffusion theory [4] we may assume

$$H(t) = 1-(1-h)S(t) \tag{23}$$

where $S(t)$ is the same function defined earlier for shrinkage i.e.

$$S(t) = \tanh\sqrt{\frac{t-t_0}{\tau_{sh}}} \tag{24}$$

The function $\rho(H)$ may be assumed of the form

$$\rho(H) = e^{aH} \tag{25}$$

The above equation on integration yields

$$C_d^2 = 2k'_{sh}\left[e^{aH(t)} - e^{aH(t')}\right] \tag{26}$$

Fitting the above expression to the test data for drying creep we obtain

$$C_d(t,t',t_0) = q_5\left[\exp\{-8H(t)\} - \exp\{-8H(t')\}\right]^{1/2} \tag{27}$$

in which

$$q_5 = 12000(f'_c)^{-1}\epsilon_{sh\infty}^{-0.6} \tag{28}$$

The additional drying creep given by this formulas is in $10^{-6}/psi$ The predictions of these formulas are compared with some important data in the literature in Fig.3.

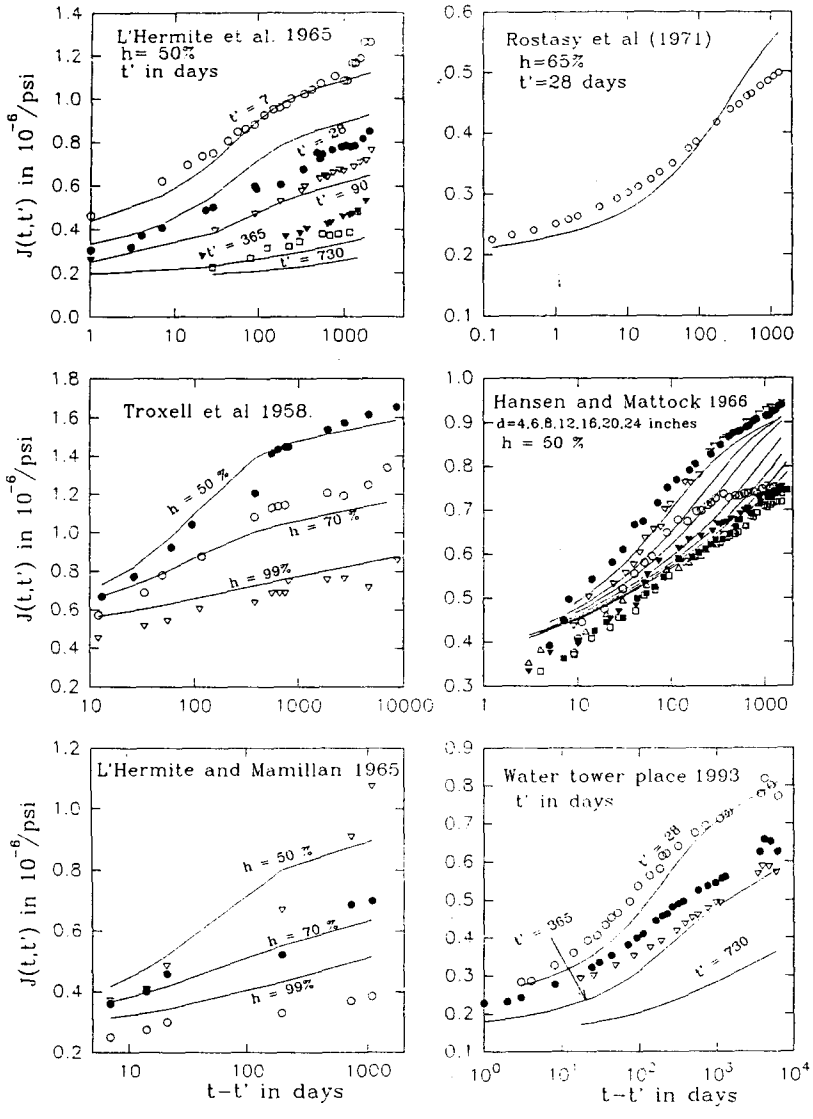

Figure 3: Predictions of the formulas for creep at drying compared with some important test data from the literature

2.4 Temperature Effect on Basic Creep

The effect of varying temperature on creep of concrete that is also drying is quite complex. The effect of constant elevated temperature on basic creep is however amenable to the development of simple code-type expressions by a simple generalization of the expressions of basic creep obtained by solidification theory. These generalized expressions are:

$$\dot{C}_0(t,t') = \frac{\partial J(t,t')}{\partial t} = R_T \left\{ \left[q_2 \left(\frac{\lambda_0}{t_T} \right) + q_3 \right] \frac{n\xi^{n-1}}{\lambda_0(1+\xi^n)} + \frac{q_4}{t_T} \right\} \tag{29}$$

where $\xi = t_T - t'_e$ and we calculate the equivalent age at loading and the equivalent creep duration to be used in the above equations as

$$t'_e = \int_0^{t'} \beta_T(t'')dt'', \qquad t_T - t'_e = \int_{t'}^{t} \beta'_T(t'')dt'' \tag{30}$$

