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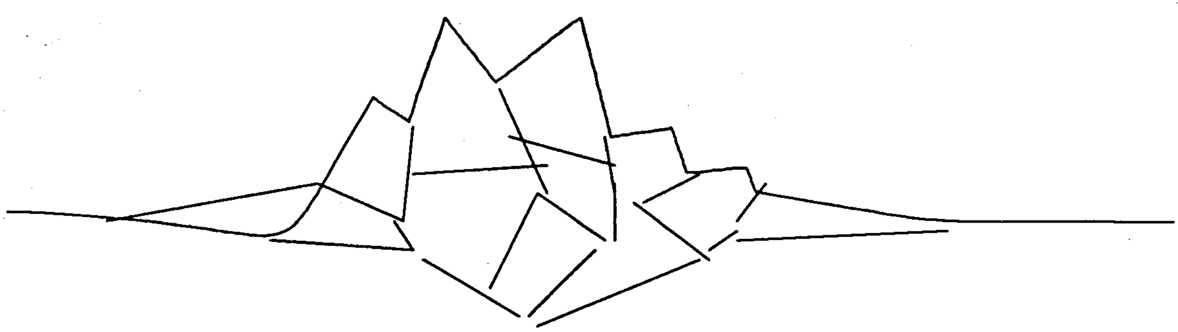

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Front cover: A first-year ridge in the Beaufort Sea near the SIMI East Camp (photo by M. Coon)
Rear cover: Site of ice fracture tests near the SIMI East Camp (photo by S. Echert)

Effect of Size on Distributed Damage and Fracture of Sea Ice

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Scope of Model, Objectives and Approach

(1) Develop a more accurate and realistic method for predicting load capacity and failure of floating sea ice plates in the Arctic and propagation of large scale fractures in these plates. (2) Establish the scaling law, determine the size effect (missing from previous models), and formulate a general fracture and damage model applicable to floating sea ice plates. The approach is theoretical and computational. The theory consists in the development of a mathematical model for fracture and damage of sea ice plates and study of its implications. Available field observations and laboratory evidence are used to calibrate the model. The computational approach uses finite element and discrete element methods.

Significant Findings to Date

Theory of Fracture Scaling -- Predictions of sea ice failure are needed for the assessment of: (1) vertical penetration of a submarine, (2) development and opening of large open water leads, (3) initiation and build up of pressure ridges, (4) ice breaking, and (5) navigation in the Arctic (Fig. 1). They have traditionally been made on the basis of strength theory, which assumes the ice to fail when the stress reaches a certain strength limit. This approach is in principle correct only for plastic materials which, after the strength limit is reached, yield at constant stress. When the material fractures, the stress that it transmits drops, and then the correct failure theory is fracture mechanics, which is obviously the case for ice. This theory has not been used in practice so far because evidence deemed to be contrary was previously found in small-scale laboratory tests of ice. Recently, however, it became clear this evidence merely proves inapplicability of linear elastic fracture mechanics to very small fractures. This is explained by the fact that sea ice, as well as other brittle heterogeneous materials develop, due to their heterogeneity, a relatively large damage zone around the crack tip. For large fractures, the size of this zone becomes small in comparison to the structure dimensions. By this argument it has been concluded that for large-scale fracture of ice, fracture mechanics is the only correct theory.

The basic difference between the strength theory of failure and fracture mechanics is in scaling. This is crucially important for extrapolating laboratory tests to real scale -- the basic problem for sea ice. The scaling is manifested by size effect. The size effect is understood as the dependence of the nominal strength of structure on the characteristic dimension D, when geometrically similar situations are compared. The nominal strength is defined as the failure load P divided by the square of D. According to the classical strength theory or plastic limit analysis, there is no size effect, that is, the nominal strength for geometrically similar situations is the same. In linear elastic fracture mechanics, the nominal strength decreases inversely to the square root of D. In the plot of logarithm of nominal strength versus $\log D$, the size effect is represented by a straight line of slope $-1/2$, as shown in Fig. 2c. The strength theory is in that plot represented by a horizontal line. The reason for this size effect is that, according to fracture mechanics, the strain energy released by the formation of fracture must be equal to the energy needed to create the fracture surfaces. The former increases as the square of fracture length a or structure size D, while the latter increases only in proportion to D.. Since both must be equal for any characteristic size D, the nominal strength must decrease with D. This is explained in Fig. 2a, b where the shaded area, which approximately represents the zone from which strain energy is released due to the fracture, is proportional to a^2 or to D^2.

For brittle heterogeneous materials such as sea ice (also called quasibrittle materials) the scaling and size effect are more complicated. In these materials, the fracture process zone at the fracture front is large, having dimension l that is not small compared to laboratory specimen size. In this zone the material undergoes distributed cracking which is approximately governed by strength theory. Therefore, the size effect represents a gradual transition from the strength theory, for which there is no size effect, to the so-called linear elastic fracture mechanics (LEFM), for which the size effect is as strong as possible (see Fig. 2c). As shown by Bažant, this transition can be approximately described by the simple law given in the figure (where D_0 and σ_0 are constants if geometrically similar situations are considered).

