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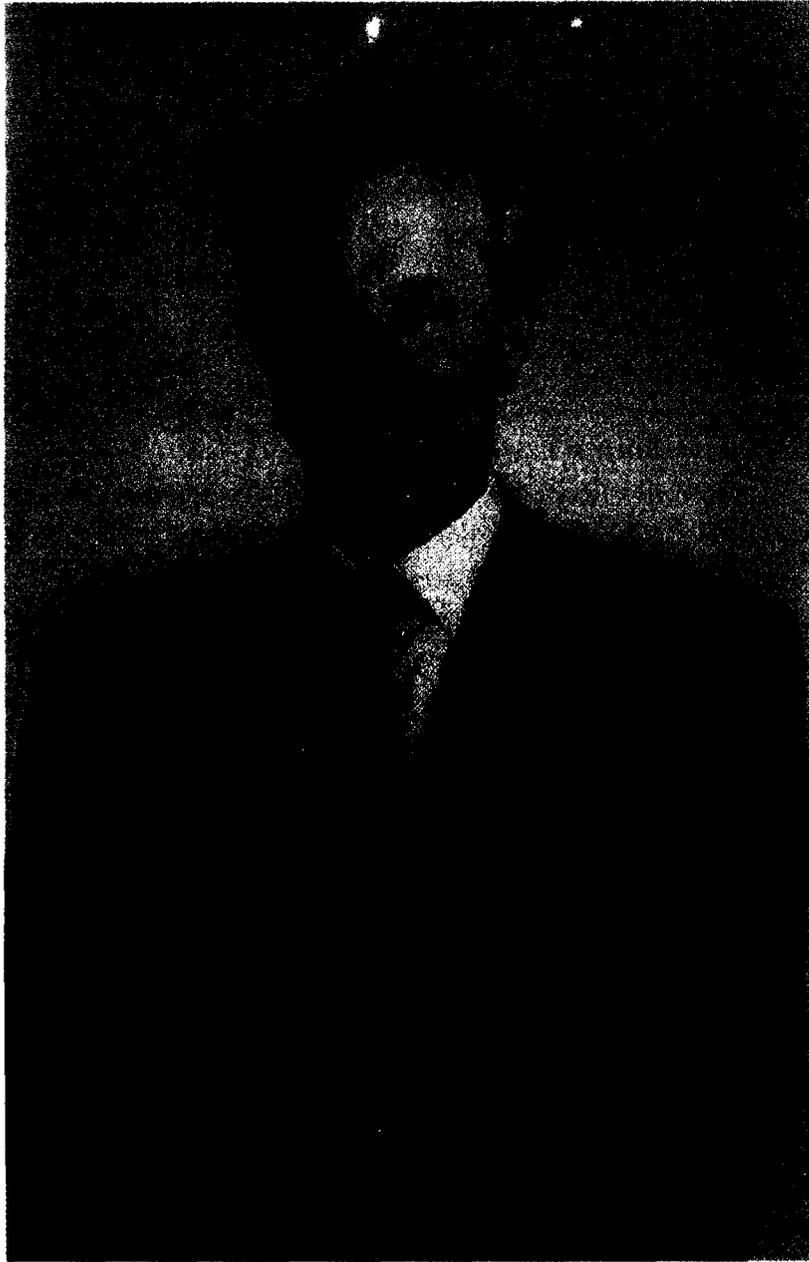
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## MICROPRESTRESS - SOLIDIFICATION THEORY FOR AGING AND DRYING CREEP OF CONCRETE

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### Abstract

A new physical theory for the effects of long-term aging and drying on concrete creep is proposed. The previously proposed solidification theory, in which the aging is explained and modeled by the volume growth (into the pores of hardened Portland cement paste) of a nonaging viscoelastic constituent (cement gel), cannot explain long-term aging because the volume growth of the hydration products is too short-lived. The paper presents a revision of solidification theory in which the viscosity of the flow term of the compliance function is treated as a tangential viscosity of a nonlinear viscous power law governing very large and highly localized microstress in the hardened cement paste. This microstress is called the microprestress because it is produced during hydration by very large and highly localized volume changes, independent of loading. An essential aspect of the theory is that the applied external load or the macroscopic continuum deformation of concrete can cause only very small changes of the microprestress, such that the response to load is determined by tangential linearization. Relaxation of the microprestress causes the

tangential viscosity to increase, which reduces long-term creep. A decrease of relative humidity in the pores causes (due to changes of capillary tension, surface tension and disjoining pressure) a large increase in the microprestress, which in turn reduces tangential viscosity and thus increases the creep rate. This explains the drying creep (Pickett effect). An improved constitutive law is formulated, verified and calibrated by comparisons of finite element solutions with test data.