Based on the activation energy theory

$$\beta_T = \exp\left[\frac{U_h}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \tag{31}$$

$$\beta'_T = \exp\left[\frac{U_c}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \tag{32}$$

$$R_T = \exp\left[\frac{U'_c}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \tag{33}$$

where T is temperature, T_0 is the reference temperature taken for the data fits to be 293 K (All temperatures are absolute temperatures), U_h is the activation energy of cement hydration, U_c is the activation energy of creep describing the acceleration of creep rate due to temperature increase and R is the gas constant. Noting that R_T is constant for a constant temperature Eq. (29) can be integrated to yield the following generalized expressions for the basic creep compliance function.

$$C_0(t,t') = R_T \left[q_2 Q(t_T, t'_e) + q_3 \ln[1 + (t_T - t'_e)^{0.1}] + q_4 \ln\left(\frac{t_T}{t'_e}\right) \right] \tag{34}$$

From the data fits we obtain $U_h/R = 5000$ K and

$$\frac{U_c}{R} = 85\,[(w/c)(c)]^{-0.27} \left(\frac{f'_c}{1000}\right)^{0.54}; \qquad \frac{U'_c}{R} = 0.18\frac{U_c}{R} \tag{35}$$

The above formulas give the compliance function in $10^{-6}/psi$. Fig. 4 shows the comparison of the predictions with the basic test data from literature. The data of Browne et al.(for original reference see [4]) is the most comprehensive data for temperature and age effect on basic creep. From the figure we can see that both the influences have been modeled correctly.

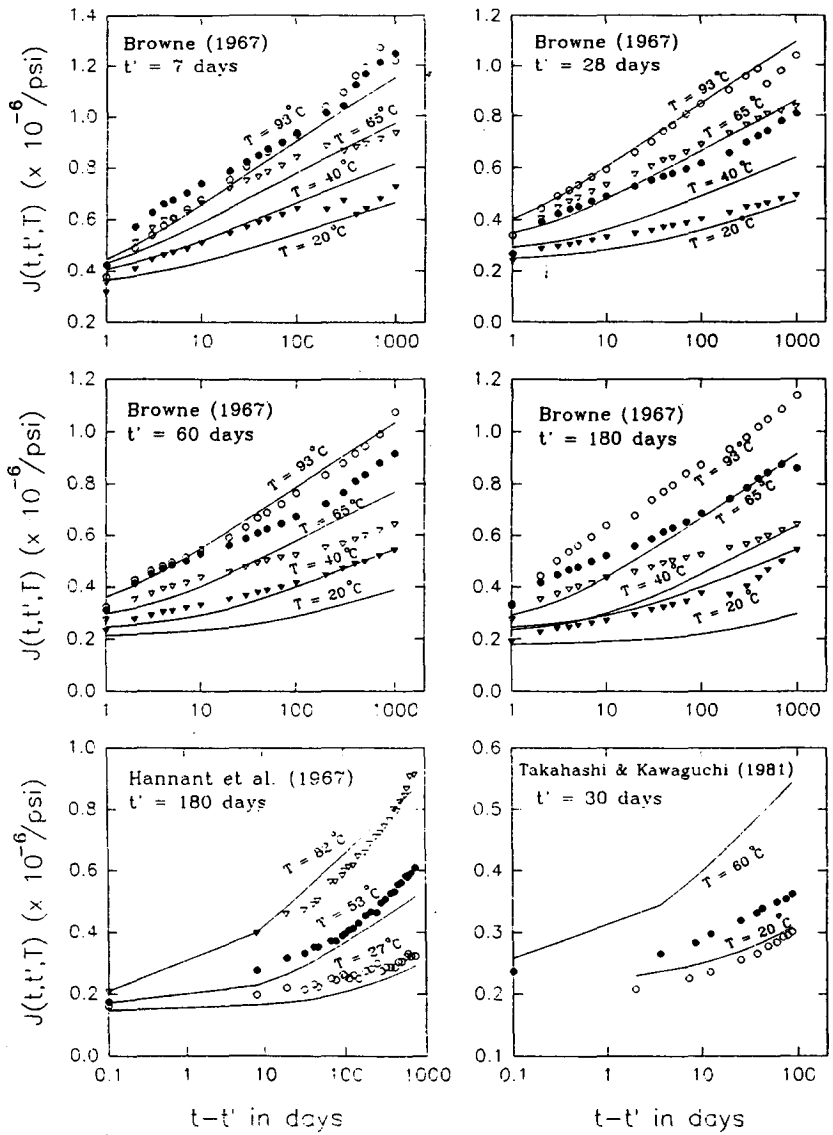

Figure 4: Predictions of the formulas for temperature effect on basic creep compared with some important test data from the literature

