

## Temperature Effect on Concrete Creep Modeled by Microprestress-Solidification Theory

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The recently developed microprestress-solidification theory on concrete creep and shrinkage is based on the current knowledge of the physical mechanisms involved. This theory was shown to provide optimal agreement with the full variety of the available experimental data on concrete creep at constant moisture content as well as shrinkage and creep at variable moisture content, their mutual coupling and the effects of aging included. The objective of the present paper is to extend this theory to creep and shrinkage at variable temperature. The mathematical formulation is outlined and selected comparisons with the test data from the literature are shown. A journal article presenting the extended theory in full is under preparation.

### 1. INTRODUCTION

It is becoming increasingly clear that errors in the prediction of creep and shrinkage effects in structures are often the cause of cracking that triggers various degradation processes. Accuracy in the modeling of these effects calls for a realistic mathematical model of the constitutive relation, which must be based on the knowledge of the intervening physical mechanisms. Such a mathematical model has recently been provided in the form of the microprestress-solidification theory. This theory was shown to provide fit a broader range of test data than the previous theories, and do so in a simpler manner. The theory, however, has been limited to creep and shrinkage at constant temperature.

The purpose of this brief article is to outline an extended theory that takes the effect of temperature variation into account, and to demonstrate some comparisons of this extended theory with test data. The extended theory will be formulated on the basis of the known physical nature of the thermal effects on various physical processes. A detailed presentation of the extended theory, with its full justification, is planned for a subsequent journal article.

### 2. REVIEW OF MICROPRESTRESS-SOLIDIFICATION THEORY

It is convenient to separate the total strain of concrete into several physical components as follows:  $\epsilon = \epsilon^i + \epsilon^{sv} + \epsilon^f + \epsilon^{sh} + \epsilon^T$  where  $\epsilon^i$  is the instantaneous strain,  $\epsilon^{sv}$  is the visco-

elastic strain,  $\epsilon^f$  is the purely viscous strain, and  $\epsilon^{sh}$ ,  $\epsilon^T$  are the strains caused by the changes of relative humidity  $h$  in the pores of concrete and of temperature, respectively.

The instantaneous strain can be considered to depend linearly on stress,  $\epsilon^i = q_1 \sigma$ . At room temperature  $T = T_0 = 23^\circ\text{C} = 296\text{K}$  and for a saturation state,  $h = h_0 = 1$ . Coefficient  $q_1$  can be considered as age independent, as has been verified by showing that the short-time creep curves for various ages at loading, plotted as a function of  $\theta^{0.1}$ , are straight and meet at  $\theta = 0$  at the same point,  $\theta$  being the load duration ([3]).

According to the solidification theory [7,8], the viscoelastic strain, which chiefly reflects the viscoelastic behavior of the C-S-H-gel particles, can be expressed in incremental form as

$$\dot{\epsilon}^{sv}(t) = \frac{\dot{\gamma}(t)}{v(t)}, \quad \gamma(t) = \int_0^t \Phi(t-\tau) \dot{\sigma}(\tau) d\tau \quad (1)$$

where  $\Phi(t-t') = q_2 \ln[1 + \xi^n]$ ,  $\xi = (t-t')/\lambda_0$  and  $v(t)^{-1} = (\lambda_0/t)^m + \alpha$ . The function  $v(t)$  represents the volume solidified material per unit volume of concrete, which grows during the hydration processes. The free parameters of the model are  $q_2$  (in  $\text{MPa}^{-1}$ ) and  $\alpha$  (dimensionless) while the others can be fixed as  $n = 0.1$ ,  $\lambda_0 = 1$  day, and  $m = 0.5$ .