### 1 Introduction

Aging, that is, the decrease of the creep compliance of concrete with the age at loading, has proven to be a formidable problem in the theory of concrete creep. Attainment of satisfactory theoretical understanding has defied seven decades of determined research efforts. Initially, the aging was simply described by empirical functions of time. However, the test data are highly scattered, and so a good creep model with prediction capability can be formulated and identified from the data only if it is based on a good, physically justified theory. Lacking that, there is too much uncertainty. A formulation violating some basic physical principles can easily result (for example, one empirically obtained creep model exhibited conflicts with the second law of thermodynamics).

The physical justification for aging must obviously be sought in the processes in the microstructure. Recently, one theory, called the solidification theory (Bažant and Prasanna, 1989a,b), apparently explained aging satisfactorily. The basic premise was that the age dependence of creep properties ought to result from changes in the composition of the system, i.e., changes in the concentrations of the constituents of the system which themselves do not change with age.

The solidification theory does not extend to drying creep and has a discrepancy in that the long term age effect on creep is more pronounced than the long term growth in the volume of hydration products.

The objective of this paper is to briefly suggest a physical mechanism which explains in a unified simple way both the aging that is not caused by volume growth and the part of Pickett effect (drying creep) that is not caused by microcracking. A detailed development of the idea, with the complete mathematical derivation are given in a separate report (Bažant et al. 1995).

## 2 New Concept and Its Motivation

The challenge is to find a mechanism of aging that does not require volume growth of the solidified constituent while the constituent has fixed properties in time. Obviously, such a mechanism cannot be associated with stiffness. But it can be associated with viscosity, and that might at the same time be helpful for modeling of the Pickett effect.

Direct dependence of viscosity on time would have to come from deposition of newly solidified layers on the pore walls. However, such a process would have to also increase stiffness, which would lead directly to the solidification theory. So one must suspect the existence of some additional viscosity increasing process other than solidification. The basic new idea advanced here (recently proposed by Bažant, 1993) is that this process is the process of relaxation of microstresses in certain viscous flow elements in the subcapillary microstructure of the cement gel. A model with the following characteristics is required to implement this idea.

1. The viscous flow elements represent very large but very localized deformations at isolated sites, called relaxation sites, which are dispersed throughout the microstructure of cement gel and have the dimensions of gel pores.
2. Only a negligible stress, compared to the total microstress, can be caused in these elements by the external load and deformation to which concrete can be subjected on the macroscale. The microstress exists even at zero load and is hence more aptly called the microprestress,  $S$ .
3. A very large stress is caused in these elements by changes of the relative humidity of water vapor in the capillary pores or by large local volume changes in the microstructure due to chemical reactions.

### 3 Microprestress Relaxation in Viscous Flow Element

Let  $S$  and  $e$  be the microprestress and the corresponding microstrain in the viscous flow element caused by the disjoining pressure (and crystal growth pressure) at zero externally applied load, and let  $\sigma$  and  $\epsilon$  be the additional stress and strain (Fig. 1) caused in these elements by the externally applied load (or macroscopic continuum stress). The basic hypothesis of the present analysis is that  $|\sigma| \ll |S|$ .

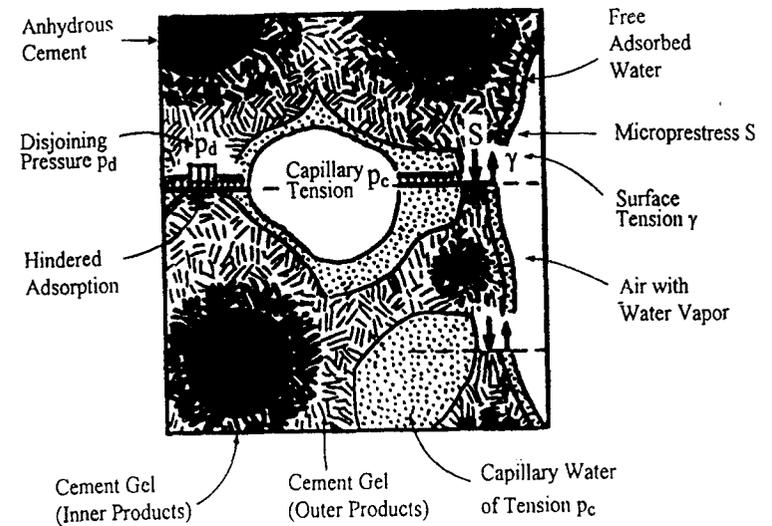


Fig.1. Idealized microstructure of hardened cement paste

The total stress  $S_T = S + \sigma$  and strain  $\dot{\epsilon}_T = \dot{\epsilon} + \dot{\epsilon}$  in the viscous flow element may be assumed to follow a power law;  $\dot{\epsilon}_T = c S_T^p$  in which superior dots denote time derivatives. It can be shown that

$$\dot{S} = -c_0 S^p \quad (1)$$

We have thus in (1) a microprestress relaxation law which does not involve time. This law, which replaces the age-dependence of viscosity, shows that the apparent increase of viscosity  $\eta$  with age  $t$  can be interpreted as a shift of the tangent point of the power law, as illustrated in Fig. 2. The long-term aging of flow viscosity is simply a consequence of relaxation of the microprestress.