3 Statistics of errors

We present the statistics of errors in terms of the coefficient of variation, $\overline{\omega}$, which was defined in [2] for a data set j as

$$\overline{\omega}_j = \frac{s_j}{\overline{J}_j} \tag{36}$$

where

$$s_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\Delta_{ij}}; \qquad \overline{J}_j = \frac{1}{n}\sum_{i=1}^{n} J_{ij} \tag{37}$$

in which J_{ij} = measured value of creep compliance function or shrinkage strain, n is number of data points in a data set and Δ_{ij} is the deviation of the predicted value of J from its measured value. Also

$$\overline{\omega}_{all} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\overline{\omega}_j^2} \tag{38}$$

Tables 1-4 show the statistics of errors of the proposed formulas as compared to test data from different sources. For comparison the statistics of errors for the current ACI [9] and the new CEB [10] formulations are also shown. To avoid the subjective bias implied in the improper spacing of data points in the logarithmic scale, where some decades may have a large number of data points and some others a much smaller number, in [2] the experimental data was hand smoothed and then points equally spaced in the log scale (creep-duration axis) on the hand smoothed curves were chosen for calculating the statistics of errors thus removing any bias due to improper spacing of data points on the log scale. In this study a different approach to remove this bias is used. Data points in each decade in the logarithmic time scale are considered as one group each group of data points is given equal weight. Thus a particular data point in a particular group is weighed inversely proportional to the number of points in that group. Since most data sets have few data points of load duration less than one day and these points are also subject to large experimental scatter, points of load duration less than ten days are treated as part of one group. The results of this statistical analysis are shown in tables 1, 2, 3 and 4. In the statistics, no distinction is made between two data sets with different ages at loading. However in the data-base the tests with age at loading from seven to twenty eight days are most numerous and there are much fewer tests where specimens were loaded at early or late ages. Therefore, statistics will not be greatly affected even if a model does not give good predictions for specimens loaded at early or late ages. To take care of this, the data sets have been grouped according to ages at loading. The data sets with ages at loading less than 10 days forming one group, 10 to 100 days the second group, 100 to 1000 days the third group and greater than 1000 the fourth group. Another grouping of the data has been done according to creep duration. The coefficient of variation of deviations of the test data from predicted values is then calculated for each of these groups. The results are shown in Table 5. This kind of calculation removes both the subjective biases discussed earlier and illustrates clearly the accuracy

of the model in different ranges of creep duration and ages at loading. Also, for this calculation, no weighting of data points is necessary because of the particular grouping of data points. From the statistics in Table 5. it is clear that the present formulation gives better predictions than both prediction models used in current codes. The present formulation is also simple enough to be adopted for a new design code.

Table 1. Coefficients of variation of errors (expressed as percentage) of the predictions of various models (Shrinkage)

	PROPOSED	ACI	CEB
Test data	$\overline{\omega}$	$\overline{\omega}$	$\overline{\omega}$
Hummel et al.	27.0	30.0	58.7
Rüsch et al.(1)	31.1	35.2	44.8
Wesche et al.	38.4	24.0	36.1
Rüsch et al.(2)	34.7	13.7	27.8
Wischers and Dahms	20.5	27.3	35.9
Hansen and Mattock	16.5	52.9	81.5
Keeton	28.9	120.6	48.3
Troxell et al.	34.1	36.8	47.4
Aschl and Stökl	57.2	61.3	44.2
Stökl	33.0	19.5	29.6
L'Hermite et al.	66.7	123.1	69.4
York et al.	30.6	42.8	8.9
Hilsdorf	11.7	24.7	29.6
L'Hermite and Mamillan	46.1	58.7	45.5
Wallo et al.	22.0	33.0	55.6
Lambotte and Mommens	39.1	30.7	31.3
Weigler and Karl	31.3	29.6	21.3
Wittman et al.	23.7	65.4	40.0
Ngab et al.	20.4	45.3	64.6
McDonald	5.1	68.8	21.4
Russel and Burg(Water tower place)	38.5	51.0	58.1
$\overline{\omega}_{all}$	34.3	55.3	46.3

Table 2. Coefficient of variation of errors (expressed as percentage) of the predictions of various models (Basic Creep)

	PROPOSED	ACI	CEB
Test data	$\overline{\omega}$	$\overline{\omega}$	$\overline{\omega}$
Keeton	19.0	37.5	42.8
Kommendant et al	15.3	31.8	8.1
L'Hermite et al.	49.4	133.4	66.2
Rostasy et al.	15.2	47.6	5.0
Troxell et al.	4.6	13.9	6.2
York et al.	5.6	37.7	12.8
McDonald	6.9	48.4	22.2
Maity and Meyers	33.8	30.0	15.7
Mossiossian and Gamble	18.6	51.5	47.3
Hansen and Harboe et al. (Ross dam)	14.1	51.2	31.1
Browne et al. (Wylfa vessel)	44.7	47.3	53.3
Hansen and Harboe et al. (Shasta dam)	22.7	107.8	43.1
Brooks and Wainwright	12.6	14.9	15.4
Pirtz (Dworshak dam)	12.5	58.2	32.5
Hansen and Harboe et al. (Canyon ferry dam)	33.3	70.2	56.9
Russel and Burg (Water tower place)	15.7	19.3	31.5
Hanson J.A.	14.1	63.3	12.1
$\overline{\omega}_{all}$	23.6	58.1	35.0