The aforementioned size effect, however, describes only fracture of sea ice plates due solely to in-plane horizontal forces, which are relatively rare. In most practical problems (see Fig. 1), the floating ice plate fractures by bending. For bending fracture (Fig. 2d), there is also size effect but, as found in this project, it is different and slightly weaker. It has been found that, in geometrically similar situations, the nominal bending strength is inversely proportional to the *3/8* power of sea ice thickness h. At first it may seem surprising that it is not proportional to the inverse square root of h. This may be understood by noting that formation of a bending fracture is equivalent to applying on the floating plate a bending moment opposite to that carried before (Fig. 2d). This produces deflections in the form of an exponentially decaying sinusoidal wave, shown (with exaggerated deflections) in Fig. 2d. The important point is that the distance between two inflection points, L, is not proportional to the plate thickness h but to $h^{3/4}$. The bending fracture releases the strain energy up to a certain fixed distance from the fracture, proportional to L (shaded area in Fig. 2f, g). Thus the energy release per unit length of the bending fracture (Fig. 2f) is proportional to $(\sigma_N h)\, h^{3/4}$, which must be equal to the energy required to create the crack surface, which is proportional to h. From this the *-3/8* power law follows. This law has been shown to be a general property of bending fractures of floating elastic plates. It applies not only to fractures caused by vertical loads, but also to fractures caused by temperature difference between the top and bottom of the floating plate. It also applies approximately to bending fractures in presence of in-plane compressive forces in the plate, which tend to cause buckling.

Vertical Penetration Through Floating Ice Plate -- The scaling law and size effect, however, are not the whole story. Complete fracture mechanics solutions have shown that the fracture patterns due to penetration of plates of different thicknesses are not exactly geometrically similar. A computer program has been written to solve the two-dimensional plate bending problem around a star-shaped system of cracks emanating from a small loaded circle (Fig. 3). The law for the growth of the star cracks as a function of the load-point displacement has been obtained and, as already known from experiments, it was found that the propagation of the radial star cracks is stable, with the load always increasing at increasing displacement. The maximum load, i.e., failure load, is obtained at the initiation of circumferential cracks at a certain distance from the loaded zone. The crack initiation, unlike crack propagation, is properly described by the strength theory, which has a different size effect, altering and mitigating the size effect associated with the radial crack growth.

Two additional phenomena further modifying the size effect have been identified. The number of radial cracks depends on the thickness of the plate, ice strength, elastic modulus and the energy that the material requires for fracture (Fig. 2c). The number of radial cracks strongly decreases with increasing stiffness, and because in a thicker plate there are fewer cracks to absorb the released energy, the size effect is made stronger by this phenomenon. Finally, in practical problems the diameter of the loaded area is not proportional to the plate thickness, because the problem is to predict the penetration force for an object of a fixed size through plates of different thicknesses. The correction for the diameter of the loaded area also increases the size effect because the ratio of the diameter to the thickness of the plate gets smaller for thicker plates.

The combination of all these phenomena superimposed on the underlying *-3/8* power law produces a strong and complicated size effect (Fig. 3d, e), for which the calculation method has been developed.

Determination of the number n of radial star cracks, mentioned above, is not an elementary problem with a solution in the literature. The difficulty lies in the initiation of fracture propagation because the energy release rate of an infinitely short crack is zero. Of course, the initiation per se is governed by the strength theory, which must be combined with fracture mechanics. Analysis showed that once the strength criterion before the crack formation is reached, cracks of a certain spacings jump to a certain initial equilibrium length a_0. One needs three conditions to determine the initial equilibrium crack length, the crack spacing, and the load at which the initiation occurs. In addition to the strength criterion, these consist in the condition that after the initial jump the rate of release of energy from the structure must equal the energy release rate required for further fracture growth (Fig. 3), and that the total energy release during the initial jump must equal the total energy required to form the initial cracks. These conditions were used in conjunction with a finite element program for the bending of a plate on elastic foundation, and provided the number of initial star cracks.

Size Effect in Thermal Bending Fracture -- Stresses caused by thermal changes of weather (Fig. 4b), especially those due to cooling in the fall, can cause floating ice plates to fracture (Fig. 4a). In the past, this problem has been solved on the basis of strength theory. However, in view of the arguments already explained, the solution must be based on fracture mechanics to correctly capture the size effect. The problem of a stationary equilibrium propagation of a semi-infinite crack in an infinite plate has been solved in detail, and the *-3/8* power size effect (Fig. 4c) has been established, as one major new result of this project. Because of long duration of thermal stresses, the relaxation due to creep of ice, which is very pronounced, has been included in the analysis and it was shown that the *-3/8* power law is approximately applicable even in the presence of creep (Fig. 4c). The analysis showed that while a temperature drop of 25°C is required to crack a plate 1 m thick, a drop of only 12°C is required to crack a plate 6 m thick, provided that similar temperature profiles get established (which takes much longer for a thicker plate). This analysis provided a possible answer to an old puzzle, first stated by Assour: the new fractures suddenly forming in the Arctic Ocean and running for distances of many miles do not follow the path of the smallest thickness of ice, that is, around the individual floes of several miles in size, separated by thin refrozen water leads. Rather, they cut straight through the thick ice. The discovery of the size effect explains why. Formation of these fractures is important for understanding the mechanism of build-up of pressure ridges, rafting, and opening of water leads (this is important also for navigation, and for surfacing as well as non-detectibility of submarines); Fig. 1.

