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EDITED BY

**N. Banthia**

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**O.E. Gjrv**

*Norwegian Institute of Technology  
Trondheim, Norway*

UNIVERSITY OF BRITISH COLUMBIA  
Department of Civil Engineering  
Vancouver, BC Canada



# SPATIAL AND TEMPORAL SCALING OF CONCRETE RESPONSE TO EXTREME ENVIRONMENTS

Zdeněk P. Bažant<sup>1</sup> and Goangseup Zi<sup>2</sup>

<sup>1</sup> W.P. Murphy Professor, Department of Civil Engineering, Evanston, IL 60201

<sup>2</sup> Graduate Research Assistant, Northwestern University

## Abstract

The paper <sup>1</sup> reviews recent researches at Northwestern University dealing with (1) the intentional spalling of concrete by very rapid microwave heating, aimed at removing a concrete layer contaminated by radionuclides, and (2) the degradation of concrete by alkali-silica reaction or other expansive reactions. In view of space limitation, only the mathematical modeling of the former is described in some detail. The problem of scaling with the change of spatial dimensions and time scales, which is common to these and other situations of response to extreme environment, is discussed in the conference presentation, documented by numerical computations, but cannot be fit into the present article.

## 1. Introduction

The simulation or prediction of the response of concrete exposed to extreme hydrothermal environments or environments produced by various chemical processes typically involves a great variety of physical and chemical processes which need to be modeled mathematically. Depending on the particular situation at hand, some processes control the outcome while other have little importance and can be ignored. Which one can be ignored can be often be decided by simple scaling analysis of the mathematical model.

The conference presentation illustrates this approach by analysis of two important problems, particularly (1) the rapid heating of concrete, which is often considered in connection with fire exposure or hypothetical nuclear reactor accidents, but is here focused on the deliberate inducement of concrete spalling by microwave radiation, and (2) the alkali-silica reaction (ASR) in supply of water from the environment can

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engender volume expansion of silica containing particles leading to severe damage to concrete (Bažant et al. 2000a,b). These and other problems involve diffusion processes of heat, moisture, etc., constitutive models for pore water, elastic and inelastic deformation, creep, and fracture modeling.

Since the scaling aspects and the modeling of damage due to ASR are discussed in other recent papers (Bažant et al. 2000a,b), and since the space is limited, the present article describes only the mathematical model for the rapid microwave heating producing spallation of concrete. The experience gained in food engineering is exploited to develop a simple formula for computing the volumetric heat generation rate in concrete reinforced by steel bars, subjected to microwave radiation. This is done by averaging the Ohmic heat generation in space as well as time.

The discretization of the governing equations to be described at the conference and in a detailed journal article currently under preparation is carried out by the finite volume method in space, and by an implicit finite difference scheme in time. The conference presentation will give a number of numerical examples and experimental comparisons which are still in progress at the time of writing.

## 2. Model for Heat and Moisture Transport

Concrete has been a common materials at nuclear facilities. Unfortunately, a thin surface layer of vast areas of concrete surface has been contaminated at those facilities over many years of operation by nuclear waste. To guarantee safe long-time working environment, the contaminated surface layer must be removed from the concrete mass and disposed of as nuclear waste.

There are several possible techniques such as the use of hammer and chisel or high pressure water jet, and various kinds of aggressive thermal treatments. The present study is focused on rapid heating by microwave radiations and deals with the modeling of heat and moisture transfer. Both the heating by conduction from the surface and the volumetric heating by penetrating electromagnetic radiation are studied, but the present article is focussed on the microwave heating. The conductive heating has been extensively studied in many works, as it is relevant to fire resistance (Bažant and Thonguthai 1978, 1979; Ahmed and Hurst 1997; Gawin et. al 1999), and so have been the relevant thermodynamic properties of concrete (Harmathy 1970; Harmathy and Allen 1973; Neville, 1973; Bažant and Kaplan 1996; Vodák et. al 1997). Therefore the characteristic of temperature profiles and pore pressure profiles are relatively well known for the case of conductive heating from a hot surface. The studies of microwave heating have been more limited, confined mainly to the determination of temperature distributions (Li et al 1993; White et. al 1995). A linear heat conservation law was used in those models and the distributed heat source which was obtained by averaging in time domain.