Table 3. Coefficient of variation of errors (expressed as percentage) of the predictions of various models (Creep at Drying)

	PROPOSED	ACI	CEB
Test data	$\overline{\omega}$	$\overline{\omega}$	$\overline{\omega}$
Hansen and Mattock	5.8	32.1	11.9
Keeton	31.4	46.3	37.9
Troxell et al.	5.9	33.0	7.9
L'Hermite et al	14.0	55.8	25.5
Rostasy et al.	6.5	20.9	14.8
York et al.	5.8	42.1	45.1
McDonald	10.9	40.4	38.9
Hummel	15.3	46.2	24.6
L'Hermite and Mamillan	20.6	62.5	15.2
Mossiossian and Gamble	11.3	71.7	30.8
Maity and Meyers	62.8	45.9	83.7
Russel and Burg (Water tower place)	10.7	41.2	19.1
$\overline{\omega}_{all}$	23.1	46.8	35.5

Table 4. Coefficient of variation of errors (expressed as percentage) of the predictions of various models (Effect of constant elevated temperature on basic creep)

Test data	$\overline{\omega}$
Johansen and Best	20.3
Arthanari and Yu	35.5
Browne et al.	27.2
Hannant D.J.	14.0
York, Kennedy and Perry	41.1
Kommendant et al.	18.8
Okajima et al.	6.9
Ohnuma and Abe	23.7
Takahashi and Kawaguchi(i)	38.8
Takahashi and Kawaguchi(ii)	43.9
$\overline{\omega}_{all}$	28.1

4 Prediction improvement by use of short-time data

Testing always represents an essential part of concrete technology and concrete quality control because of the significant variability in the properties of the components of concrete from region to region. The predictions of the proposed formulas are amenable to significant improvement if the composition parameters are identified using the results of short-time testing. Also, the same procedure can be applied to identify the composition parameters for certain specialized and high strength concretes if some short-term test results are available. It may be noted here that several different types of admixtures, superplasticizers and pozzolanic materials have been found to have widely varying effects on shrinkage and creep of concrete [14,16]. There can thus be considerably different effects of different combinations of these additives on shrinkage and creep. Therefore, for creep sensitive structures using specialized concretes short-term testing to determine material parameters which can then be used in the proposed expressions is useful. On the other hand general empirical formulas for composition parameters for concrete with additives are difficult to formulate and are expected to be not very reliable simply because of the large number of additives in use currently, different combinations of which can give rise to significantly different creep and shrinkage effects. Although short-term testing is recommended for determining the material parameters for specialized concretes the present model can be easily used in conjunction with the method in [14] for the prediction of long-term deformations of specialized concretes in the absence of short-term data. Identification of material parameters for improvement of prediction of creep using short time test data is easy in the present formulation. For basic creep the material parameters q_1, q_2, q_3 and q_4 can be easily determined from linear regression. There is however some caution to

Table 5. Statistics of errors of various models compared for different ranges of age at loading and creep duration. All relevant data in the ACI-CEB data base used. Age at loading and creep duration are given in days.

Proposed Model

ω	t' ≤ 10	10<t'≤100	100<t'≤1000	t'> 1000
t-t'≤ 10	17.8	24.0	19.8	
10<t-t'≤100	13.7	23.1	25.3	29.3
100<t-t'≤1000	13.9	20.5	22.6	33.6
t-t'> 1000	12.7	14.6	17.8	

ACI Model

ω	t' ≤ 10	10<t'≤100	100<t'≤1000	t'> 1000
t-t'≤ 10	60.3	30.7	33.3	
10<t-t'≤100	45.7	36.7	49.9	97.1
100<t-t'≤1000	34.6	39.9	51.7	93.9
t-t'> 1000	36.8	39.9	40.9	

CEB Model

ω	t' ≤ 10	10<t'≤100	100<t'≤1000	t'> 1000
t-t'≤ 10	40.5	23.1	11.2	
10<t-t'≤100	25.8	23.5	21.2	40.8
100<t-t'≤1000	17.5	22.8	25.0	41.3
t-t'> 1000	11.6	20.5	24.7	

be exercised. At times it may happen that the linear regression may give negative values of one or more of the parameters q_1, q_2, q_3 and q_4. This would however be unacceptable because all four contributions to basic creep have to be additive. The problem may be remedied in some cases by hand smoothing the data before performing the regression. If this does not work one of the parameters q_1, q_2, q_3 and q_4 may be set to zero and regression performed for the remaining parameters. If the data is for a concrete loaded at very early age, q_3 may be set to zero and if the data is for a concrete loaded at late ages, q_4 may be set to zero if the regression with all four parameters gives negative values of some of the parameters. For the case of creep at drying there is another additional material parameter namely q_5 to be determined from test data by linear regression.