Part-Through Cracks and Dome Effect in Penetration -- The penetration solution just described has been obtained under the assumption that the bending crack cuts through the entire thickness of the ice plate all the way to the crack front. In reality, the bending crack opens gradually, that is, at the front there is a crack only to a portion of the plate thickness, and this portion increases with the distance from the crack front, although closing of cracks may also occur due to compression forces. This requires a modification of the aforementioned solutions, which is important for not too long cracks. Furthermore, the rotation of crack faces in the opposite directions is opposed by contact stresses. Both phenomena, i.e., the part-through cracks and the contact stresses, produce in-plane compressive reactions from the ice plate, which oppose the bending fracture. They also shift vertically the location of the resultant of the in-plane forces, with the result that the penetration force is partially resisted not by plate bending but by a dome effect, the dome being represented by the surface of compression resultant location in the plate. Mathematically, this phenomenon causes a coupling between the plate bending problem and the in-plane elastic deformation of the plate. The problem has already been formulated mathematically and a solution, by means of finite elements, is in progress. In this solution, the part-through cracks are assumed to have a horizontal crack front within each finite element, propagating upwards. The propagation is based on a linear elastic fracture mechanics solution of the dependence of the in-plane normal force and bending moment across the cracked section on the additional in-plane displacement and bending rotation caused by the crack. This dependence has been obtained numerically. For initiating the vertical propagation of the crack in each element, a strength limit is assumed. A nonlinear optimization algorithm is used for solving the highly nonlinear equation system that arises, for each loading step. Fig. 5 shows examples of some results already obtained.

Nonlocal Damage and Random Particle Simulation -- Fracture mechanics is an idealized theory in which the fracture front is either a point or a line segment. All quasibrittle materials, including sea ice, have at fracture front a zone of distributed cracking. Its continuum-type modeling is possible but rather difficult. A simpler approach is to force a certain proper spacing of microcracks in the fracture process zone by assuming a suitable discrete micro-structure of the material in the fracture process zone. Such a micro-structure can be obtained by modeling the material as a system of particles. The spacing of the particles enforces a certain spacing of the cracks, as dictated by the preexisting inhomogeneities in ice (such as spacing of preexisting thermal cracks, effect of bottom roughness, spacing of warmer ice regions under snow drifts, spacing of larger brine pockets, etc.). The particles are considered to interact through central forces, whose law is selected so as to give correct elastic properties of the particle system, correct strength limit and correct fracture energy. A large computer program for particle simulation of large scale in-plane fracture of sea ice plate has been written. The program uses an explicit time step algorithm, which is very powerful and makes it possible to solve large systems (systems with over 120,000 degrees of freedom, and thousands of loading steps, have been handled on a desk-top work station).

Fig. 6 shows an application of this approach to the simulation of compression fracture caused by the impact of a large floe (several miles in size) traveling at various velocities and hitting a fixed obstacle. The obstacle can be an oil drilling platform or a bridge pier. There is one well recorded observation in which such an ice floe impacted a small island (Hans island, west of Greenland), having a vertical rock wall at its shore. The fracture sequence and pattern obtained in the particle simulation resembles well that observed in this event.

The particle simulation has also been used to study the size effect due to fracture mechanics. It was shown that the calculated maximum loads for specimens of various sizes follow the transitional curve in Fig. 2c and approach the asymptotic power law for large sizes.

Prediction of Fracture Characteristics by Micromechanical Analysis -- Experimental determination of the fracture energy and characteristic process zone size for large scale fracture of sea ice is difficult because its direct measurement requires specimens of enormous sizes (over 10 m). Therefore, prediction or at least a crude estimation of the fracture characteristics from the properties of the microstructure would be extremely useful. To gain a better understanding of this problem and determine some basic relations, fracture of ice specimens of various sizes (such as those used recently by Dempsey's team in the Arctic) have been carried out by the random particle method (Fig. 7). The size effect curve shown in Fig. 2c has been simulated. The location of the final asymptote of this curve makes it possible to calculate the fracture energy, and the location of the intersection of the horizontal and inclined asymptotes makes it possible to determine the characteristic size of the process zone. An important point is the dependence of these characteristics on the fracture characteristics of the interparticle force interaction, particularly the microstrength limit for the inter particle force and the microductility number representing the ratio of the interparticle displacement at full break to the displacement at peak force (Fig. 8). It has been established that the macroscopic fracture energy decreases with an increasing coefficient of variation of the microstrength.

