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EXPERIMENTAL-ANALYTICAL SIZE-DEPENDENT PREDICTION OF MODULUS OF RUPTURE OF CONCRETE

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Abstract

The procedure for experimental-analytical prediction of flexural strength is reviewed. It allows consideration of size effect phenomena with only minor modification of standard laboratory test procedure. Two possibilities are suggested: testing with one size only and testing with two sizes. The application of this procedure to real case, a massive concrete subway tunnel constructed recently in Prague, is shown. This real case study demonstrates feasibility of the proposed approach.

Keywords: size effect, modulus of rupture, testing of concrete, flexural strength

1 Introduction

The recently accumulated extensive test data on the modulus of rupture, analytical studies and numerical simulations, all clearly indicate that the flexural strength of concrete, called the modulus of rupture, significantly decreases with increasing size of the beam, Bažant and Novák (2000a,b). The present paper describes a method to incorporate the size effect into the existing test standards. The proposed method is based on a recently established size effect formula that describes both the deterministic-energetic size effect caused by stress redistribution within the cross section due to finite size of the boundary layer of cracking near at the tensile face of beam, and the classical Weibull-type statistical size effect due to randomness of the local strength of material, Bažant and Novák (2000c), Bažant and Novák (2001). Two alternatives of the test procedure are formulated. In the first alternative, beams of only one size are tested, as in the current standards, and the size effect on the mean modulus of rupture is approximately predicted on the basis of the existing information for all concretes on the average. In the second alternative, beams of two sufficiently different sizes are tested. The latter is more tedious but gives a much better prediction of size effect for the concrete.

The aim of this paper is to present fundamentals of both alternative for practical usage and to show the application for size-dependent prediction of modulus of rupture of concrete in case of real concrete structure - large subway tunnel under Vltava river in Prague. The modification of standard testing procedure according to ASTM C78-94 and C293-94 has been recently proposed and the whole procedure can be naturally applied also for other standards for testing of plane concrete beams in three-point and four-point bending. Note, that testing of flexural strength in Czech laboratories follows ČSN-ISO 4013 and it is in principle same like above mentioned ASTM standards.

2 Energetic-Statistical Size Effect Formula

The concept of modulus of rupture is based on the elastic beam theory. The modulus of rupture, f_r, is defined as the maximum normal stress in the beam calculated from the maximum (ultimate) bending moment M_u under the assumption that the beam behaves elastically;

$$f_r = \frac{6\,M_u}{bD^2} \tag{1}$$

where D, b = beam depth and width. Except for the asymptotic case of an infinitely deep beam, the whole cross section of a concrete beam does not remain elastic up to the maximum load, and so f_r represents merely the nominal strength, $f_r = \sigma_N$, which is a parameter of the maximum load having the dimension of strength.

The inelastic behavior before the maximum load is caused by the development of a sizable boundary layer of cracking whose depth is approximately constant, dictated by the maximum aggregate size (Bažant and Planas 1998). The cracking causes energy release and stress redistribution which increases the moment capacity of the cross section. Since in a deeper beam the cracking layer occupies a smaller percentage of beam depth, there is less stress redistribution, and so the nominal strength decreases with an increasing beam depth. This represents a size effect.

The size effect on the modulus of rupture has been shown to follow the energetic-statistical formula (Bažant and Novák 2000c):

$$f_r = f_r^0 \left[\left(\frac{D_b}{D} \right)^{rn/m} + \frac{rD_b}{D} \right]^{1/r} \tag{2}$$

where f_r^0, D_b, r and m are positive constants, representing unknown empirical parameters; and n is the number of dimensions in geometric similarity, $n = 2$ or 3 (D_b has approximately the meaning of a boundary layer of cracking). Since r and m can be prescribed on the basis of the information on all concretes studied in the literature, there are only two parameters, namely f_r^0 and D_b, to be identified from tests of the given concrete. For this purpose, testing beams of only one size while ignoring the size effect, as currently specified in standards, is insufficient. One must either test beams of two sufficiently different sizes, or make a size effect correction based on prior knowledge.