Prediction improvement by the use of short time data for shrinkage is a more involved problem as it involves nonlinear regression. Also the estimation of the final shrinkage $\epsilon_{sh\infty}$ may not be possible using short time shrinkage data just by minimizing the squares of deviation of the values calculated by the formula from the short term test results unless the duration of the test is long enough so that the slope of the shrinkage versus drying time curve in logarithmic time scale begins to decrease. This problem is illustrated in Fig. 5 taken from [7] from which it is clear that very different final values of shrinkage used with the proposed shrinkage time curve or the ACI 209 time curve based on the Ross' hyperbola can give equally good fits of short-time shrinkage data. In view of this problem of the ill-posedness of the shrinkage extrapolation problem in mind, reliable estimates of final shrinkage from tests on standard shrinkage specimens would call for testing durations greater than one year. Some kind of accelerated testing would be needed to get a reliable estimate of final value of shrinkage from short-time tests. This can be achieved by testing on thinner specimens at slightly higher temperatures to reduce the duration of test. Another possible approach would be to utilize weight loss measurements to fix one of the two parameters to be determined from the test data. This problem of a reliable short-term shrinkage test the results of which could be extrapolated to yield improved long term prediction is currently being studied at Northwestern university.

5 Conclusions

In conclusion, it should be emphasized that improvement in the mathematical modeling of such a complex phenomenon as creep and drying effects in concrete can be achieved only through better understanding of the physical mechanisms involved and derivation of the model from the law describing such a mechanism. Various progress of this type has been made before. This paper presents a further improvement in which the additional creep due to drying is calculated by simplified integration of the local constitutive description of stress-induced shrinkage. The improved version follows the guidelines previously formulated by RILEM Committee 107 to a greater extent than previous versions (BP and BP-KX models).

Further improvement is achieved in the formulas for the effect of composition on the parameters in the model. This effect has so far defied mathematical argument, however, trends observed empirically and perceived intuitively on the basis

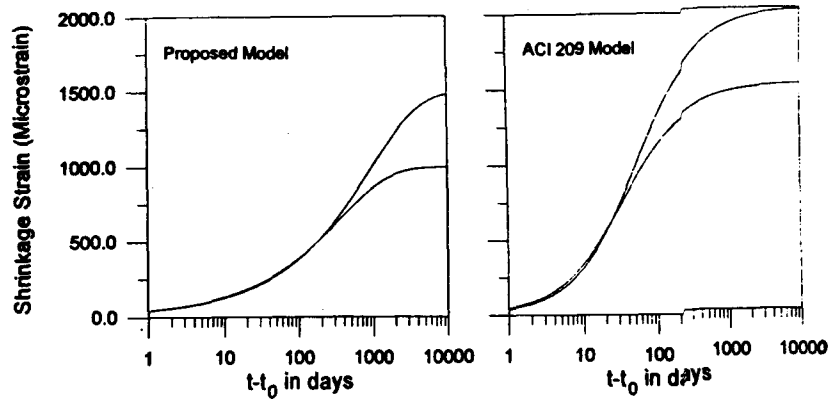

Figure 5: Example of shrinkage-time curves giving nearly the same development of shrinkage for short times but very different final values (left: $\epsilon_{sh} = 1500\tanh(\hat{t}/1500)^{1/2}$ and $1000\tanh(\hat{t}/600)^{1/2}$; right: $\epsilon_{sh} = 2000\hat{t}/(55+\hat{t})$ and $1500\hat{t}/(35+\hat{t})$

of many experiments have been used successfully, together with empirical data fitting and optimization. The most improvement has been achieved by means of sensitivity analysis, which showed which empirical parameters are important and which are not. This made possible considerable simplification of the formulas for the effect of composition.

Statistical evaluation of the proposed formulas on the basis of all relevant test data available in the literature is very important. In this regard, it is necessary to suppress the bias involved in the selection of specimen ages, test durations and reading times made by the experimenter. A statistical procedure to do that is proposed. It offsets the bias caused by the crowding of data points in some regions of the space of load duration and age at loading, and sparsity of data in other regions.

Improvement of long-time predictions can be achieved on the basis of extrapolation of short-time measurements. This can be used quite effectively for creep, but for shrinkage and drying creep the extrapolation problem is much harder. The reasons for this are discussed and some means to overcome the difficulties are pointed out.