Simulation of Acoustic Emissions in Fracturing Sea Ice -- Recently, Dempsey's team conducted measurements of sea ice fracture properties on the Arctic Ocean near Resolute and Barrow. The size effect method, which has been developed by Bažant and is based on the size effect described by the curve in Fig. 2c, was one of the main approaches. Specimens of sizes ranging from less than 1 m up to 80 × 80 m and thickness 1.8 m have been broken in a controlled manner. Fracture patterns similar to those observed in these experiments have been achieved. The force and displacement data are not yet available for analysis, however, the acoustic records of fracture have already been made available (by Farmer and Xie). These acoustic data include the record of a hydrophone located in sea water beneath the advancing fracture. An innovative method of simulating the acoustic record has been developed, exploiting the existing program for random particle simulation of fracture. The interparticle breaks serve as microenergy releases and cause emission of acoustic waves. Analytical solutions of the wave propagation problem, including refraction at the plate-water interface, reflection from top of ice plate and from the sea bottom, and leaking of planar wave into sea water, have been incorporated in the random particle program. The acoustic records are being simulated; for example the record of power (energy) flux at the hydrophone (proportional to the square of pressure) versus time. The acoustic signals from the crack jumps in ice have been determined from a modified Farmer-Xie's acoustic model for shallow water in the frequency domain. The acoustic pressure time histories were synthesized by inverse Fourier transform. The synthesized acoustic signal shows statistical resemblance to the statistical record in the experiments (Fig. 9). A size effect has been detected in the acoustic emissions in the sense of a root-mean-square of the acoustic pressure (Fig. 9). This kind of size effect can be related to the energy released from the ice floe of various thicknesses and sizes. By inversion of the solution it may be possible to judge the size of the fracture from the received acoustic signal. This simulation demonstrates the possibility of varying the assumed microfracture characteristics so as to obtain a very similar record and thus determine which microfracture characteristics yield the correct results.

Statistical Weibull-Type Size Effect -- At the beginning of the project, the effect of randomness of material strength on the failure load has been studied. A detailed report would require a lengthy discussion. Suffice to say that the conclusion was that the statistical size effect is important only for failures that occur at fracture initiation but has only a minor influence on failures that occur after large stable crack growth. This comprises most, although not all, problems of sea ice fracture. A correction to the size effect law arising from these statistical effects has also been determined and was shown to be relatively minor under the aforementioned conditions.

PLANS AND FUTURE DIRECTIONS

In continuation of this program, the following problems should be solved: (1) Application of the present methods to the analysis of the experiments of Dempsey's team at Resolute and Barrow; (2) determination of the relations between macro- and micro- fracture characteristics; (3) fracture solutions for part-through cracks and cracks with large process zones; (4) generalization of fracture and failure models for sea ice to rate dependence; (5) determination of the relation between acoustic emission records and fracture process in sea ice; (6) enhancement of understanding of the size effect in sea ice, particularly with regard to the cohesive crack model; (7) formulation of a complete theory for crack initiation and crack spacing; (8) determination of the effect of creep and temperature on the evolution of ice fracture; (9) analysis of the possible role of fractal nature of crack surfaces in the size effect on sea ice fracture.

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Main Papers in Refereed Journals

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9. Bažant, Z.P. (1995). "Can size effect in fracture be caused by fractal nature of crack surfaces?" *Int. J. of Fracture*, submitted to.

10. Bažant, Z.P., and Li, Zhengzhi (1995). "Acoustic emissions in fracturing sea ice plate simulated by particle system." To be submitted to *J. of Engrg. Mech.*

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11. Bažant, Z.P., Xi, Y., and Reid, S.G. (1991). "Statistical size effect in quasi-brittle structures: I. Is Weibull theory applicable?" *ASCE J. of Engrg. Mech.*, 117(11), 2609-2622.