Concrete is a kind of hygrothermal material whose mechanical behavior depends very strongly on temperature change as well as humidity change. The humidity change causes shrinkage and drying creep, and of course it affects the relative wa-

ter vapor pressure and capillary tension in the cement paste pores, which drive moisture transport. Therefore it is required to analyze not only the temperature profiles but also the pore pressure profiles. The moisture content, temperature and pore pressure are coupled thermodynamically, and so a coupled heat and moisture transfer model must be used. The basic framework of this model is inherited from the previous work (Bažant and Thonguthai 1978).

The equations of mass and heat conservation are

$$\frac{\partial w}{\partial t} + \nabla \cdot \mathbf{J} = I_w, \quad \frac{\partial}{\partial t} \{(wC_w + \rho C) T\} + \nabla \cdot \mathbf{q} = I_H \quad (1)$$

where  $w = \phi \rho_w$  = water content,  $\phi$  = porosity,  $\rho_w$  = specific mass of water,  $I_w$  = source of water,  $\mathbf{J}$  = water flux vector,  $C_w$  = specific heat of water,  $\rho$  = mass of concrete,  $C$  = specific heat of concrete,  $T$  = temperature,  $\mathbf{q} = \mathbf{q}_{cd} + \mathbf{q}_{cv}$  = total heat flux vector,  $\mathbf{q}_{cd}$  = conductive heat flux vector,  $\mathbf{q}_{cv}$  = convective heat flux vector, and  $I_H$  = source of heat. Although the general approach within the frame of irreversible thermodynamics is well known (Nield and Bejan 1992; Coussy 1994), the problem in its full detail is very complex and must, therefore, be simplified. The heat capacity of concrete  $C$  can be divided into the heat capacities of solid part  $C_s$  and of dehydration heat  $C_d$ . Further complications stem from  $C_s$ , which includes the heat of chemical conversion of various components of concrete during heating because concrete is a chemical mixture of many components. The complete description seems very complicated. However, in concrete, the latent heat effects in concrete appear less important if one takes account of the aggregates (Harmathy 1973). Therefore, the heat capacity of oven dried concrete is used as  $C$ , and the term  $wC_w$  term is dropped. The water flux  $\mathbf{J}$  and the conductive heat flux  $\mathbf{q}_{cd}$  are expressed as linear combinations of the gradient of water content  $w$  and the gradient of temperature  $T$ . If pore pressure is taken as the driving force, the cross effect of temperature on water flux can be dropped (Bažant and Kaplan 1996).

$$\mathbf{J} = -\frac{a}{g} \nabla P, \quad \mathbf{q}_{cd} = -k \nabla T \quad (2)$$

where  $a$  = permeability,  $g$  = gravity acceleration,  $P$  = pore pressure, and  $k$  = heat conductivity; (2), which is the classical Darcy's law, can be applied to non-saturated concrete provided that  $P$  is interpreted as the pressure of pore vapor (Bažant and Najjar 1972; Bažant 1975). The heat convection by water negligible. When concrete is heated, the water which is chemically bound becomes free and is released into the pores. This is reflected in the source term  $I_w$ . Below 100°C,  $I_w$  is negative due to hydration. The boundary conditions for the mass and heat transfer at the surface are  $\mathbf{n} \cdot \mathbf{J} = B_w(p_b - p_{en})$  and  $\mathbf{n} \cdot \mathbf{q} = B_T(T_b - T_{en})$  where  $\mathbf{n}$  = unit outward normal of the boundary surface,  $p_{en}$  = the partial pressure of the adjacent environment,  $T_{en}$  = the environmental temperature,  $p_b$  = surface partial pressure,  $T_b$  = surface temperature,  $B_w$  = surface emissivity of water, and  $B_T$  = surface emissivity of heat.

The formulation of the equation of state of pore water and of permeability is complex problem and because of space limitations the reader is referred to Bažant and

Thonguthai (1978, 1979) and Bažant and Kaplan (1996).