Concrete also exhibits a purely viscous strain (flow strain), visualized by a dashpot, which is completely irrecoverable and is perceived as a result of shear slips at the interface between gel particles. The flow rate is

$$\dot{\epsilon}^f(t) = \frac{\sigma(t)}{\eta(S)} \quad (2)$$

The flow viscosity  $\eta(S)$  is a function of the so-called microprestress  $S$ ,  $1/\eta(S) = 2cS$  [4]. The microprestress represents a measure of the tensile peaks of the microstresses in the hardened cement paste microstructure. The prefix 'pre' refers to the fact that the microprestress  $S$  is a microstress field that is generated in the microstructure independently of the applied load and unaffected by it. The tensile microstresses develop in order to balance the disjoining pressures (and pressures analogous to crystal growth pressures) which act across the hindered adsorbed water layers in the cement paste microstructure. Creep consists of slips due to bond ruptures and restorations of the bonds at adjacent sites. The slips occur at creep sites which are the sites of the peaks of tensile microprestress.

The microprestress relaxes in time, but chemical volume changes and movements of hindered adsorbed water (or interlayer water) cause a build-up of the microprestress. According to [4], the evolution of the microprestress at each point of structure can be visualized by a Maxwell model;

$$\dot{S}(t)/C_S + S(t)/\eta(S) = \dot{\epsilon}_S \quad (3)$$

where  $\dot{\epsilon}_S$  is the variation in time of the total strain of the Maxwell model due to humidity and temperature changes.

### 3. TEMPERATURE AND HUMIDITY EFFECTS

A temperature rise has two physically different effects:

1. It accelerates the chemical processes of hydration, thereby accelerating the aging of concrete, which reduces the rate of the creep.
2. It increases the rate of bond breakages causing creep.

In general the second effect is predominant and the overall effect is an increase of creep. Nevertheless, for very young concretes, it can happen that a decrease of creep is observed. The pore humidity plays a similar role. The hydration rate and creep processes are both slowed down by a decrease of humidity.

It is possible to describe these two effects by introducing the concepts of equivalent time  $t_e$  and reduced time  $t_r$ , which characterize, respectively, the acceleration or deceleration of hydration reaction and creep process at the microstructural level. Following [2],

$$t_e(t) = \int_0^t \beta(\tau) d\tau, \quad \beta(t) = \beta_T(t) \beta_h(t) \quad (4)$$

$$\beta_h(t) = \left\{ 1 + [a - a_h(t)]^4 \right\}^{-1}, \quad \beta_T(t) = \exp \left\{ \frac{Q_h}{R} \left( \frac{1}{T_0} - \frac{1}{T(t)} \right) \right\} \quad (5)$$

and

$$t_r(t) = \int_0^t \psi(\tau) d\tau, \quad \psi(t) = \psi_T(t) \psi_h(t) \quad (6)$$

$$\psi_h(t) = \alpha_h + (1 - \alpha_h) h(t)^2, \quad \psi_T(t) = \exp \left\{ \frac{Q_v}{R} \left( \frac{1}{T_0} - \frac{1}{T(t)} \right) \right\} \quad (7)$$

where  $T$  = absolute temperature,  $T_0$  = reference temperature,  $h$  = humidity (relative vapor pressure) in the capillary pores of cement paste,  $Q_h$  and  $Q_v$  = the activation energies for the hydration and viscous processes respectively;  $T_0 = 296 K$ ,  $Q_h/R \approx 2700 K$ ,  $Q_v/R \approx 5000 K$ ,  $a = 5$  and  $\alpha = 0.1$  [2].

Eqs. 1 are valid only for  $h(t) = 1.0$  and  $T(t) = T_0$  while for general temperature and humidity history, according to the foregoing relations, we must write

$$\dot{\epsilon}^{ev}(t) = \frac{\dot{\gamma}(t)}{v [t_e(t)]}, \quad \gamma(t) = \int_0^t \Phi [t_r(t) - t_r(\tau)] \dot{\sigma}(\tau) d\tau \quad (8)$$

The temperature and humidity influence the rate of the viscous strain as well as the viscoelastic strain. Since the aging due to hydration is taken into account separately by  $v(t)$ , only the reduced time  $t_r$  is needed and Eqs. 2 and 3 may simply be rewritten as

$$\frac{d\epsilon^f}{dt_r} = \frac{\sigma(t)}{\eta(S)} \quad (9)$$

$$\frac{1}{C_S} \frac{dS}{dt_r S^2} + \frac{S}{\eta(S)} = \frac{d\epsilon_S}{dt_r S} \quad (10)$$