Data fitting with the new formula (2) reveals that, for concrete and mortar, the Weibull modulus $m \approx 24$ rather than 12, the value generally accepted so far (Bažant and Novák 2000c). This means that, for extreme sizes, the nominal strength (modulus of rupture) decreases, for two-dimensional (2D) similarity ($n = 2$), as the $-1/12$ power of the structure size, and for three-dimensional (3D) similarity, as the $-1/8$ power (in contrast to the $-1/6$ and $-1/4$ powers that have generally been assumed so far). Fitting of the formula to the main test data sets available in the literature showed an excellent agreement, with a rather small coefficient of variation of errors of the formula compared to the test data. Furthermore, the new formula was verified numerically by the nonlocal Weibull theory (Bažant and Novák 2000a,b).

3 Experimental-analytical prediction of modulus of rupture

The entire procedure of the standard test method can be retained. Only the size effect consideration needs to be added. Two levels of size effect consideration are proposed: (1) Testing with only one specimen size and taking the size effect into account based on prior knowledge, and (2) testing with two specimen sizes. The latter is more accurate but involves more work. For both levels, the values $m = 24$, $r = 1.14$, $n = 2$ shown to be suitable for all concretes on the average (Bažant and Novák 2000c), should be used.

3.1 Testing with only one specimen size

1. When the ease of testing is important, one specimen size suffices; at least $D_1 = 76$ mm (3 in.), but better $D_1 = 305$ mm (12 in.). The uncertainty of the test results depends on the size selected, as numerically verified by nonlocal Weibull theory (Bažant and Novák 2000a,b). The scatter is much higher for smaller sizes; e.g, the coefficient of variation of deviation of the formula from test data, $\omega \approx 0.3$ for $D_1 = 76$ mm, while $\omega \approx 0.1$ for $D_1 = 305$ mm). Therefore, more specimens are desirable if the smaller size is used, but generally it is recommended that the number of specimens of one size should not be less than six.

2. Using the existing formula in standards , the modulus of rupture can be determined as the mean value, f_1 (in MPa), corresponding to the selected size D_1.

3. The parameter D_b of the size effect formula (2) is then approximately estimated as a function of the characteristic length l_0:

$$D_b \approx \delta_1 \, 10^{0.15+(l_0/l_1)}, \qquad \delta_1 = 1 \text{ mm}, \qquad l_1 = 53 \text{ mm} \tag{3}$$

The characteristic length l_0 is usually not known, and a rough estimate may then be obtained as.

$$l_0 \approx d_a \, (d_a/\delta_1)^{1/3}, \qquad \delta_1 = 1 \text{ mm} \tag{4}$$

where d_a is the maximum aggregate size in mm.

Justification of both formulas was given by Bažantand Novák (2000c) based on numerical results of nonlocal Weibull theory.

4. Knowing D_b, one can estimate

$$f_r^0 = f_1 \left[\left(\frac{D_b}{D_1} \right)^{rn/m} + \frac{rD_b}{D_1} \right]^{-1/r} \tag{5}$$

All the parameters of the energetic-statistical formula (2) for size-dependent prediction of modulus of rupture are thus determined. For any size D , modulus of rupture f_r can be easily calculated.

3.2 Testing with two specimen sizes

1. When more accurate results are desired, two specimen sizes need to be used, e.g., $D_1 = 76$ mm (3 in.) and $D_2 = 305$ mm (12 in.) Other sizes can be also used, but

one should note that the sizes selected must not be very close (such as $D_1 = 76$ mm and $D_2 = 100$ mm). If the sizes are not very different, the problem of identification of material constants tends to be ill-posed (Bažant and Li 1996, Planas et. al. 1995, Bažant and Planas 1998). The number of specimens should be chosen as already mentioned.

2. According to the existing formula in standards, the values of modulus of rupture are calculated for each individual size; f_1 for size D_1 and f_2 for size D_2.