### 3. Model for Rapid Microwave Heating of Concrete

The approach to mathematical modeling of microwave heating and its effects on concrete will now be described, based on an internal research note by G. Zi (2001) at Northwestern University. The electromagnetic waves are governed by Maxwell equations which describe the evolution of the coupled electric and magnetic fields. The Maxwell equations can be decoupled since a dielectric source (ferromagnetic material) can be neglected (von Hippel 1954). The energy carried by electromagnetic waves is described by the Poynting vector  $\mathcal{P}$  (Cheng 1983);

$$-\int_S \mathcal{P} \cdot dS = -\int_S \mathbf{E} \times \mathbf{H} \cdot dS \quad (3)$$

$$= \frac{\partial}{\partial t} \int_V \left( \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right) dV + \int_V \sigma E^2 dV$$

$$= \frac{\partial}{\partial t} \int_V (w_e + w_m) dV + \int_V p_\sigma dV \quad (4)$$

which gives the power density transmitted by the electromagnetic wave;  $\mathbf{E}$  = electric field strength vector,  $\mathbf{H}$  = magnetic field strength vector,  $\epsilon = \epsilon' - i\epsilon''$  = complex dielectric permittivity,  $\epsilon'$  = dielectric permittivity,  $\epsilon''$  = dielectric loss,  $\mu = \mu' - i\mu''$  = complex magnetic permeability,  $\mu'$  = magnetic permeability, and  $\mu''$  = magnetic loss;  $\sigma$  = dielectric conductivity,  $w_e$  = electric energy density,  $w_m$  = magnetic energy density,  $p_\sigma$  = Ohmic power dissipation,  $t$  = time, and  $V$  = volume. If a medium is lossless ( $\sigma = 0$ ), the Ohmic power dissipation term vanishes. When a medium is exposed to microwave radiation, the last term describes the heat generation, i.e., the Ohmic heat dissipation in (4) and (4). For the present problem of microwave heating of concrete, it suffices to consider only transmitted electromagnetic waves that are parallel, with normal incidence at surface. Then the dielectric field depends only on the normal coordinate  $x$  (distance from concrete surface) and time  $t$ ;

$$\mathbf{E} = \mathbf{E}_0 e^{i\omega t - \gamma x}, \quad \mathbf{H} = \mathbf{H}_0 e^{i\omega t - \gamma x} \quad (5)$$

where  $\omega = 2\pi\nu$  = angular velocity,  $\nu$  = frequency,  $\gamma = i\omega\sqrt{\epsilon\mu} = \alpha + i\beta$  = complex propagation factor,  $\alpha$  = attenuation factor, and  $\beta$  = phase factor. A part of an electromagnetic wave is reflected at the interface of two media, and a part is transmitted as a refracted wave (which may be computed by Fresnel's equation, the reflection and refraction angle being given by Snell's law; von Hippel 1954). If the wave is incident to the interface of two dielectric media in normal direction, the wave forms a standing wave pattern and can be calculated simply from the continuity condition at each interface (Cheng 1983; Wait 1985). For calculations, one must distinguish strips orthogonal to the surface which do or do not include a steel bar. The wave solution in concrete may be written as

$$E_y = E_{0y} C_1 \left[ e^{-\gamma_1 x} + R_1 e^{\gamma_1 (x-2l)} \right], \quad (6)$$

$$H_z = \frac{E_{0y}}{\eta_1} C_1 \left[ e^{-\gamma_1 x} - R_1 e^{\gamma_1(x-2l)} \right] \text{ for } 0 < x < l$$

where  $\eta = \gamma/i\omega\epsilon = \sqrt{\mu/\epsilon}$  = intrinsic impedance (generally a complex number),  $C_1$  = magnitude factor, and  $R_1$  = effective reflection factor. From the continuity condition for each interface,

$$R_0 = \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1}, \quad R_1 = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad (7)$$

$$C_1 = \frac{C_0}{1 - R_0 R_1 e^{-2\gamma_1 l}} \text{ with } C_0 = \frac{2\eta_1}{\eta_0 + \eta_1} \quad (8)$$

At the interface of concrete with a steel bar, almost all of the wave is reflected since the intrinsic impedance of a metal is very small compared to concrete. So the reflection factor  $R_1 \approx -1$  (von Hippel 1954).