### 4 Microprestress Caused by Humidity Change

The hardened cement paste, serving as the binder in concrete, is a strongly hydrophylic porous material whose pores have enormous internal surface (about  $500 \text{ m}^2$  per  $\text{cm}^3$ ). This causes capillary and adsorption forces to dominate the stress levels in the microstructure (Powers and Brownyard, 1946-1947; Powers, 1968).

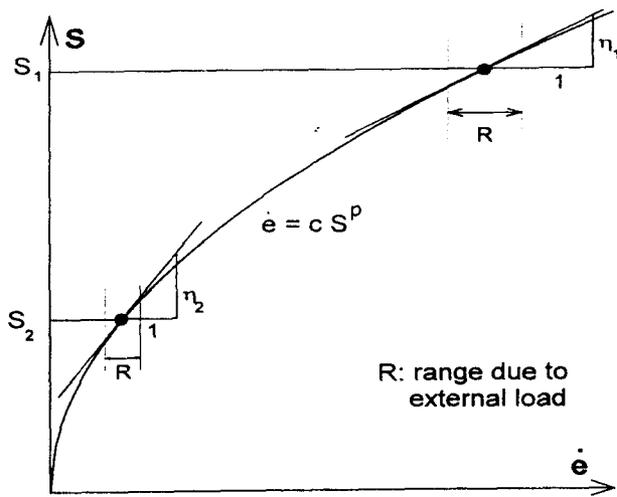


Fig. 2. Tangential linearization of power type viscous law

The surface tension, capillary tension, disjoining pressures and crystal growth pressures must be balanced by stress elsewhere in the solid framework of cement gel. From microscopic observations it is known that there exist very localized very narrow necks between lumps of hydration products (Fig. 1) in which the balancing stress  $S$  must be far larger than the strength of concrete on the macroscale. These necks are imagined to represent the aforementioned microscopic viscous flow elements, prestressed by the surface tension, capillary tension, disjoining pressures and crystal growth pressures. The existence of such microprestressed elements in the cement gel microstructure, in which the stress is controlled by physical phenomena independent of the stress acting on the macroscale, is a central hypothesis of the present mathematical model. The surface tension, capillary tension and disjoining pressure depend on  $h$  generally as follows:

$$\gamma = C_1 R M^{-1} T \ln h + \gamma_1 \quad (2)$$

(e.g. Eq. 44 in Bažant, 1972);  $R$  = gas constant,  $M$  = molecular weight of water,  $T$  = absolute temperature, and  $C_1, \gamma_1$  = constants. Thus the rate  $\dot{\gamma}$  is proportional to  $\dot{h}/h$ .

According to our previous discussion and the model in Fig. 1, the changes of microprestress must balance the changes of surface tension, capillary tension and disjoining pressure. So the increment of microprestress rate

$\dot{S}$  due to  $\dot{h}$  must be proportional to  $\dot{\gamma}$ , and thus to  $\dot{h}/h$ . Therefore, (1) may be generalized for humidity variation as follows:

$$\dot{S} = -c_0 S^p - c_1 \frac{\dot{h}}{h} \quad (3)$$

where  $c_1$  is a constant.

Changes of  $\gamma$  must also cause elastic deformation of the solid skeleton in cement gel, which represents a part of the free shrinkage (Powers, 1968), i.e., shrinkage that occurs at no macroscopic stress. Because  $\dot{\gamma}$  is proportional to  $\dot{h}$ , the free shrinkage is properly described by the following simple rate equation

$$\dot{\epsilon}_{sh} = k_{sh} \dot{h} \quad (4)$$

$k_{sh}$  is an empirical shrinkage coefficient which can be taken approximately as constant.

## 5 Combination of Solidification and Microprestress Models

Aside from the aging due to relaxation of microprestress, there is of course aging due to volume growth of hydration products, which dominates at early ages. From extensive studies of test data (Bažant and Prasanna, 1989b; Bažant and Baweja, 1995) and theoretical analysis (Carol and Bažant, 1992), it transpired that this type of aging can be described by either a Kelvin chain or a Maxwell chain spring-dashpot model in which the relaxation times  $\tau_\mu$  ( $\mu = 1, 2, \dots$ ), are constant and all the spring moduli and dashpot viscosities vary in proportion. The simple proportional variation of all the spring moduli is a logical consequence of the fact that this type of aging is due to volume growth. The analysis of the available test data shows that the effects of microprestress and of volume growth can be assumed to be separated, acting on different parts of the chain model (Fig. 3). Putting all the sources together, the compliance function of concrete can be expressed as:

$$J(t, t') = q_1 + C_1(t, t') + q'_1 S^{p-1} \quad (5)$$

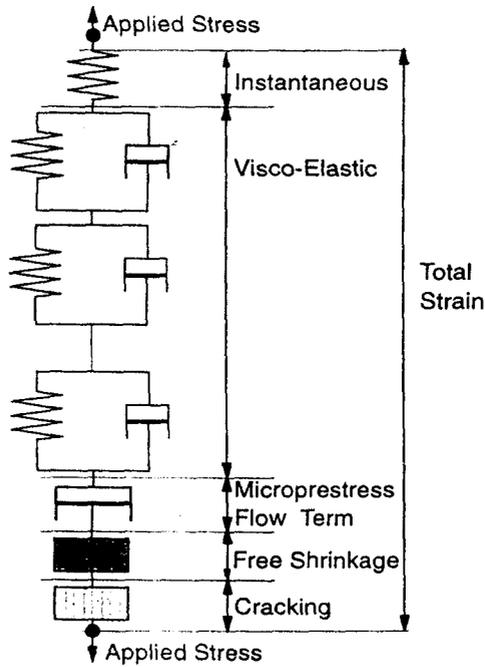


Fig. 3. Rheologic model for concrete creep

in which  $J(t, t')$  = strain at time  $t$  caused by a unit uniaxial stress acting since time  $t'$ ;  $C_v(t, t')$  = viscoelastic compliance affected by the volume growth of hydration products, characterized by function  $v(t)$ ,  $q'_4 = c_p = q/k_1$  = new constant introduced for brevity of notation,  $q_1$  = constant characterizing the asymptotic instantaneous strain (the age independence of  $q_1$  is justified by Fig. 6b in Bažant and Baweja, 1995 b);

As shown before (Bažant and Prasannan 1989b, Bažant and Baweja 1995), it is more convenient to write the detailed expression in terms of the compliance rate  $\dot{j}(t, t') = \partial J(t, t') / \partial t$ . Co-opting for  $C_v(t, t')$  the expressions previously formulated and justified for the solidification theory, the compliance rate may be written as (Bažant and Prasannan, 1989b; Bažant and Baweja, 1995):

$$\dot{j}(t, t') = \frac{n(q_2 \lambda_0^m t^{-m} + q_3)}{(t - t') + \lambda_0^n (t - t')^{1-n}} + q'_4 (p - 1) S^{p-2} \dot{S} \quad (6)$$

in which  $q_2, q_3, m, n$  and  $\lambda_0$  are empirical constants. According to extensive data fitting, one can always take  $\lambda_0 = 1$  day,  $m = 0.5$  and  $n = 0.1$ .

## 6 Constitutive Relation, Cracking and Numerical Algorithm

Before writing the multiaxial creep law, note that the uniaxial Hooke's law  $\epsilon = \sigma/E$  is generalized to multiaxial stress by the matrix relation  $\epsilon = E^{-1} G \sigma$ . For axisymmetric stress  $\sigma = (\sigma_r, \sigma_\theta, \sigma_z, \sigma_{rz})^T$ ,  $\epsilon = (\epsilon_r, \epsilon_\theta, \epsilon_z, \gamma_{rz})^T$  are column matrices of stress and strain components in cylindrical coordinates  $(r, \theta, z)$  superscript  $T$  denotes matrix transpose, and

$$G = \begin{bmatrix} 1 & -\nu & -\nu & 0 \\ -\nu & 1 & -\nu & 0 \\ -\nu & -\nu & 1 & 0 \\ 0 & 0 & 0 & 2(1 + \nu) \end{bmatrix}$$

Because of linearity of the creep law, and because the creep Poisson ratio  $\nu$  can be assumed to be constant, the generalization to a multiaxial creep law may be accomplished by replacing, in the uniaxial creep constitutive law,  $\sigma$  with  $G\sigma$  and  $\epsilon$  with  $\epsilon$  (Bažant, ed., 1988). Thus, e.g., for constant stress matrix  $\sigma$ , the strain history is  $\epsilon = GJ(t, t')\sigma$ . Since the creep Poisson's ratio can be assumed to be constant, the matrix generalization of the uniaxial constitutive law,  $\epsilon(t) = \int_0^t J(t, t') d\sigma(t') + \epsilon_{sh}(t)$  may be written as

$$\epsilon(t) = \int_0^t J(t, t') G d\sigma(t') + \epsilon_{sh}(t) \quad (8)$$

in which  $\epsilon_{sh} = (\epsilon_{sh}, \epsilon_{sh}, \epsilon_{sh}, 0)^T$  for the axisymmetric case.

Like the physical explanation and derivation of the solidification theory, large-scale numerical computation, too, is best approached by approximating the compliance function with the viscoelastic Kelvin chain or the Maxwell chain rheologic model. For uniaxial stress, the constitutive law based on Kelvin chain and the corresponding step-by-step solution algorithm were formulated in Bažant and Prasannan (1989b). We will now present their matrix generalization for multiaxial stress, either axisymmetric or general, using the same method of matrix generalization as for age-dependent Kelvin or Maxwell chain (Bažant 1988).