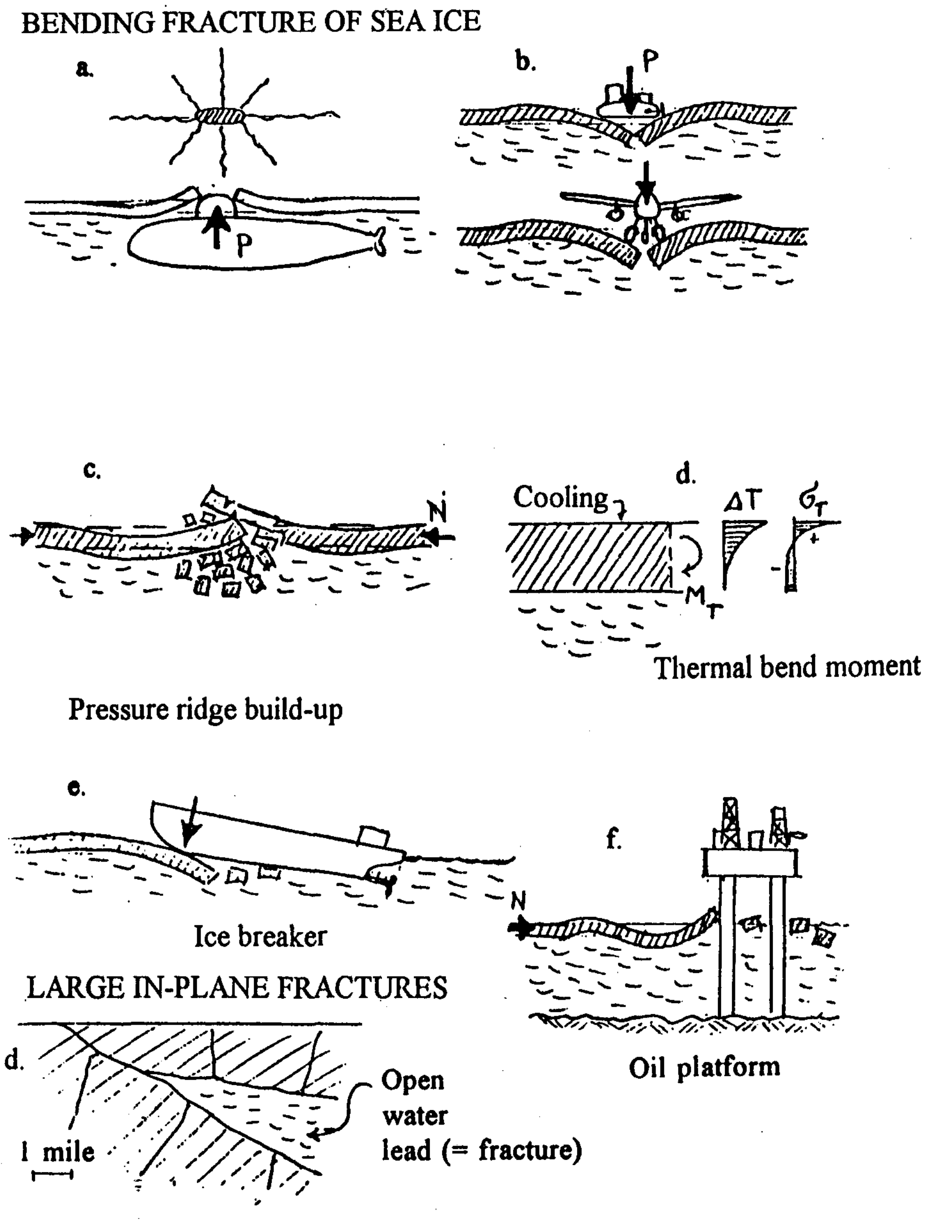

FIG. 1 Typical fracture problems for sea ice.

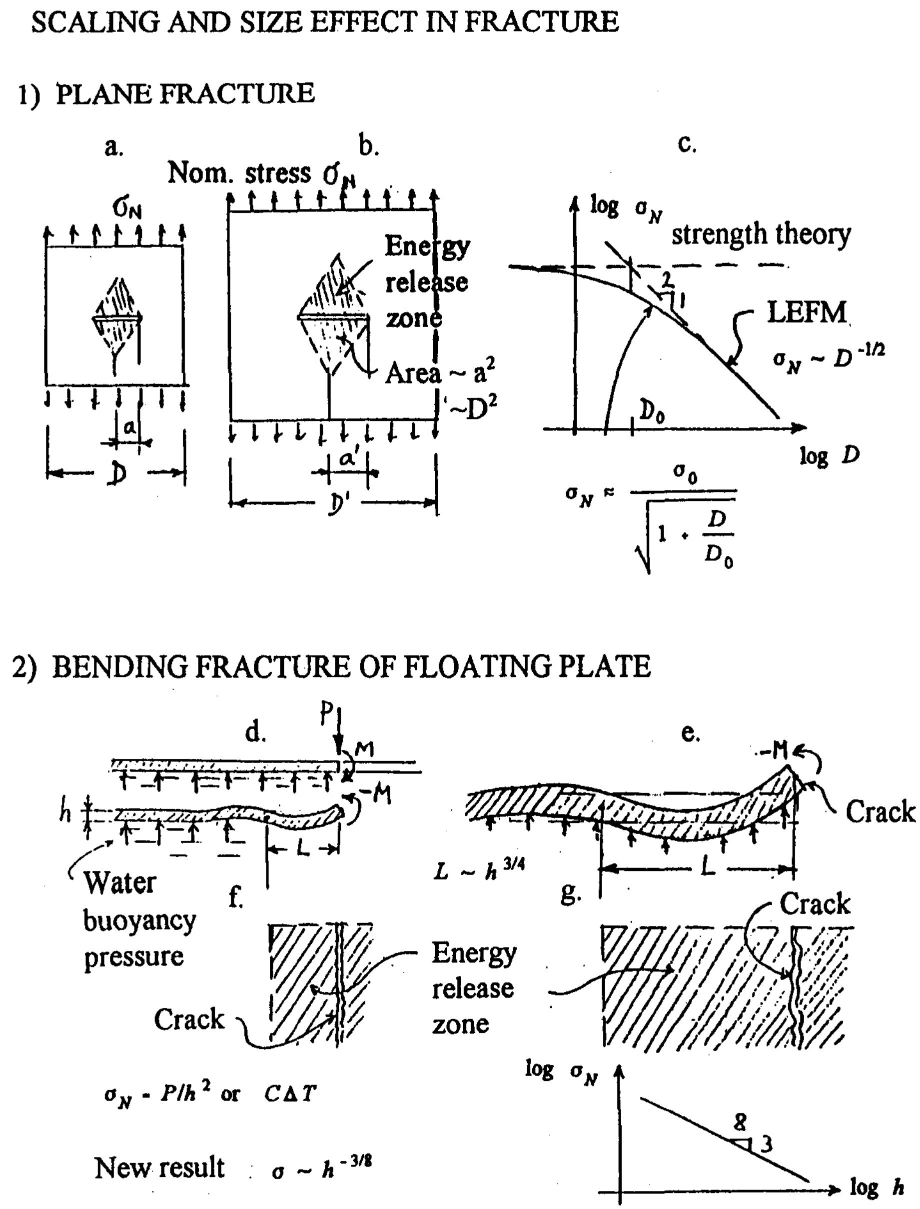

FIG. 2 Explanations of scaling and size effect for in-plane and bending fractures