The actual solution can be taken as the real part of  $E$ . After some mathematical manipulations,

$$\text{Re}(E) = C' (m \cos \omega t + n \sin \omega t) - C'' (m \sin \omega t - n \cos \omega t) \quad (9)$$

$$m = e^{-\alpha x} \cos \beta x + e^{\alpha(x-2l)} (R_1' \cos \beta(x-2l) - R_1'' \sin \beta(x-2l)) \quad (10)$$

$$n = e^{-\alpha x} \sin \beta x - e^{\alpha(x-2l)} (R_1' \sin \beta(x-2l) + R_1'' \cos \beta(x-2l)) \quad (11)$$

where  $C_1 = C_1' + iC_1''$ ,  $R_1 = R_1' + iR_1''$ , and  $E_y$  is replaced by  $E$  for simplicity. The volumetric heat generation can be computed using (4) and (9). Because the frequency is so high that the propagation speed of a heating front is negligible, the heat generation may be averaged over the time period  $T$  (Li et al. 1993);

$$I_{ave} = \frac{1}{T} \int_0^{T=2\pi/\omega} \sigma [\text{Re}(E)]^2 dt \quad (12)$$

$$= \frac{1}{2} \sigma E_0^2 \|C_1\|^2 \left[ e^{-2\alpha x} + \|R_1\|^2 e^{2\alpha(x-2l)} + 2e^{-2\alpha l} \{R_1' \cos 2\beta(x-l) - R_1'' \sin 2\beta(x-l)\} \right] \quad (13)$$

The coefficients of  $R_1'$  and  $R_1''$  are the real part and the imaginary part of the reflection factor. If the wave is circularly polarized, the coefficient 1/2 in (13) disappears.

In a concrete wall, the electromagnetic wave is reflected and scattered mainly by steel reinforcing bars. In view of the heterogeneity of a concrete wall, one may take the spatial average of (13) over the wave number  $2\pi/\beta$ , where  $\beta$  is phase factor. The averaged heat generation rate is obtained as

$$I = \frac{1}{2} \sigma E_0^2 \|\bar{C}_1\|^2 \left[ e^{-2\alpha x} + \|R_1\|^2 e^{2\alpha(x-2l)} \right] \quad (14)$$

where

$$\bar{C}_1 = C_0 \left[ 1.0 + \frac{\beta}{2\pi\gamma_1} \ln \left\{ \frac{1 - R_0 R_1 e^{-2\gamma_1(l+\frac{\pi}{2})}}{1 - R_0 R_1 e^{-2\gamma_1 l}} \right\} \right] \quad (15)$$

$$\approx C_0 \left[ 1.0 + \frac{\beta}{2\pi\gamma_1} R_0 \exp(-2\gamma_1 l) \left\{ \exp\left(-\frac{2\pi\gamma_1}{\beta}\right) - 1 \right\} \right] \quad (16)$$

Note that if the reflection is not considered, (14) becomes the classical *Lambert's law*, widely used in food engineering;

$$I_{Lambert} = I_0 e^{-2\alpha x} \quad (17)$$

where  $I_0$  = heat generation rate on surface;  $\frac{1}{2}\sigma_0 E_0^2 \|\bar{C}_1\|^2$  in (14) is interpreted as the surface heat generation rate. Lambert's law is known to be inaccurate at high temperature because latent heat during evaporation of water is ignored (Komolprasert and Ofoli 1989). While the latent heat is important for food materials, which generally have a high water content, it is not important for concrete because the water content is much lower, usually less than 6%.

The heat generation in the unreinforced and reinforced strips of concrete wall may be averaged, which gives

$$I = pI_R H(d-x) + (1-p)I_U \quad (18)$$

where  $p$  = the area of rebar in unit area. By (7)–(8), and (14), the heat generation of each strip is expressed

$$I_R = \frac{1}{2}\sigma E_0^2 \|\bar{C}_R\|^2 \left[ e^{-2\alpha x} + \|\bar{R}_R\|^2 e^{2\alpha(x-2d)} \right] \quad \text{reinforced} \quad (19)$$

$$I_U = \frac{1}{2}\sigma E_0^2 \|\bar{C}_U\|^2 \left[ e^{-2\alpha x} + \|\bar{R}_U\|^2 e^{2\alpha(x-2L)} \right] \quad \text{unreinforced} \quad (20)$$

with

$$\bar{C}_R = \frac{C_0}{1 + R_0 e^{-2\gamma_1 d}}, \quad R_R = -1 \quad \text{reinforced}$$

$$\bar{C}_U = \frac{C_0}{1 - R_0^2 e^{-2\gamma_1 L}}, \quad R_U = R_0 \quad \text{unreinforced}$$

where  $H(x)$  = Heaviside step function.