In view (8) and the analogy of  $\epsilon = E^{-1}G\sigma$  with  $\epsilon = \sigma/E$  the matrix generalization of the uniaxial formulation of the Kelvin chain model may be obtained by replacing  $\epsilon$  with  $\epsilon$ ,  $\sigma$  with  $G\sigma$ ,  $\epsilon_{sh}$  with  $\epsilon_{sh}$ ,  $\gamma_{\mu}$  with  $\gamma_{\mu}$  etc. According to this matrix analogy, the equations for the Kelvin chain model given by

$$E_{\mu}(\gamma_{\mu} + \tau_{\mu}\dot{\gamma}_{\mu}) = G\sigma; \quad \gamma = \sum_{\mu=1}^N \gamma_{\mu} \quad (9)$$

in which  $\gamma$  = column matrix of strains of age-independent solid constituent  $= [\gamma_r, \gamma_{\theta}, \gamma_z, \gamma_{rz}]^T$  for the axisymmetric case,  $\gamma_{\mu}$  = similar column matrices of partial strains corresponding to the individual Kelvin chain units ( $\mu = 1, 2, \dots, N$ ); and  $E_{\mu}$ ,  $\tau_{\mu}$  = spring modulus and retardation time of the  $\mu$ th Kelvin unit in the chain (Fig. 3), with  $\tau_{\mu} = \eta_{\mu}/E_{\mu}$  where  $\eta_{\mu}$  = viscosity of the dashpot of the  $\mu$ th Kelvin unit ( $\mu = 1, 2, \dots, N$ ). The value of  $\tau_{\mu}$  must be suitably chosen as described in Bažant, ed., (1988), and the corresponding values of  $E_{\mu}$  are then determined by fitting the solution of (9) for unit uniaxial stress  $\sigma = 1$ , i.e.  $\epsilon(t) = \sum_{\mu} E_{\mu}^{-1}(1 - e^{-(t-t')/\tau_{\mu}}) + A_0$ , to the compliance function  $J_{\epsilon}(t-t')$  non-aging solidifying constituent. This is best done by discretizing a continuous retardation spectrum (Bažant and Xi, 1995).

Fig. 4 shows a graphical interpretation of this method. For each retardation time,  $\tau_{\mu}$ , the corresponding  $A_{\mu}$  is given by the area of the rectangle centered at  $\tau_{\mu}$ , as shown in Fig. 4. To include the tail for  $\log t \rightarrow -\infty$ , an extra term  $A_0$  is needed. Although only about 5 Kelvin units suffice

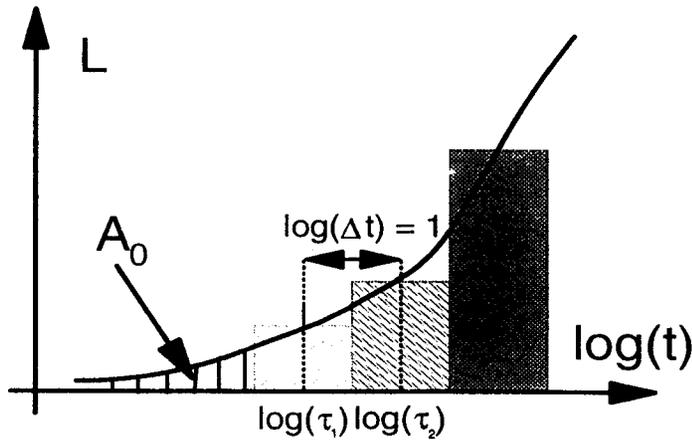


Fig 4. Discretization of continuous retardation spectrum of solidifying constituent

for practical problems to avoid bumpy curves, the present calculations used 10 Kelvin units, and then  $A_0 = 0.223$ . The viscoelastic strain  $\epsilon^v$  is given for the uniaxial stress by (5) in Bažant and Prasanna (1989a), which for uniaxial stress and variable humidity in the pores reads  $\epsilon^v = \phi_h \dot{\gamma} / v(t)$  where  $\phi_h$  is a function of  $h$  which decreases if  $h$  decreases because basic creep is smaller when the specimen has been brought to equilibrium at a lower environmental humidity. The previously verified form of function  $\phi_h$  is  $\phi_h(h) = \alpha_h + (1 - \alpha_h) h^2$  where  $\alpha_h$  is a material parameter ( $0 < \alpha_h < 1$ ).