Since  $dt_r = \psi(t) dt$  we can set  $dt_{r,S} = \psi_S(t) dt$ . Taking also into account the expression of the viscosity in function of the microstress, equations 9 and 10 become

$$\dot{\epsilon}^f(t) = \frac{\psi(t)}{\eta(S)} \sigma(t) \quad (11)$$

$$\dot{S}(t) + \psi_S(t) \alpha_0 S(t)^2 = C_S \dot{\epsilon}_S(t) \quad (12)$$

The reduced time for the microstress evolution process  $\psi_S$  is defined according to Eqs. 6, 7 in which  $\alpha_h$  is replaced by  $\alpha_S$  and  $Q_v$  by  $Q_S$ . Analysis of the available experimental data has shown the values  $\alpha_S = 0.1$  and  $Q_S/R \approx 3000 K$  to be appropriate.

In Eq. 12, the rate  $\dot{\epsilon}_S$  on the right-hand side depends on the capillary tension, surface tension, crystal growth pressure and disjoining pressure [4], which are sensitive to temperature and humidity changes. Since the distances between micropores and capillary pores are very short we can assume that the thermodynamic equilibrium is maintained at any time. For that case, all these variables can be expressed in the form  $f(h, T) = f_0 - C_0 RT M^{-1} \ln h$  ([4]), and so we can set  $\epsilon_S = -C_1 RT M^{-1} \ln h + \epsilon_{S,1}$  and then  $\dot{\epsilon}_S = -C_1 R M^{-1} (\dot{T} \ln h + T \dot{h} h^{-1})$ . So the governing equation of the microstress problem is

$$\dot{S} + \psi_S \alpha_0 S^2 = -k_1 (\dot{T} \ln h + T \dot{h} h^{-1}) \quad (13)$$

where  $k_1 = C_S C_1 R/M$  (in  $MPa K^{-1}$ ). Equation 13 gives results that depend on the sign of  $\dot{T}$  and  $\dot{h}$ . In particular, negative variations of  $h$  (drying) and positive variations of  $T$  (heating) lead to an increase of the microstress peaks and so to an increase of the rate of creep flow. On the other hand, wetting ( $\dot{h} > 0$ ) and cooling ( $\dot{T} < 0$ ) would lead to a decrease of the existing microstress peaks and thus to a decrease of the rate of creep. Many experimental results, however, indicate an increase of creep to be caused by both drying and wetting [18],[16] (this a picture that is plausible but not absolutely certain since the test results have not yet been analyzed by finite elements taking into account nonhomogeneous stress and humidity states in the specimens). A more complex behavior has been evidenced for the effect of cooling. Many experimental results show that the creep increases under cooling [20],[15] while some others lead to the opposite conclusion [12],[13].

Let us consider this problem in the context of the microstress theory. The source of the flow component of creep is assumed to be the shear slips between the cement gel particles at overstressed 'creep sites', caused by breakages of bonds bridging the hindered adsorbed water layers (the bonds must of course be getting restored at adjacent sites because no decrease of elastic stiffness is observed). The rate process theory, based on Maxwell distribution of thermal energies of atoms, indicated that the frequency of breakages of such bridges must increase with an increase of tensile microstress.

Mathematically, this phenomenon is represented by the evolution of the scalar  $S(x, t)$  which characterizes the tensile microstress peaks. When cooling is reversed to heating or vice versa, the stresses at the previous microstress peaks get reduced and the corresponding microstress peaks get deactivated, however, new microstress peaks arise at different creep sites in the microstructure, which again tends to augment the creep rate. Thus it is reasonable to assume that the difference between heating and cooling is chiefly in the distribution of the creep sites but not in any drastic change in their total number. This explanation does not conflict with experimental observations, some of which show an increase and others a decrease of creep upon cooling. It is of course true that, when the temperature drops, the viscoelastic strain rate, controlled by activation energy, must always decrease (whether or not the aging due to hydration is completed). But the total

effect on the observed creep rate may be either a decrease or an increase depending on whether the activation energy effect or the microprestress effect prevails.