The frequency of the microwaves considered for the decontamination process ranges from about 2.45GHz to about 10.4GHz (White et al. 1995). In this frequency range, the attenuation depth (or skin depth—the depth  $\alpha$  over which the field strength decays to 1/e of its original value) is very small compared to the typical wall thickness. Hence, the reflection from the opposite face of the wall can be neglected. With this simplification,

$$I = \frac{1}{2}\sigma E_0^2 \left[ \{p\|\bar{C}_R\|^2 H(d-x) + (1-p)\|\bar{C}_U\|^2\} e^{-2\alpha x} \right. \\ \left. + p\|\bar{C}_R R_R\|^2 e^{2\alpha(x-2d)} H(d-x) \right] \quad (21)$$

$$= I_{0U} \left[ \{(1-p) + p\rho H(d-x)\} e^{-2\alpha x} \right. \\ \left. + p\rho\|\bar{R}_R\|^2 H(d-x) e^{2\alpha(x-2d)} \right] \quad (22)$$

where  $\rho = I_{0R}/I_{0U}$ ,  $I_{0R}$  = surface heat generation when the medium 2 is a metal, and  $I_{0U}$  = surface heat generation without reinforcement.  $I_0$  could be measured experimentally to extrapolate this results obtained above to much more complicated real situation.

## References and Bibliography

1. Ahmed, G. N. and Hurst, J. P. (1997), "Coupled Heat and Mass Transport Phenomena in Siliceous Aggregate Concrete Slabs Subjected to Fire" *Fire and Materials*, 21, 161-168.
2. Bažant, Z. P. (1975), "Pore Pressure, Uplift, and Failure Analysis of Concrete Dams" *Proceedings, International Commission of Large Dams Symposium on Criteria and Assumptions for Numerical Analysis of Dams, Swansea, September 1975*, Swansea, University College, 781-808.
3. Bažant, Z. P. and Kaplan, M. F. (1996), *Concrete at High Temperatures*, Longman.
4. Bažant, Z.P., Zi, G., and Meyer, C. (2000a). "Fracture mechanics of ASR in concretes with waste glass particles of different sizes." *J. of Engrg. Mechanics* ASCE 126 (3), 226-232.
5. Bažant, Z.P., and Steffens, A. (2000b). "Mathematical model for kinetics of alkali-silica reaction in concrete." *Cement and Concrete Research* 30 (3), 419-428.
6. Bažant, Z. P. and Najjar L. J. (1972), "Nonlinear water diffusion in nonsaturated concrete" *Materiaux et constructions*, 5(25), 3-20.
7. Bažant, Z. P. and Thonguthai, W. (1978), "Pore Pressure and Drying of Concrete at High Temperature" *Journal of the Engineering Mechanics Division, Proceedings of the ASCE*, 104(EM5), 1059-1079.
8. Bažant, Z. P. and Thonguthai, W. (1979), "Pore Pressure in Heated Concrete Walls: Theoretical Prediction" *Magazine of Concrete Research*, 31(107), 67-76.
9. Celia, M. A., Bouloutas, E. T., and Zarba, R. L. (1990), "A General Mass-Conservative Numerical Solution for the Unsaturated Flow Equation" *Water Resources Research*, 26(7), 1483-1496.
10. Cheng, D. K. (1983), *Field and wave electromagnetics*, Addison-Wesley publishing company, London.
11. Christopoulos, C. (1995), *The transmission-line modeling method*, IEEE Microwave Theory and Techniques Society, Oxford University Press, Oxford.
12. Coussy, O. (1994), *Mechanics of Porous Continua*, translated by Ulm, F., John Wiley & Sons Ltd, NY.
13. Dolande, J. and Datta, A. (1993), "Temperature profiles in microwave heating of solids: a systematic study" *Journal of Microwave Power and Electromagnetic Energy*, 28(2), 58-67.
14. Gawin, D., Majorana, C. E. and Schrefler, B. A. (1999), "Numerical analysis of hygro-thermal behaviour and damage of concrete at high temperature" *Mechanics of Cohesive-Frictional Materials*, 4, 37-74.
15. Harmathy, T. Z. (1970), "Thermal Properties of Concrete of Elevated Temperature" *Journal of Materials*, JMLSA, 5(1), 47-75.
16. Harmathy, T. Z. and Allen L. W. (1973), "Thermal Properties of Selected Masonry Unit Concrete" *ACI Journal*, 70(15), 132-142.