The rate of the vector of total strains,  $\epsilon$ , is

$$\dot{\epsilon} = \dot{\epsilon}^v + \dot{\epsilon}^f + \dot{\epsilon}_{sh} + \dot{\epsilon}_{cr} \quad (10)$$

where  $\dot{\epsilon}^f$  = rate of the vector of flow strains;  $\dot{\epsilon}^v$  includes the instantaneous elastic strain rate.  $\dot{\epsilon}_{sh}$  = rate of vector of shrinkage strain, and  $\dot{\epsilon}_{cr}$  = rate of vector of cracking strains. For concrete exposed to a drying environment,

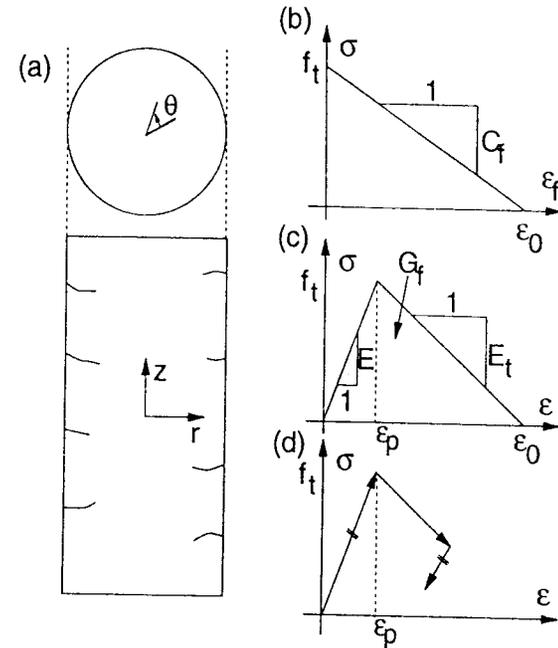


Fig. 5. (a) Crack orientations considered; (b) relation of normal stress to cracking; (c) Stress-strain law with post-peak strain softening; and (d) rule assumed for unloading of cracks

it is necessary to consider distributed cracking. If the specimen or structure is large and drying times long, it is also necessary to consider localization of cracking into continuous cracks and use fracture mechanics, but this situation will not be considered here. The sum of elastic and cracking strain increments for the axisymmetric case may be written as  $\dot{\epsilon} = C_s \dot{\sigma}$ , where the  $C_s$  (Fig. 5) is the well known elastic crack compliance matrix (Bažant and Oh, 1983). Without cracking it would, in particular, be impossible to model the large strain difference between creep specimen and its companion shrinkage specimen that appears initially after the start of drying (See Fig. 6, 7). Time is subdivided by discrete times  $t_i$  ( $i=1,2,\dots$ ) into time steps  $\Delta t = t_{i+1} - t_i$ . The distributions of pore humidity  $h$  and microstress  $S$  are solved first, in advance of structural analysis. The solution of  $h$  is reviewed in Appendix I. The solution of  $S$  is obtained step-by-step by integrating the first order differential equation (3). To this end, for step  $(t_i, t_{i+1})$ , (3) is approximated as

$$\Delta S = -c_0 S_{i+1/2}^p \Delta t - c_1 \frac{\Delta h}{h_{i+1/2}} \quad (11)$$

where  $\Delta h$  and  $\Delta S$  are the increments over  $\Delta t$  and the value of  $S_{i+1/2}$  is taken as  $S_i$  for the first iteration and  $S_i + \Delta S/2$  for the second iteration of step  $(t_i, t_{i+1})$  (this is actually the Runge-Kutta procedure). For uniaxial stress, an efficient step-by-step solution algorithm, was given by Eqs. 5-14 in Bažant and Prasanna (1989b). In view of (8), this algorithm may be generalized to multiaxial stress by replacing  $\sigma$  with  $G\sigma$ ,  $\epsilon$  with  $\epsilon$ , etc. Adding the matrix generalization of the viscous flow strain,  $\Delta \epsilon^f$ , of the instantaneous elastic strain,  $q_i G \Delta \sigma$ , of the shrinkage strain,  $\Delta \epsilon_{sh}$ , and of the cracking strain  $\Delta \epsilon_{cr} = C_s \Delta \sigma$ , we obtain the incremental quasi-elastic stress-strain relation  $\Delta \sigma = E(\Delta \epsilon - \Delta \epsilon'')$ : where  $E$  is the incremental quasi-elastic stiffness matrix of the material, and  $\Delta \epsilon''$  is the total inelastic strain increment during  $\Delta t$ .

### 6.1 Comparison with Test Data

In creep measurements in drying environment, it is standard not to report the total deformation but report separately the drying shrinkage strain of load-free companion specimen and the compliance representing the difference in strain per unit stress between the loaded creep specimen and the load-free companion specimen. The creep value thus defined, however, is affected by microcracking of the companion shrinkage

specimen, which relieves the tensile stresses due to drying and strongly reduces the overall shrinkage deformations (Fig. 6). Thus, when the deformations measured on the companion specimen (1 in Fig. 6), are subtracted from the deformations measured on the loaded and drying specimen, it is not only the true creep that remains but also the cracking deformations of the companion specimen (Fig. 7). This explains why some reported drying creep measurements show a large strain increase immediately after loading, compared to the non-drying loaded specimen (e.g., Mamillan and L'Hermite 1965). Fig. 8 shows the development of microstress with time with or without drying. The development of deformations and stresses over the section of an unloaded drying specimen are shown in Fig. 9. Cracking, albeit milder, appears even in the creep specimen that is subjected to compressive load (e.g., Granger, Acker and Torrenti, 1994). Therefore it is not easy to separate cracking in the companion drying specimen from cracking in the loaded drying specimen. With this in view the total measured deformations on the sealed and drying specimens have been fitted by the present model as shown in Fig. 10.