3. The unknowns parameters f_r^0 and D_b of the size effect formula (2) are then solved from the following system of two nonlinear equations which ensues by writing (2) for $D = D_1$ and $D = D_2$ and solving f_r^0 from each:

$$f_r^0 = f_1 \left[\left(\frac{D_b}{D_1} \right)^{rn/m} + \frac{rD_b}{D_1} \right]^{-1/r} \tag{6}$$

$$f_r^0 = f_2 \left[\left(\frac{D_b}{D_2} \right)^{rn/m} + \frac{rD_b}{D_2} \right]^{-1/r} \tag{7}$$

Equating the last two expressions yields a formula for D_b;

$$D_b = \left[\frac{f_1^r \, D_1 \, D_2^p - f_2^r \, D_1^p \, D_2}{r \, (f_2^r \, D_2 - f_1^r \, D_1)} \right]^{1/p}, \quad p = 1 - \frac{rn}{m} \tag{8}$$

4. Parameter f_r^0 is then evaluated from (6) or (7). The energetic-statistical formula (2) for size-dependent prediction of the modulus of rupture is thus completely determined. For any size D, the modulus of rupture f_r can be easily calculated.

4 Application: Large concrete subway tunnel

4.1 General remarks

An extension of the subway system in Prague, Czech Republic, is crossing Vltava River. A couple of large concrete tunnels, which are curved in plan as well as in elevation, are cast in the dry dock excavated in the bank of the river. After casting of the first tube, it was launched into the trench excavated in the river bed. The procedure was repeated for the second tube and then the dry dock was used for erection of the definitive tunnels connected to the underwater parts. Each tunnel was cast in segments 12 m long. The total number of 14 segments formed a large tube 168 m long. The cross-section had the outside width 6.48 m and the height also 6.48 m. The thickness of walls and the top and bottom slabs was about 0.7 m. During the launching the tube was suspended in the one third of the length from its front on the pontoon and at the back end the tube was supported by hydraulic telescopic sliding shoes. The tube formed a simply supported beam with the span 112 m with overhanging cantilever 56 m long. The structure was designed to be watertight without any waterproofing. It was necessary to eliminate any cracking of concrete.

During the launching the tube was subjected to two actions; i) the self weight and ii) the bouyancy of the water. Both these actions exhibit a statistical scatter due to the deviations in dimensions and in the density of concrete. The actual load is a difference

of the two actions and therefore it is extremely sensitive to the variablity of them. It was essential to know the realistic strength of the tube in bending due the requirement of watertightness. The size effect plays an important role in the specification of the flexural strength of the tube and thus in assessment of the reliablity of the entire project. When the concrete strength (analysed for the dimensions of the tube) was known it was possible to determine the allowed load of the tunnel. The precise dimensions of the cross-section and the allowable tolerances were fixed according the resistance of the tunnel against cracking.