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Appendix I: RILEM TC-107 Guidelines for Formulation of Functions Characterizing Creep and Shrinkage

1. The creep of concrete in the service stress range can be characterized in terms of the compliance function. Its use is made possible by the fact that the creep of concrete (in contrast to creep of metals, ice or clay) can be approximately considered as *linear* with regard to stress, following the *principle of superposition.* The principle of superposition agrees with test results very well if there is no drying. At drying, and especially if cracking takes place, there are appreciable departures from the principle of superposition, but they have to be neglected in simple design code formulations because of their complexity.

2. The design codes should specify the *compliance function* $J(t, t')$ rather than the creep coefficient ϕ (ϕ = ratio of creep to the initial 'elastic' strains, t = current age of concrete, t' = age at loading). For structural creep analysis, of course, it is often more convenient to use the creep coefficient, but its value can always be easily calculated from the compliance function specified in the code ($\phi = EJ - 1$, E = conventional 'elastic' modulus, characterizing the truly instantaneous deformation plus short-time creep). One reason for preferring J is that the E–values specified in the codes are not defined on the basis of the initial strains measured in typical creep tests. A more profound reason is that concrete creep in the range of short load durations from 0.1 s to .1 day is already quite significant, which means that $1/E$ is inevitably an arbitrarily chosen point t_0 on the smoothly rising creep curve for unit stress. Depending on t_0, the corresponding E–values vary widely (and different creep data correspond to very different choices of t_0). But what matters for the results of structural creep analysis is the values of J, not ϕ and E. If the creep coefficient ϕ is given to the structural analyst, there is always the danger that he might combine it with some non-corresponding value of E, which then implies an incorrect J. When J is specified, this kind of mistake is prevented.

3. Considering concrete as a homogeneous material, creep and shrinkage should be considered, strictly speaking, as phenomena associated with a point of that continuum. The evolution of such *intrinsic creep* and *intrinsic shrinkage* is affected by factors such as the specific moisture content in the pores or temperature, which can vary from point to point in a cross section of the structure. Therefore, the compliance function and shrinkage in general will be non uniform throughout the cross section and the intrinsic creep will not be linearly distributed. If, however, the cross section remains plane (which usually is true for long prismatic members such as beams or columns), some internal stresses will be generated to make the strain at each point of the cross section conform to a linear strain distribution.

   For most practical purposes, however, the *mean (or average)* creep and shrinkage can be defined for the cross section as a whole. They have the meaning of the average cross-section compliances or average shrinkage strains, regardless of the associated internal stress and the inherent cracking at each particular point. Although, strictly speaking, such average creep or shrinkage depend on the type of loading (i.e., the ratio of the bending moment to the normal force), approximate average properties can be established for all types of loading. This is in general the meaning of the formulas characterizing creep and shrinkage in contemporary design codes. This type of formulas are useful, but it must be emphasized that they cannot be considered as material properties unless members in a sealed condition and at constant uniform temperature are considered. Rather, they are the properties of the cross section. Consequently, the formulas depend on the properties of the cross section, such as its size and shape, and are influenced by the non homogeneity of creep and shrinkage within the cross section. This inevitably causes that good prediction formulas are much more complicated than the constitutive law for a material point.

4. *Drying* plays a fundamental role in creep and shrinkage. It is the direct cause of shrinkage—if no drying occurs, no shrinkage occurs (except for autogeneous shrinkage, which is of chemical origin and is usually negligible). Drying also affects creep, increasing the creep strain significantly with respect to a similar situation without drying. In a cross section, the drying process is governed by the nonlinear theory of moisture diffusion through the pores in concrete. Its effects increase as the environmental relative humidity decreases. Of course, no drying occurs when the specimen is sealed. In the case of immersion in water (relative humidity 100%) there is swelling or negative shrinkage, which is usually rather small (the inhibition of water also causes some small non uniformity in pore humidities throughout the cross section, residual stress and possibly microcracking).

5. *Basic creep* is defined as the creep at no drying and at constant temperature. Under such conditions, the behavior at all points in the cross section is the same, and therefore the basic creep can be considered as an intrinsic material property. *Drying creep* is defined as the increase of the creep strain over the basic creep, when drying takes place. This component of creep vanishes for sealed conditions, which is approximately at 100% environmental relative humidity (the small humidity drop called self-desiccation, caused by hydration, is neglected). Therefore, under sealed conditions, the creep for any cross section should be the same and equal to the basic creep regardless of cross-section size and shape.

6. *Diffusion theory,* linear as well as nonlinear, indicates that the drying times $\hat{t}$ required to achieve a similar degree of overall drying in a cross-section (i.e., the same relative water loss) increase with the square of its size. The shrinkage formulas must give shrinkage as a function of the relative time $\theta =$

$\hat{t}/\tau_{sh}$ rather than the actual drying time $\hat{t}$ where τ_{sh} represents a coefficient with the meaning of the time necessary to achieve a certain percentage of the final shrinkage, e.g., the *shrinkage half-time* or the time necessary to achieve half of the final shrinkage; τ_{sh} should be proportional to the square of the cross-section size. Furthermore, τ_{sh} should depend on the shape of the cross section as indicated by the solution of diffusion theory. The incorporation of τ_{sh} implies that, for a fixed duration of drying, the mean shrinkage in the cross section should decrease with increasing size, with practically no shrinkage at all beyond a certain size.

