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Energetic and Statistical Size Effects in Fiber Composites and Sandwich Structures: A Précis of Progress

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Abstract: *The conference lecture gives an overview of the problems of scaling and size effect in solid mechanics, which have not come to the forefront of attention until late in the last century. The classical view that any observed size effect was statistical was reversed during the 1980s. As is now widely accepted, quasibrittle materials including concrete, rock, tough ceramics, sea ice, snow slabs and composites exhibit major size effects on the mean structural strength that are deterministic in nature, being caused by stress redistribution and energy release associated with stable propagation of large fractures or with formation of large zones of distributed cracking. The lecture begins by reviewing the general asymptotic properties of size effect implied by the cohesive crack model or crack band model, and highlights the use of asymptotic matching techniques as a means of obtaining scale-bridging size effect laws representing a smooth transition between two power laws. Attention is focused on size effects observed in fiber-polymer composites failing either by tensile fracture or by propagation of compression kink bands with fiber microbuckling. The size effects in polymeric foams and sandwich structures are also discussed. A nonlocal model for incorporating the Weibull-type statistical size effect due to local strength randomness into the energetic size effect theory is outlined next, and the predictions of the combined nonlocal energetic statistical theory are compared to experimental evidence. Finally, a new probabilistic analysis of the size effect on the statistical distribution of nominal strength of structures is presented and discussed from the viewpoint of the extreme value statistics. In closing, some implications for the design of hulls, bulkheads, decks, masts and antenna covers for very large ships, and for the design of large load-bearing aircraft fuselage panels, are pointed out.*

Because of space limitation, the compact article which follows[2] summarizes only one of the ideas covered in the conference lecture—a new mathematical model for the size effect on the probability distribution of nominal strength of quasibrittle structures.

The recently developed and experimentally verified nonlocal generalization of Weibull statistical theory is taken as the basis of analysis, and there is not enough room to review here the concepts; see Bažant and Xi (1991), Bažant and Novák (2000a,b,c), Bažant (2001b) (2002). Considering the nonlocal averaging domains in a nonlocal model of a structure to be analogous to the links of a chain, the failure probability of a structure is:

$$P_f = 1 - \exp\left(-\int_V \left\langle \frac{\hat{\sigma}(\boldsymbol{x})}{s_0} \right\rangle^m \frac{\mathrm{d}V(\boldsymbol{x})}{V_r}\right) \tag{1}$$

Here V = volume of structure, m = Weibull modulus, s_0 = scaling parameter, $\sigma(\boldsymbol{x})$ = maximum principal stress at coordinate vector $\boldsymbol{x}$, $\hat{\sigma}$ = nonlocal stress, V_r = representative volume of material; $\langle x \rangle$ = max(0,x); and superior ˆ denotes nonlocal quantities. Introduce dimensionless coordinates and size-independent variables:

$$\boldsymbol{x} = D\boldsymbol{\xi}, \quad V_0 = l^n, \quad V = l^n v, \quad \mathrm{d}V(\boldsymbol{x}) = l^n \mathrm{d}v(\boldsymbol{\xi}), \quad \sigma(\boldsymbol{x}) = \sigma_N S(\boldsymbol{\xi}) \tag{2}$$

where D = structure size; n = number of spatial dimensions (n = 1, 2 or 3); l = characteristic length of material; $V_r = l^n$; $\boldsymbol{\xi}$ = dimensionless coordinate vector; and $\sigma_N = P/bD$ = nominal strength of structure (P = maximum load, b = structure width. Consider geometrically similar structures of different sizes D, for which the corresponding points have the same dimensionless coordinate $\boldsymbol{\xi}$; then

$$-\ln(1 - P_f) = \left(\frac{\sigma_N}{s_0}\right)^m \left(\frac{D}{l}\right)^n \int_v \langle S(\boldsymbol{\xi})\rangle^m \mathrm{d}v(\boldsymbol{\xi}) \tag{3}$$

The influence of structure geometry on the size effect is here delivered by means of function $S(\boldsymbol{\xi})$.