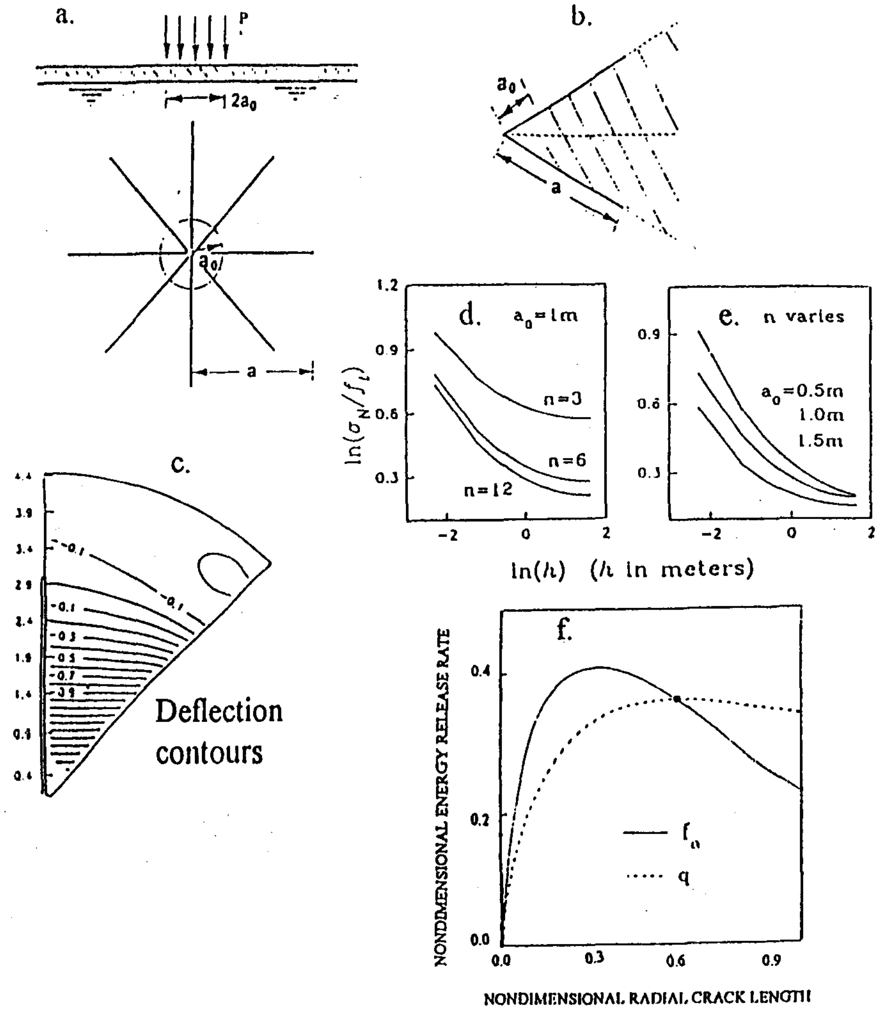

FIG. 3. Penetration fracture of sea ice plate under vertical load: (a) star-crack pattern, (b) plate wedge limited by two radial cracks, (c) typical calculated deflection contours of cracked plate, (d-e) decrease of nominal strength with plate thickness at constant loaded-area radius for various numbers of star cracks, and for various radii of loaded area and the most dangerous variation of the number of cracks, (f) diagrams of the incremental and average energy release rates, giving at their intersection and the initial crack length and the initial crack spacing.

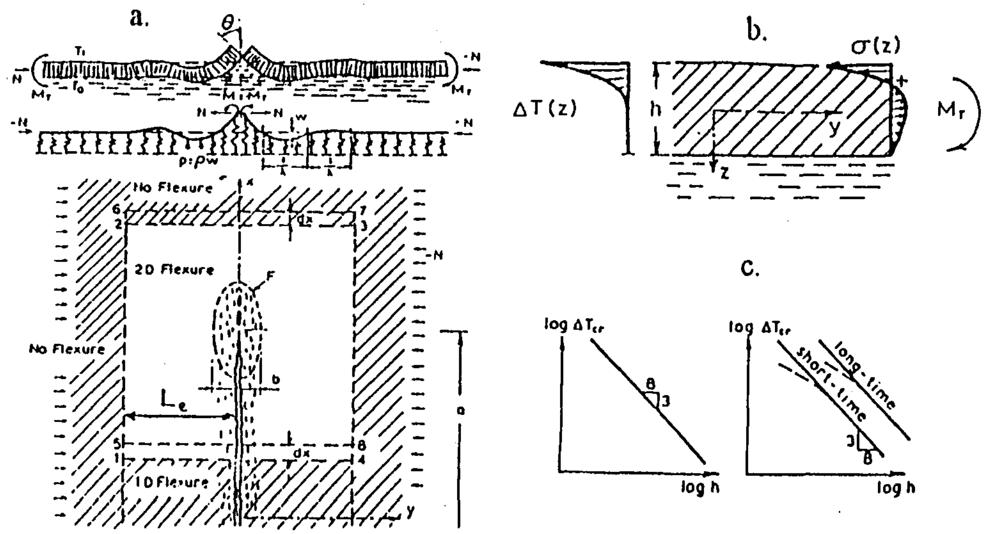

Fig. 4 Thermal bending fracture of floating ice plate: (a) Unloading that causes release of stored energy, (b) temperature and cooling stress profiles, and (c) effect of plate thickness *h* on critical temperature drop of short or long duration.