7. *Diffusion theory* further indicates that, asymptotically for short drying times $\hat{t}$, the shrinkage strain ϵ_{sh} should grow in proportion to $\sqrt{\hat{t}}$. This property agrees well with tests[1] Also, according to the diffusion theory, the final shrinkage value ϵ_{sh_∞} should be asymptotically approached as an exponential of a power function of time, that is, the difference of the shrinkage strain from the final value should asymptotically decrease as $\exp(-ct^n)$ where c and n are constants and t = time.

8. Similarly, the mean *drying creep* of a cross section, for a fixed load duration, should also decrease with increasing size and almost vanish at a certain size beyond which only basic creep remains. It is reasonable that the decrease of shrinkage and drying creep with the cross section size be governed by the same law, since both are caused by the same physical process, that is, the drying in the cross section. Therefore, like shrinkage, the drying creep term should have the following properties:

   (a) It should be specified as a function of θ rather than $\hat{t}$.

   (b) Asymptotically for short times, it should be proportional to $\sqrt{\hat{t}}$.

   (c) Also, it should asymptotically approach the final value as $\exp(-kt^n)$ where k, n = constant.

9. Because the *basic creep* and the *drying creep* have different properties, depend on different variables and originate from different physical mechanisms, they should be given by separate terms in the creep model. Therefore, the formula for the creep curves must contain a separate term for the drying creep, which is additive to the basic creep term. Although other more complicated possibilities might be conceivable, a summation of the basic and drying terms in the creep formula appears to be acceptable. Unlike the basic creep term, the drying creep term should approach a final asymptotic value.

[1]This is true, however, only for properly conducted tests, in which the first length reading should be taken right after the stripping of the mold, within no more than approximately 15 minutes. Unfortunately, in many measurements the first reading has been taken too late, even as late as several hours after the stripping. In that case, a significant initial shrinkage deformation (possibly 3% to 10% of the final shrinkage) has been missed. Such distorted data may then give the false impression that the initial shrinkage curve does not follow the $\sqrt{\hat{t}}$–law.

10. The model of *basic creep* should be able to fit the existing experimental data throughout the entire ranges of the ages at loading and the load durations. It should exhibit the following characteristic features:

    (a) According to test results, the creep curves do not possess any final asymptotic value.

    (b) A power function of the load duration, with the exponent around 1/8, fits very well the data available for short durations, while a linear function of the logarithm of the load duration, with a slope independent of the age at loading, fits very well the data available for long durations. The creep model should satisfy these properties asymptotically.

    (c) The transition from the short-time and long-time asymptotic creep curves, which is quite gradual, should be centered at a certain creep value rather than at a certain creep duration, which means that for an older concrete the transition occurs later.

11. The *aging* property of creep, that is the decrease of creep strain at a fixed load duration with the age at loading, is well described by a power function of age with a small negative exponent, approximately -1/3. This function satisfies the experimental observation that a significant aging effect continues through a very broad range of ages at loading, certainly from the moment of set to over 10 years. This property contrasts with the age effect on strength or elastic modulus, which becomes unimportant after several months of age. For this reason, the functions suitable for describing those properties (e.g. $t/(t + \text{const.})$ do not work well for the age effect on creep.

12. Although not strictly necessary on the theoretical basis (i.e., thermodynamic restrictions), it is appropriate and convenient that the creep model exhibit *no divergence* of the creep curves for different ages at loading. There is no clear experimental evidence of such a phenomenon, and models showing such divergence can lead to various unpleasant difficulties when used for complex load histories in combination with the principle of superposition (for instance, giving a reversal of the creep recovery that follows load removal).

13. The creep formulation should also have a form suited for numerical computation. In the interest of efficient numerical solution of large structural systems, it is necessary to expand the creep formula (i.e. the compliance function) into a Dirichlet series, which makes it possible to characterize creep in a *rate-type* form corresponding to a Kelvin or Maxwell chain model. Every compliance function can be expanded into Dirichlet series (or converted to a Maxwell chain model), but for some there are more difficulties than for others because the expansion process:

    - requires a computer solution rather than a simple evaluation from a formula,
    - leads to an ill-posed problem with a nonunique solution, and

- yields negative values of the chain moduli within short periods of time.

Creep formulations that avoid these unpleasant characteristics should be preferred for the codes.

The source of the aforementioned difficulties is the age-dependence of creep properties, which in general requires considering the chain moduli to be age-dependent. These difficulties disappear with a creep formulation in which the chain moduli are age-independent, same as in classical viscoelasticity. Such a desirable property of the creep model can be achieved if the age dependence is introduced separately from the chain model, by means of some transformations of time. More than one such time transformation is known to be necessary, and it should preferably be based on some reasonable physical model for the *solidification process* that causes the age dependence of creep.