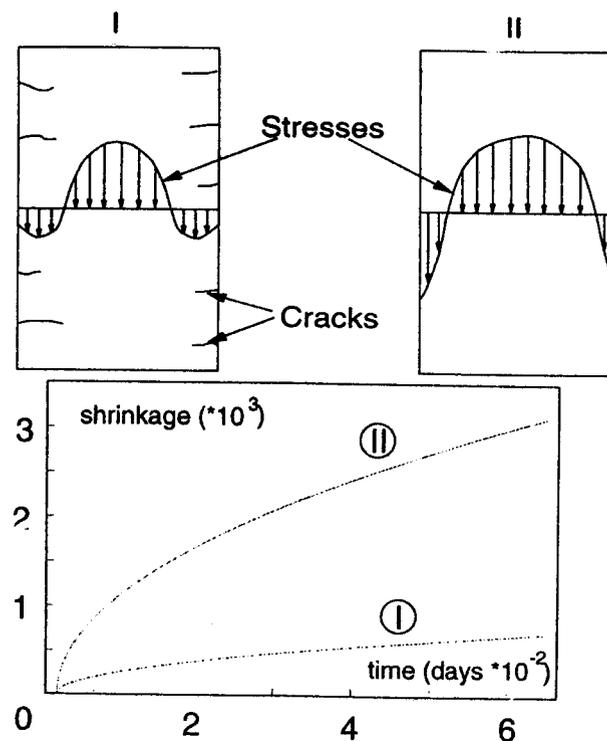


Fig. 6. Stresses and shrinkage in cracked and uncracked specimen

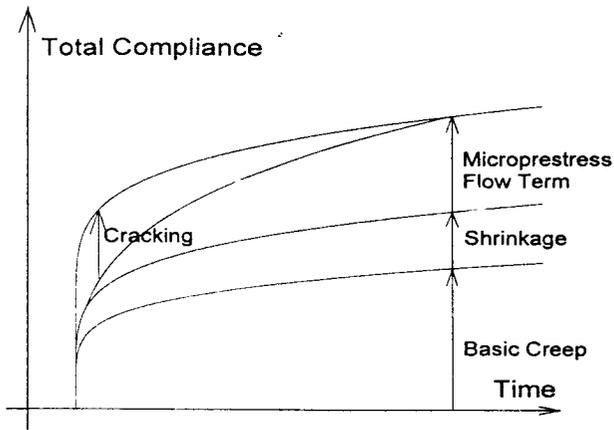


Fig. 7. Contributions from various mechanisms of drying creep

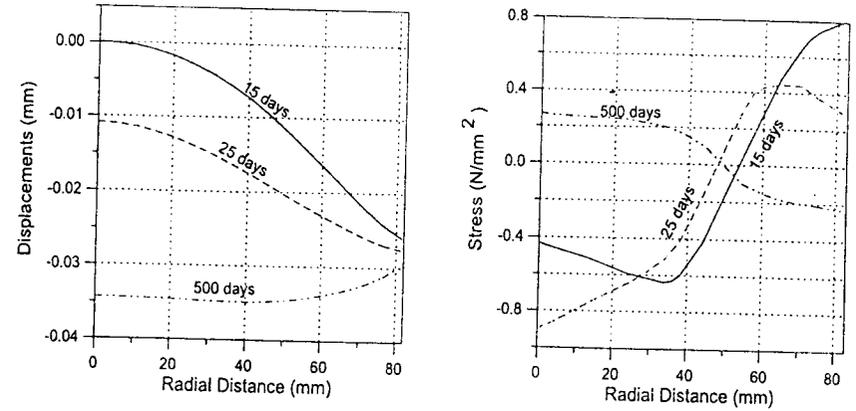


Fig. 9. Displacements and Stresses in the drying specimen

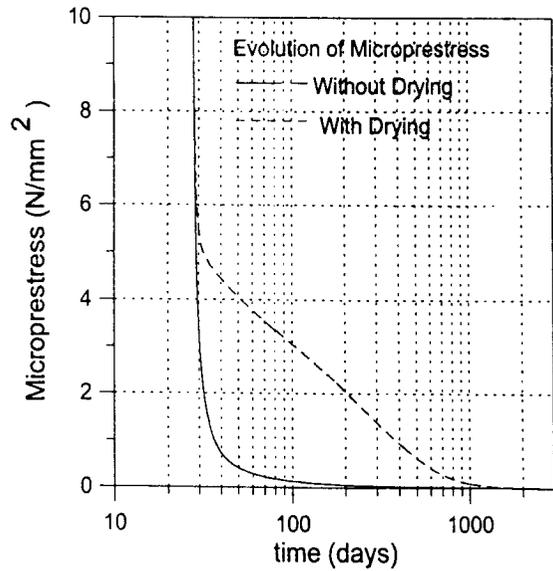


Fig. 8. Evolution of microstresses with and without drying

## 7 Conclusions

The introduction of the new concept of microprestress, assumed to be caused by large highly localized volume changes in the microstructure independent of loading and affected by capillary tension, surface tension and disjoining pressure has enabled the new formulation to be free of two shortcomings of the solidification theory. First, the long-term aging is now