17. Komolprasert, V. and Ofoli, R. Y. (1989), "Mathematical modeling of microwave heating by the method of dimensional analysis" *Journal of Food Processing and Preservation*, 13, 87-106.
18. Lagos, L. E., Li, W., Ebadian, M. A., White, T. L., Grubb, R. G., and Foster, D. (1995), "Heat transfer within a concrete slab with a finite microwave heating source" *International Journal of Heat and Mass Transfer*, 38(5), 887-897.
19. Li, W., Ebadian, M. A., White, T. L., and Grubb, R. G. (1993), "Heat transfer within a concrete slab applying the microwave decontamination process" *Journal of Heat Transfer Transactions of the ASCE*, 115, 42-50.
20. Neville, A. M. (1973), *Properties of Concrete*. A Halsted Press Book, John Wiley and Sons, New York, N. Y.
21. Ni, H. (1997), "Multiphase moisture transport in porous media under intensive microwave heating", ph. D. Thesis, Cornell University.
22. Ni, H., Datta, A. K., and Torrance, K. E. (1999), "Moisture transport in intensive microwave heating of biomaterials: a multiphase porous media model" *International Journal of Heat and Mass Transfer*, 42, 1501-1512.
23. Subedi, P. and Chatterjee, I. (1993), "Dielectric mixture model for asphalt-aggregate mixtures" *Journal of Microwave Power and Electromagnetic Energy* 28(2), 68-72.
24. Taoukis, P., Davis, E. A., Davis, H. T., Gordon, J. and Talmon, Y. (1987), "Mathematical modeling of microwave thawing by the modified isotherm migration method" *Journal of Food Science*, 52(2), 455-463.
25. Wait, J. R. (1985), *Electromagnetic wave theory*, Harper & Row, publishers, New York.
26. Meredith, R. J. (1998), *Engineer's handbook of industrial microwave heating*, The Institution of Electrical Engineers, London.
27. Metaxas, R. C. and Meredith, R. J. (1983), *Industrial microwave heating*, IEEE Power Engineering Series 4, Peter Peregrinus Ltd, Exeter.
28. Patankar, S. V. (1980), *Numerical Heat Transfer and Fluid Mechanics*, McGraw-Hill Book Company, NY.
29. Vodák, F., Černý, R., Drchalová, J., Hošková, Š., Kapičková, O., Michalko, O., Semerák, P., and Toman, J. (1985), "Thermophysical Properties of Concrete for Nuclear-Safety Related Structures" *Cement and Concrete Research* 27(3), 415-426.
30. von Hippel, A. R. (1954), *Dielectric Materials and Applications*, MIT press, Cambridge MT.
31. Wei, C. K., Davis, H. T., Davis, E. A., and Gordon, J. (1985), "Heat and mass transfer in water-laden sandstone: microwave heating" *AIChE Journal* 31(5), 842-848.
32. White, T. L., Foster D. Jr., Wilson, C. T., and Schaich, C. R. (1995), *Phase II Microwave Concrete Decontamination Results*, ORNL Report of contract DE-AC05-84OR21400.
33. Zeng, X. and Faghri A. (1994), "Experimental and numerical study of microwave thawing heat transfer for food materials" *Journal of Heat Transfer Transactions of the ASCE*, 116, 446-455.