14. An important requirement for any creep and shrinkage model is *continuity* in the general sense. Small variations of the dimensions, environmental conditions, loading times, etc., should lead to small variations in the creep and shrinkage predictions, without no finite jumps.

15. Because the available creep data do not, and cannot, cover all possible practical situations, it is important that even a simple prediction model be based to the greatest extent possible on a sound theory. In that case one has the best chance the creep model would perform correctly in such experimentally unexplored situations. This means the model should be based on, and agree with, what is known about the basic *physical processes* involved—particularly diffusion phenomena, solidification process, theory of activation energy, etc.

16. Even though the creep and shrinkage model for a design code must be sufficiently simple, it should be compared to *all the test data* that are relevant and exist in the literature. The model parameters should be calibrated on the basis of these data by optimum fitting according to the method of least squares. This task is made feasible by the computer and is greatly facilitated by the existence of a comprehensive *computerized data bank* for concrete creep and shrinkage (such a bank was compiled in 1978 by Bažant and Panula, was extended by Müller and Panula as part of a collaboration between ACI and CEB creep committees, and is now being further updated, extended and refined by a subcommittee of RILEM Committee TC107 chaired by H. Müller). No model, even the simplest one, should be incorporated in a code without evaluating and calibrating it by means of such a comprehensive data bank.

17. *Selective* use of test data sets from the literature does not prove validity of any model (unless a sufficiently large number of data sets were selected at random, by casting dice). Many examples of the deception by such a practice have been documented. For instance, as shown by Bažant and Panula (1978), by selecting 25 among the existing 36 creep data sets from the literature, the coefficient of variation of errors of the BP creep model dropped from 18.5% to 9.7%, and by selecting only 8 data sets, it further dropped to 4.2%; likewise, by selecting 8 among the existing 12 shrinkage data sets, the coefficient of variation dropped from 31.5% to 7.9% (no reason for suspecting the ignored data sets from being faulty could be seen from the viewpoint of experimental method).

18. Design codes should specify, as a matter of principle, the *coefficient of variation* ω of prediction errors of the model compared to all the data that exist in the aforementioned data bank. The values of ω should also be given separately for basic creep, creep at drying, and shrinkage. The value of ω automatically results from the computer comparison of the model with the data bank.

19. Design codes should require that creep-sensitive structures be analyzed and designed for a specified *confidence limit*, such as 95%. This means designing the structure in such a way that the probability of not exceeding a certain specified value of response (for example, maximum 50-year deflection, maximum bending moment or maximum stress) would be 95%. The response values that are not exceeded with a 95% probability can be easily calculated with a computer by the sampling method, in which the statistical information is obtained by repeatedly running a deterministic creep analysis program for a small number of randomly generated sets (samples) of the uncertain parameters in the creep prediction model.

20. The notoriously high uncertainty of long-time creep and shrinkage predictions can be significantly reduced by recalibrating the most important coefficients of the model according to short-time tests of creep and shrinkage of the given concrete, and then extrapolating to long times. This practice should be recommended in the code.

21. The unknown material parameters in the model should be as few as possible and should preferably be involved in such a manner that the identification of material parameters from the given test data (i.e., data fitting) be amenable to *linear* regression. This not only greatly simplifies the identification of material parameters and extrapolation of short-time tests, but it also makes it possible to obtain easily the coefficients of variation characterizing the uncertainty of the material parameters and the predictions. However, this desirable property should not be imposed at the expense of accuracy of the model. It seems that for creep this property can be achieved without compromising accuracy, but for shrinkage this does not seem to be the case.

One important influencing factor, namely temperature, has been ignored in the foregoing guidelines. The same has been true of all the existing design codes or recommendations. Although the effect of constant and uniform temperature is known quite well (it can be described by two activation energies, one controlling the rate of creep and the other the rate of hydration), there is a problem in

regard to the statistical variability of environmental temperature, and its daily and seasonal fluctuations. In the future codes and recommendations, however, the effect of temperature ought to be taken into account.

Appendix II

Approximate formula for calculating the function $Q(t,t')$ used in the expression for the basic creep compliance function.

$$Q(t,t') = Q_f(t')\left[1 + \left(\frac{Q_f(t')}{Z(t,t')}\right)^{r(t')}\right]^{-1/r(t')} \tag{39}$$

with

$$Z(t,t') = (t')^{-m}\ln[1 + (t-t')^n] \tag{40}$$

$$Q_f(t') = [0.086(t')^{2/9} + 1.21(t')^{4/9}]^{-1} \tag{41}$$

$$r(t') = 1.7(t')^{0.12} + 8 \tag{42}$$

See Bažant and Kim [5]. For a table of values of $Q(t,t')$ for different values of age at loading, t', and creep duration, $t - t'$, calculated using numerical integration see Bažant and Prasannan [3].