To take this behavior into account, an absolute value can be introduced into the right-hand-side of Eq. 13 [11];

$$\dot{S} + \psi_{Sc_0} S^2 = k_1 \left[ \dot{T} \ln h + T \dot{h} h^{-1} \right] \quad (14)$$

This equation leads to positive values of the microprestress  $S$  for any temperature-humidity history, while Eq. 13 can lead to meaningless negative values of  $S$  for certain evolutions of  $T$  and  $h$ .

#### 4. HYGROMETRIC AND THERMAL STRAIN

The free hygrometric strain (shrinkage or swelling) and free thermal strain are defined as the strains caused in a very small material element by changes of pore humidity and temperature at zero stress. Obviously, the hygrometric strain is an abstract property; in normal size specimens it cannot be observed directly because it is hardly possible to change pore humidity without producing self-equilibrated shrinkage stresses. Therefore, this property must be inferred indirectly, by inverse analysis of shrinkage data. The free hygrometric strain (shrinkage or swelling) is caused by the variations in capillary tension, surface tension and disjoining pressure induced by the movement of moisture into and out of the pores of the cement gel. Its rate may be expressed as  $\dot{\epsilon}^{sh}(t) = k_{sh} \dot{h}(t)$ . In general  $k_{sh}$  depends on  $h$  and  $\dot{h}$ , but as an approximation we can assume  $k_{sh} = \text{constant}$ .

The thermal strain rate is proportional to the temperature rate  $\dot{\epsilon}^T(t) = \alpha(T) \dot{T}(t)$  where  $\alpha(T)$  is the coefficient of thermal expansion. This coefficient can be estimated from the coefficients of thermal expansion of cement paste and aggregate. It depends on many factors but in general the influence of cement paste is quite small, and so the thermal expansion coefficient of concrete must be governed chiefly by the aggregates. Thus we can consider  $\alpha$  for concrete under moderate temperature to be approximately constant; the usual value is  $10^{-5} \text{ } ^\circ\text{C}^{-1}$ .

#### 5. ANALYSIS OF EXPERIMENTAL TEST DATA

To validate the model and calibrate the unknown material parameters, the relevant experimental data from the literature have been numerically simulated. The identification of parameters  $q_1$ ,  $q_2$ ,  $\alpha$  and  $q_4$  must be done by fitting the test data on basic creep at room temperature while parameters  $c$  and  $k_1$ , which govern the evolution of microprestress, must be identified from transient analysis in which temperature and humidity distributions vary in time.

The tests of the effect of temperature on basic creep have been analyzed first. After casting, the specimens have been protected against water loss, and for this reason  $h \approx 1$  (precisely,  $h = 0.98$  has been assumed, because of self-desiccation). Just before loading, the specimens were heated to the test temperature and then the evolution of strain under the load was recorded while keeping the temperature constant.

Fig. 1a, 1b, 1c and 1d show the compliance function  $J(t, t')$  for different sets of data. The comparison between experimental and numerical data shows a satisfactory agreement.