We restrict attention to large enough structures such that the nonlocal averaging domain, roughly of the same size as the fracture process zone (FPZ), is small compared to D. The nonlocal stress

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$\hat{\sigma}$ within the zone of localized damage (distributed cracking) may be assumed to be approximately uniformly distributed and equal to the elastically calculated stress that existed at the center of this zone before the stresses have redistributed due to damage; approximately,

$$\hat{\sigma} \approx \sigma_N \left(S_{max} - gS' \frac{l}{D} \right) \tag{4}$$

where S_{max} = maximum within the structure before cracking damage occurs; S' = magnitude of the dimensionless gradient of S at the maximum stress point in the direction toward the FPZ center; and g = geometry factor ($S' \neq 0$ assumed). Further we may assume the nonlocal stress to be uniform throughout the FPZ and equal to (??). This is the simplest way to capture the effect of stress redistribution (and the corresponding energy release) caused by FPZ formation. Based on this simplifying idea, the integration in (??) can be subdivided into two domains, domain $\mathcal{Z}$ of the FPZ and domain $\mathcal{R}$ of the rest of the structure volume, and so (??) provides:

$$-\ln(1-P_f)\left(\frac{s_0}{\sigma_N}\right)^m \left(\frac{l}{D}\right)^n = \int_{\mathcal{Z}} \left\langle S_{max} - gS'\frac{l}{D} \right\rangle^m \mathrm{d}v(\boldsymbol{\xi}) + \int_{\mathcal{R}} \langle S(\boldsymbol{\xi})\rangle^m \mathrm{d}v(\boldsymbol{\xi}) \tag{5}$$

$$= \left\langle S_{max} - gS'\frac{l}{D} \right\rangle^m v_{\mathcal{Z}} + S_{\mathcal{R}}{}^m \tag{6}$$

where $v_{\mathcal{Z}} = \int_{\mathcal{Z}} \mathrm{d}v(\boldsymbol{\xi})$, $S_{\mathcal{R}}{}^m = \int_{\mathcal{R}} \langle S(\boldsymbol{\xi})\rangle^m \mathrm{d}v(\boldsymbol{\xi})$; $v_{\mathcal{Z}}$ = dimensionless FPZ volume for $D = l$.

For sufficiently large sizes,

$$-\ln(1-P_f)\left(\frac{s_0}{\sigma_N}\right)^m \left(\frac{l}{D}\right)^n = v_Z\, S_{max}{}^m \left(1 - \frac{gS'}{S_{max}}\frac{l}{D}\right)^m + S_R{}^m \tag{7}$$

$$\approx v_Z\, S_{max}{}^m \left(1 - m\frac{gS'}{S_{max}}\frac{l}{D}\right) + S_R{}^m \tag{8}$$

$$= S_0{}^m \left(1 - m\kappa\frac{l}{D}\right) \approx S_0{}^m \left(1 - \kappa\frac{l}{D}\right)^m \tag{9}$$

where $S_0 = (v_Z S_{max}{}^m + S_R{}^m)^{1/m}$, $\kappa = v_Z g S' S_{max}{}^{m-1}/S_0{}^m$.

Because of (??), only the first two terms of the asymptotic series expansion σ_N in terms of powers of $1/D$ can be expected to be realistic. Therefore, any other approximation that shares the same first two terms of this expansion is equally valid. We exploit this fact to find an approximation that also exhibits realistic asymptotic properties for $D \to 0$, which consist of a finite positive small-size limit σ_N $D \to 0$ (which should be approached linearly as $D \to 0$). Matching these properties, we can find asymptotic matching approximation. For small enough l/D, $1 - \kappa\frac{l}{D} \approx [1 + r\kappa(l/D)]^{-1/r}$ where r = empirical positive constant. This approximation is second-order accurate in l/D, which is verified by setting $\zeta = r\kappa l/D$ and $q = -1/r$, and noting the binomial series expansion $(1+\zeta)^r = 1 + q\zeta + q(q-1)\zeta^2/2! + q(q-1)(q-2)\zeta^3/3! + \ldots$. For the sake of matching the small-size asymptotic properties of the cohesive crack model, we may further set: $l/D \approx l/(\eta l + D/$ where η = empirical coefficient of the order of 1. Since, for $D \gg l$, $l/(\eta l + D) \approx l/D$, the first two large-size asymptotic terms remain unaffected by the foregoing approximation. Eq. (??) now provides $P_f = 1 - \mathrm{e}^{-(\sigma_N/s_1)^m}$ where