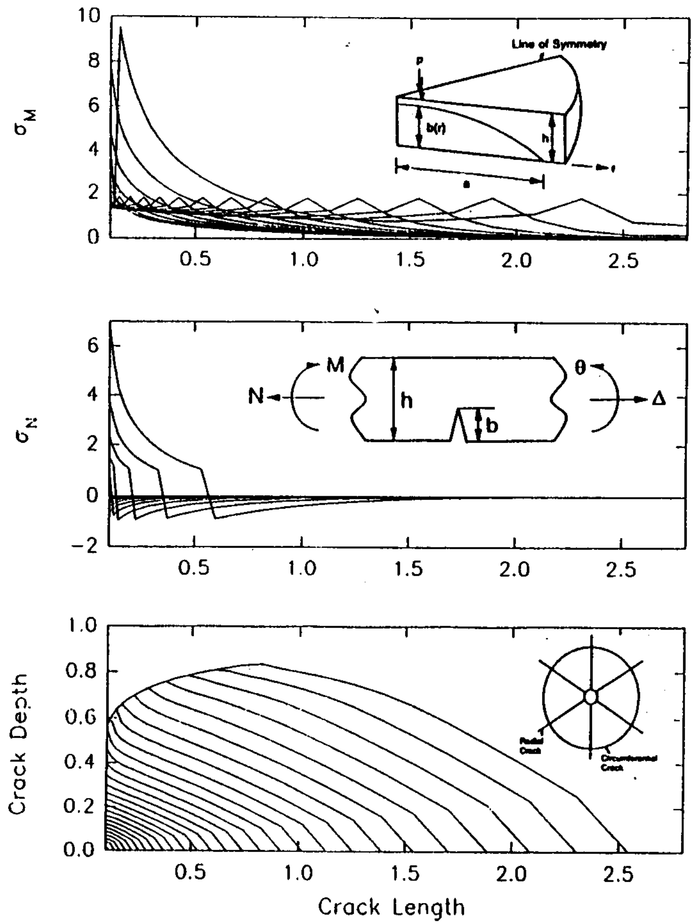

Fig. 5 (a) Distribution of bending stress along the crack length for crack tips at various locations; (b) distributions of the average normal stress along the crack length for crack tips at various locations; and (c) distributions of the vertical crack depth along the crack for crack tips at various locations. (Note that high compression stresses are transmitted across the partially cracked cross sections even far behind the crack tip.)

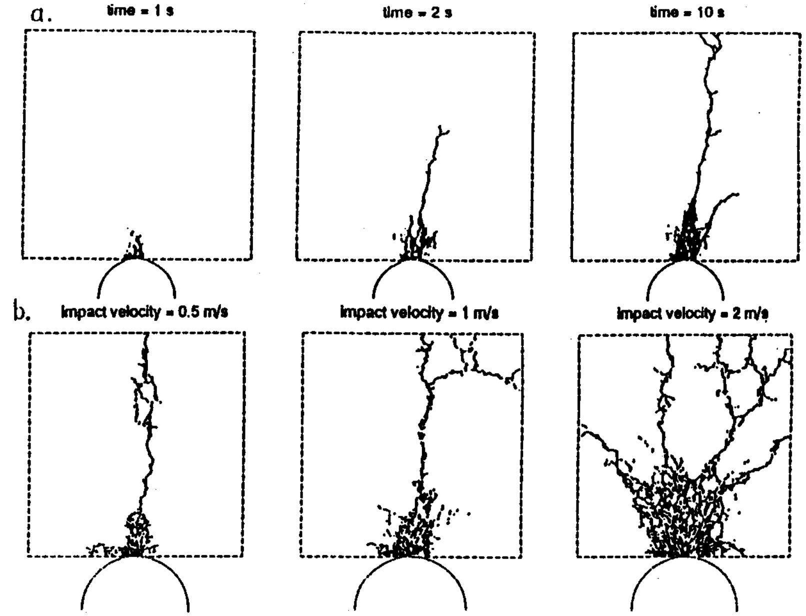

Fig. 6 Simulation of impact of ice floe on rigid circular obstacle by discrete element method. Top: fracture patterns at various times; botton: for various velocities.

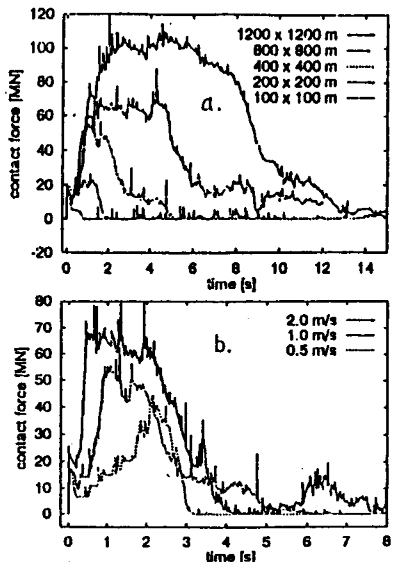

Fig. 7 Historics of caluclated contact force for Fig. 1, for various floe sizes and various impact velocities.