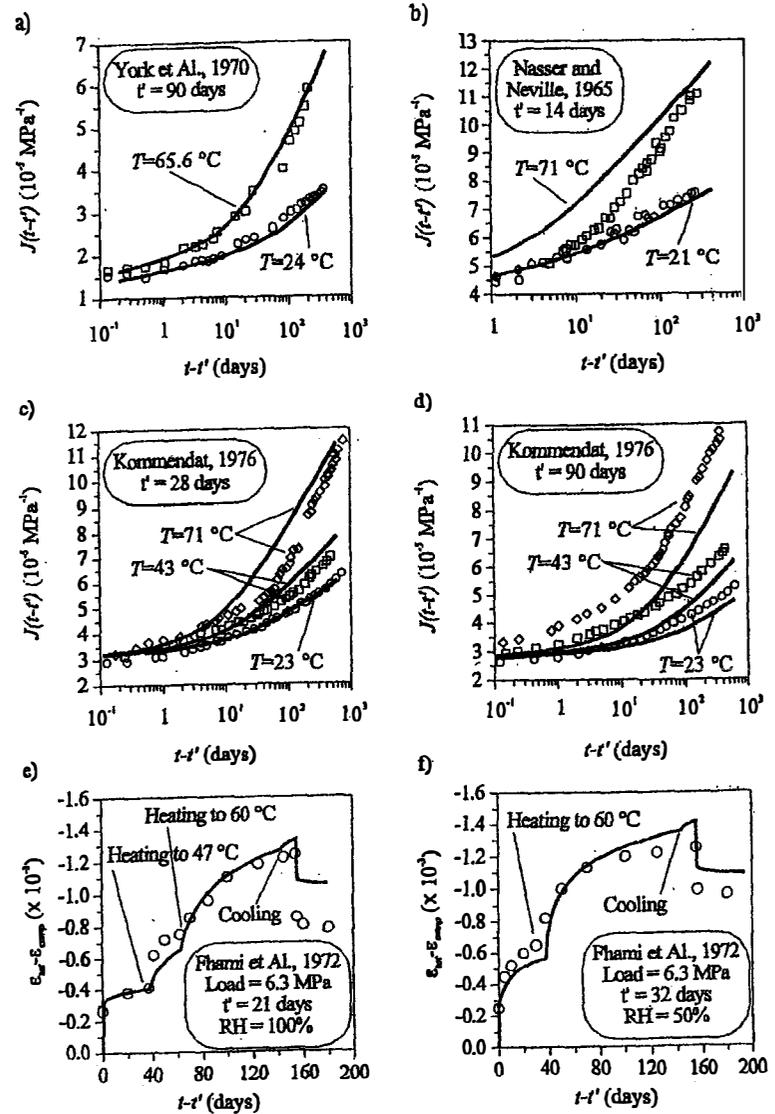


Table 1  
Values of parameters used for numerical simulations

	$q_1/10^{-5}$	$q_2/10^{-4}$	$\alpha$	$q_4/10^{-5}$	$c/10^{-3}$	$k_0$
York et Al, 1970	0.04	0.5	0.27	6.0	1.0	3.0
Nasser, 1965	1.5	0.8	0.3	5.0	1.0	3.0
Kommendat, 1976	2.0	0.7	0.08	7.0	1.0	3.0
Fhami, 1972	2.5	1.0	0.015	6.0	3.0	3.0

Consider now the transitional thermal creep tests in which the temperature varies while concrete is under load. The experimental data by Fahmi et al. [15] have been analyzed. Two different environmental histories have been used. In the first, temperature is varied while the water content is kept constant (which means that  $h$  decreases slightly due to selfdesiccation;  $h = 0.98$  is assumed). In the second, the specimen is allowed to dry at environmental relative humidity 50%. In these cases, the parameters  $c$  and  $k_1$  have an important effect. Their values have been optimized in order to get the best fit the experimental data. Figs. 1e and 1f compare the experimental and numerical results. They both show the difference between the total strain measured under load ( $\epsilon_{tot}$ ) and the total strain measured on a load-free companion specimen ( $\epsilon_{comp}$ ). Since the companion specimen normally suffers distributed cracking, due to the non-uniformity of temperature and humidity distributions inside the specimen, the simulation of cracking should be introduced into the model. For these particular tests, however, the effect of cracking can be considered negligible because the specimens were very small.

For all simulations the optimized parameters are reported in table 1.

## 6. CONCLUSIONS

1. The previous microprestress-solidification theory, already applied to drying creep, is here improved to capture also the effect of temperature changes. The physical cause of creep rate changes at variable temperature is the build-up of microprestress (microstress peaks at creep sites) as a result of temperature changes. The theory gives creep acceleration for both increasing and decreasing temperature, and also for both drying and wetting.
2. The relevant experiments reported in the literature have been numerically simulated and a satisfactory match of the test results has been achieved. The data in the literature, however, are very limited and further experimental verification and calibration will be needed, especially for extensions to high temperature.

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