$$s_1 = \frac{s_0}{S_0}\,\theta^{n/m}\,(1 + r\kappa\theta)^{1/r}, \qquad \theta = \frac{l}{\eta l + D} \tag{10}$$

The foregoing approximations have not affected the first two asymptotic terms of the series expansion of σ_N in terms of the powers of $1/D$, while at the same time the value of $\lim_{D\to 0}\sigma_N$ is finite and $-\infty < \lim_{D\to 0}(\mathrm{d}\sigma_N/\mathrm{d}D) < 0$, as required by the cohesive crack model or crack band model.

Similar to the classical Weibull theory, the size effect law for the mean nominal strength:

$$\overline{\sigma}_N = \int_0^1 \sigma_N \mathrm{d}P_f = s_1\,\Gamma\left(1 + \frac{1}{m}\right) \tag{11}$$

where s_1 = function of D. The standard deviation and the coefficient of variation are:

$$\delta_N{}^2 = \int_0^1 (\sigma_N - \bar{\sigma}_N)^2 \mathrm{d}P_f = s_1{}^2 \Gamma\left(1 + \frac{2}{m}\right) - \bar{\sigma}_N{}^2; \quad \omega_N = \frac{\delta_N}{\bar{\sigma}_N} = \sqrt{\frac{\Gamma(1+2/m)}{\Gamma^2(1+1/m)} - 1} \quad (12)$$

It is noteworthy that, according to the large-size asymptotic approximations made, ω_N is independent of D, similar to the classical Weibull theory.

In analogy to the classical Weibull theory, the mean and standard deviation of σ_N are given by:

$$\bar{\sigma}_N = \int_0^\infty \sigma_N \mathrm{d}P_f(\sigma_N) = s_D \Gamma(1+1/m), \quad \delta_N{}^2 = \int_0^\infty \sigma_N{}^2 \mathrm{d}P_f(\sigma_N) - \bar{\sigma}_N{}^2 = s_D{}^2 \Gamma(1+2/m) - \bar{\sigma}_N{}^2 \quad (13)$$

Accordingly, the coefficient of variation, ω_N, of σ_N (for $D \gg l$) is given by

$$\omega_N^2 = \frac{\delta_N{}^2}{\bar{\sigma}_N{}^2} = \frac{\Gamma(1+2/m)}{\Gamma^2(1+1/m)} - 1 \quad (14)$$

We see that, if $D \gg l$ then ω_N is asymptotically independent of structure size D, which is the same as in the classical Weibull theory.

When D/l is not large enough, (??) with (??) is doubtless invalid because the failure of small structures containing a cohesive crack or crack band approaches, for $D \to 0$, the case of elastic body with a plastic crack. In that case the failure must be simultaneous along the entire failure surface, rather then propagating. Because all the bonds in the microstructure are being severed almost simultaneously, the failure probability should obey not the weakest-link model but Daniels' (1945) 'fiber-bundle' (parallel coupling) model, for which the distribution of nominal strength converges, for $N \to \infty$, to a gaussian distribution having a mean that is asymptotically independent of N and a the coefficient of variation that decreases as $1/\sqrt{N}$. In our problem, N may be considered analogous to D. Therefore, the size effect on mean σ_N should asymptotically vanish for $D \to 0$ and the coefficient of variation of σ_N should asymptotically decrease as $1/\sqrt{D}$.