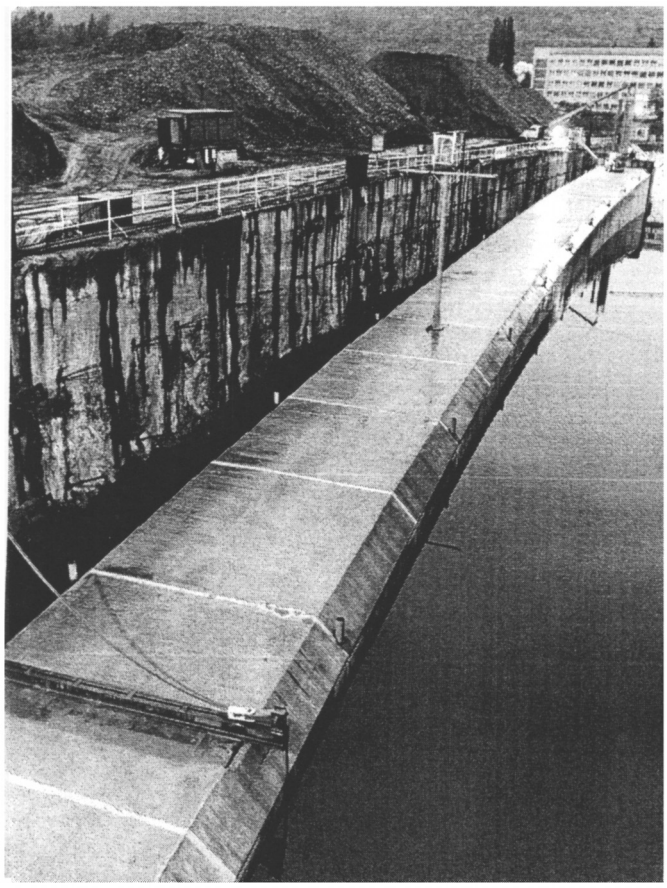

Figure 1 Subway tunnel on the bank of river

4.2 Flexural strength size-dependent prediction

Standardized test of modulus of rupture has been used, 8 beam specimens of one size D_1 = 100 mm in 3-point bending. This was made possible by the experimental-analytical procedure which permits one-size testing only. Basic statistical assessment of test data resulted in estimation of mean value f_1 = 7.444 MPa and standard deviation 0.3 Mpa. Based on maximum aggregate size d_a = 22 mm characteristic length l_0 can be roughly estimated using approximate heuristic formulae (4) (l_0 = 61.645 mm). Then parameters D_b and f_r^0 was calculated using (3) and (5) (D_b = 20.654 mm, f_r^0 = 6.875 MPa). Thus

the size effect formulae was determined, size effect curve is plotted in Figure 2. Modulus of rupture has been use for some simplified calculations of tunnel subjected to bending. From size effect curve in Figure 2 it was confirmed that for maximum geometry 6.48 m modulus of rupture corresponds to value 4.28 Mpa. This value has been found satisfactory. Nevertheless this value is significantly smaller than the value obtained from tests (7.444 Mpa).

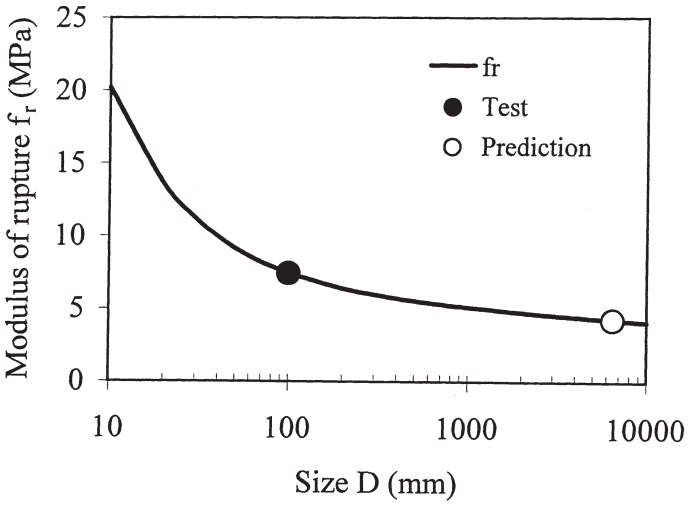

Figure 2 Size effect curve for concrete of tunnel

Conclusions

1. It is proposed to incorporate size effect for the standard modulus of rupture testing. The proposed method includes both the deterministic (energetic) and statistical size effects. Two alternatives of the test procedure are formulated.

2. The approach has been used for practical problem - flexural strength prediction of large concrete subway tunnel constructed recently in Prague.

3. To make the determination of the parameters of energetic-statistical size effect formula particularly easy, a spreadsheet form, available from the authors, has been developed using the standard Microsoft Office package employing the Microsoft Excel. All that one needs to do is to open the file with the name FSCtest.xls and type input parameters . The output parameters (parameters of the size effect formula) are calculated automatically. The illustrative size effect figures are plotted automatically when the input parameters are changed, the important points being highlighted.

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