explained by the long-term relaxation of microprestress rather than volume growth of hydration products which practically ceases after one year. Second, the phenomenon of Pickett effect which was not addressed by solidification theory can now be explained in a unified manner by an increase in microprestress engendered by drying.

Good agreement with test data, as good as with the previous solidification theory, can be achieved with the proposed model. Cracking must of course be taken into account in simulating the tests, and the diffusion problem of drying must be solved.

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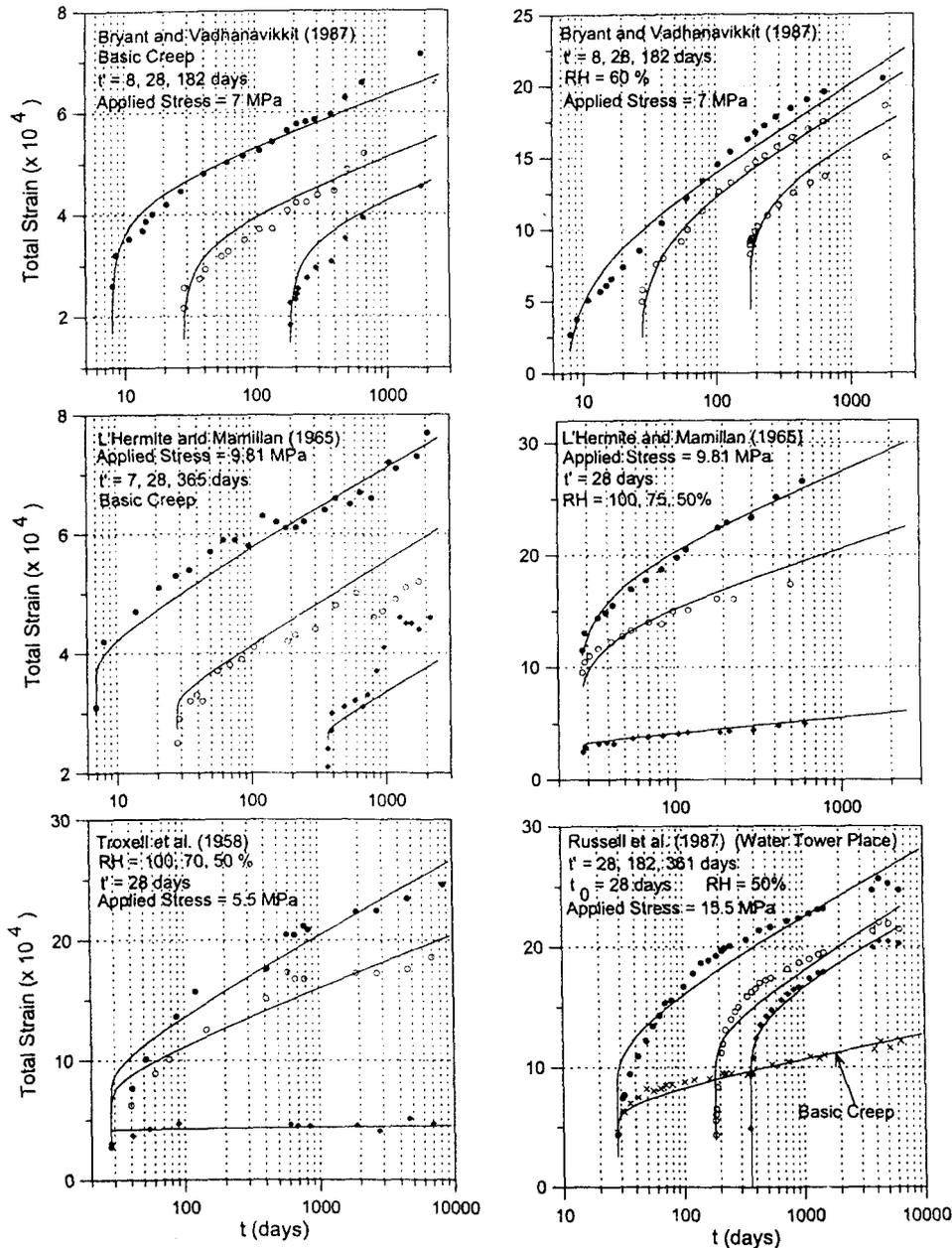


Fig. 10. Fits of some important test data for basic creep and total strain

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