For D varying from 0 to ∞, we may expect a continuous transition from Gaussian distribution of fiber bundle model to Weibull distribution of weakest link model. Theoretically this is a difficult problem, and we will treat it by approximate asymptotic matching. To this end, we introduce the inverse of the cumulative probability distribution: $\sigma_N(P_f) = \bar{\sigma}_N(D)\Phi(P_f, D)$. For $D/l \to \infty$, function Φ represents the inverse Weibull distribution:

$$\Phi(P_f, D) = \Phi_W(P_f) = [-\ln(1 - P_f)]^{1/m} \qquad (D/l \to \infty) \quad (15)$$

The inverse of the Gaussian distribution for the small size limit may be written as $\Phi(P_f, D) = \Phi_G(P_f, D) = 1 + \Psi^{-1}(P_f)\sqrt{D/\chi l}$. Subscripts $_G$ and $_W$ stand for 'gaussian' and 'Weibull'; $\Psi^{-1}(P_f)$ is the inverse of the unit cumulative gaussian distribution the mean of which is zero and standard deviation is 1; and χ is a dimensionless empirical constant of the order of 1. The following asymptotic matching approximation is suggested (Fig. ??):

$$\Phi(P_f, D) = \frac{(\eta l)^u \Phi_G(P_f, D) + D^u \Phi_W(P_f)}{(\eta l)^u + D^u} \quad (16)$$

where u = empirical constant (the value $u = 1/2$ would seem logical since it would exactly cancel the size dependence of the coefficient of variation of Daniels' fiber bundle model dominating for $D/l \to 0$).

With this, the objective of outlining a coherent formulation for size effect on the entire probability distribution of nominal strength, consistent with fracture mechanics, has been completed.

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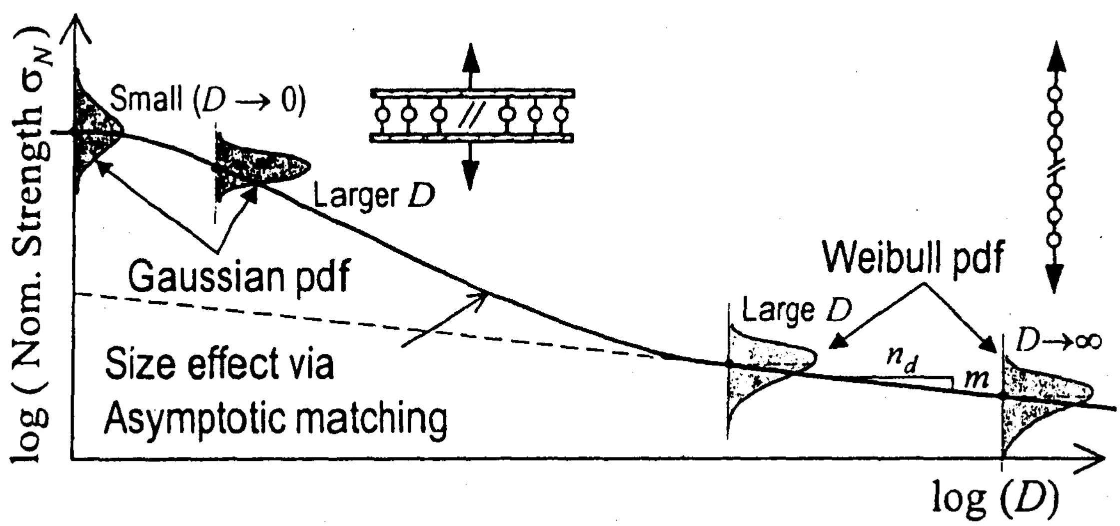

Fig. 1 Mean size effect curve of quasibrittle structure of positive geometry, containing no notch and no pre-existing crack, and evolution of the probability distribution of nominal strength with structure size.