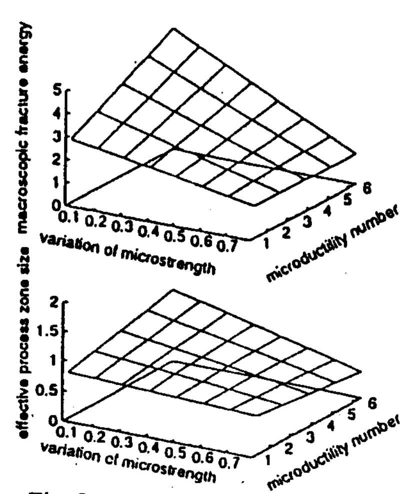

Fig. 8 Calculated dependence of macro-fracture energy and effective process-zone size on statistical coefficients of variation microstrength of ice and microductility.

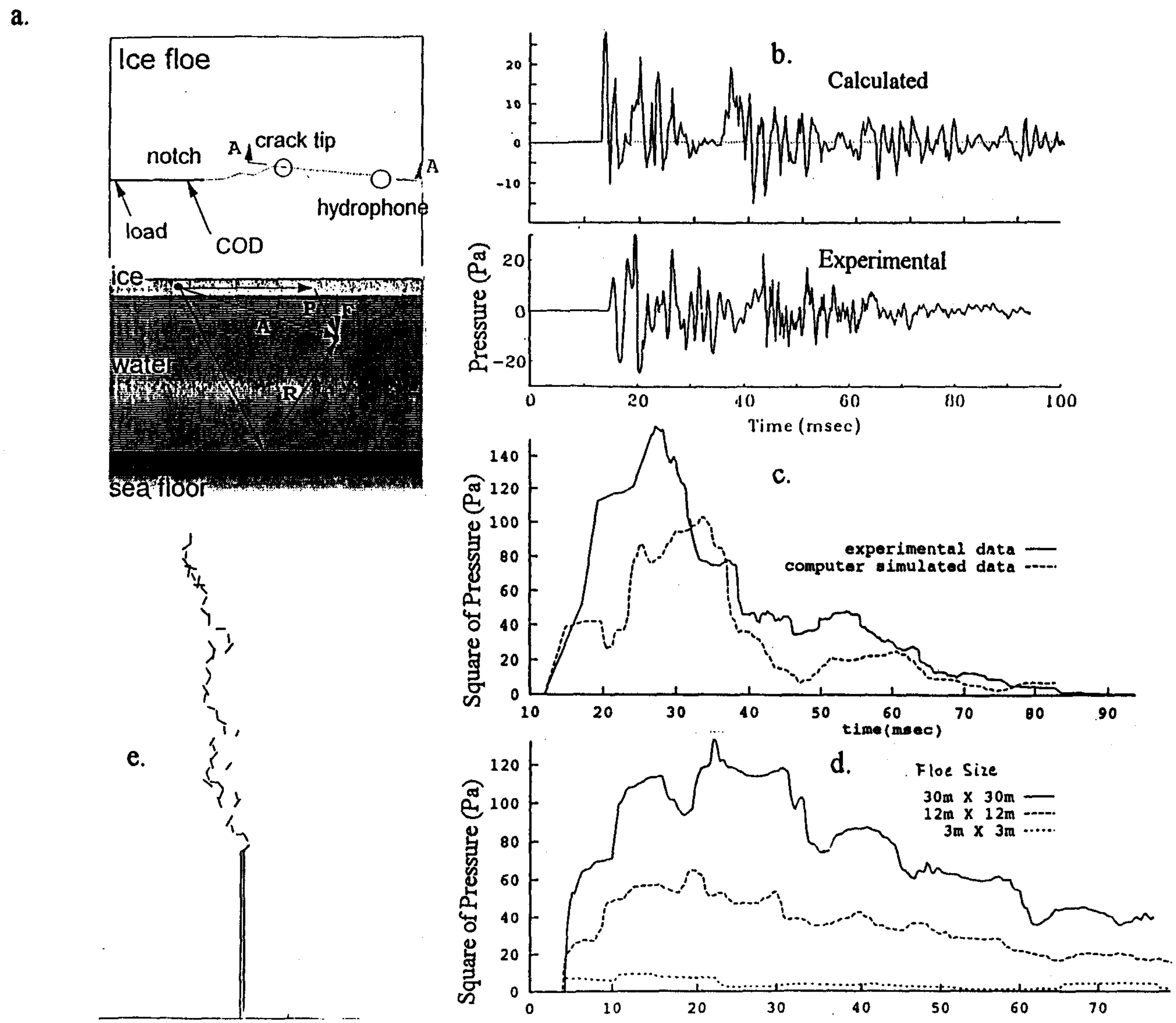

Fig. 9 Computers fracture simulation of acoustic signals obtained in Dempsey's tests at Resolute: (a) Plan view of ice specimen and acoustic paths in vertical section, (b) Calculated and recorded time history of pressure at geophone, (c) Calculated and recorded history of square of pressure, (d) Calculated historics of the square of pressure for fracturing of specimens of different sizes, and (e) Plan view of individual crack jumps causing acoustic emissions.