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Seismic analysis of an earth dam based on endochronic theory

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ABSTRACT

A two-dimensional finite element program using triangular plane strain elements has been developed to determine the stress and displacement fields that result in an earth dam subjected to earthquake loading. The governing equations are derived from the principle of virtual work, and they account for the existence of two phases (solid and fluid) with coupling and damping due to relative accelerations and velocities. Endochronic (nonlinear inelastic) and linear elastic constitutive relationships are used to describe the soil behavior, and a step-by-step integration in time is performed by an implicit method. Two different earthquake acceleration records with approximately the same maximum acceleration were employed in the endochronic analysis. Calculated results demonstrate the importance of utilizing appropriate constitutive relationships and the effect caused by the nature of the input earthquake record.

Endochronic theory and the concept of soil as a two-phase medium are combined to conduct a dynamic finite element analysis of an earth dam. Calculations are made (a) with both endochronic and linear elastic constitutive relationships using a modified version of the N-S component of the El Centro accelerogram from the 1940 earthquake and (b) with an endochronic constitutive relationship using a deconvoluted and enriched version of the Pacoima accelerogram from the 1971 San Fernando earthquake. Treatment of the soils as two-phase media accounts for the coupling between phases and allows the stress distribution to be determined in both the solid and fluid phases; as such, it is a more accurate and realistic representation of the actual behavior of soils. Virtually all of the available models that have been applied to this problem are based on the concept of a one-phase medium, and the pore pressure response is determined explicitly.

ENDOCHRONIC CONSTITUTIVE LAW FOR SOILS

Meaningful advances in our analytical ability to solve field problems in soil dynamics requires, among other things, the formulation of more realistic and generally applicable constitutive relationships for soils. Toward this end endochronic theory, which was first applied to describe the mechanical behavior of metals (24), has been extended to characterize the response of concrete (3) and soils (2, 4, 11, 15). In endochronic theory the notion of an intrinsic time scale is formulated to account for the independent dissipative effects of accumulated strain and external time. The intrinsic time parameter, z, depends on external time (strain rate) only for rate-dependent materials, such as cohesive soils, and it should be a monotonically increasing function of strain and external time. Assuming that the development of inelastic strains is gradual, the intrinsic time increment, dz, can be expressed as a function of the strain increment, de_{ij}, and the external time increment, dt, as

$$(dz)^2 = \frac{d\zeta}{z_1} + \left[\frac{dt}{\tau_1}\right]^2 \quad (1a)$$

$$d\zeta = F(\underset{\sim}{\epsilon},\underset{\sim}{\sigma},\zeta)d\xi \quad (1b)$$

$$d\xi = \sqrt{\tfrac{1}{2}\, de_{ij}de_{ij}} \quad (1c)$$

where z_1 and τ_1 are material constants; $e_{ij} = \epsilon_{ij} - \delta_{ij}\epsilon_{kk}$ is the deviator of the strain tensor, ϵ_{ij}; F is the strain hardening-softening function; and ξ is the distortion measure.

Assuming that the source of inelasticity in soils is the irreversible rearrangement of grain configurations associated with deviatoric strains, it is convenient to characterize the accumulation of rearrangements by an appropriate variable, termed the rearrangement measure. Since the increments of irreversible (inelastic) strain are caused by interparticle rearrangements, they must be proportional to increments of the rearrangement measure, and the proportionality coefficient may, in general, depend on the state of strain and stress. However, it has been observed in both quasi-static and cyclic tests on soils (2, 15) that this irreversible rearrangement diminishes after a certain point. In order to express this phenomenon, it is more suitable to consider the strain hardening-softening relationships as a composite function, such as

$$F(\underset{\sim}{\epsilon},\underset{\sim}{\sigma},\zeta)d\epsilon = \frac{d\eta}{f(\eta)} \quad (2a)$$

$$d\eta = F_1(\underset{\sim}{\epsilon},\underset{\sim}{\sigma})d\xi \quad (2b)$$

where the parameter η assures the continuous accumulation of inelastic strains and $f(\eta)$ serves as a limiting function that incorporates the critical state concept.

Most soils also manifest inelastic volumetric strains, termed densification or dilatancy, as a result of shear. It is assumed that the inelastic volume change is due only to the existence of shear, and plastic volumetric strains due to changes in the hydrostatic stress (that is, consolidation) are excluded from the formulation. The densification-dilatancy measure, λ, can be expressed in the following differential form as a function of the stress and strain invariants and the accumulated value of λ:

$$d\lambda = L(\underset{\sim}{\epsilon},\underset{\sim}{\sigma},\lambda)d\xi \quad (3)$$

where the rate dependence of the inelastic volume change can be included in the formulation, as in the case of intrinsic time, thereby leading to

$$(d\lambda)^2 = (d\bar{\lambda})^2 + (\sigma_c dt/\tau_2)^2 \quad (4)$$

where τ_2 is a constant material parameter and σ_c is the consolidation stress. The use of internal variables, such as the rearrangement measure and the densification-dilatancy measure with rate dependence, is equivalent (at least conceptually) to approximating on a macroscopic level the changes in the microscopic state of the material. The intrinsic functions, such as F and L, must be determined separately for different materials; this must be done in a semi-empirical manner based on some selected set of data.

As treated herein, the soils are assumed to be incrementally isotropic and statistically homogeneous on a sufficiently large scale. As a consequence, the incremental stess-strain relations may be expressed separately in terms of the deviatoric and volumetric components of the stress and strain tensors, such that

$$de_{ij} = \frac{ds_{ij}}{2G} + \frac{s_{ij}}{2G}dz \quad (5a)$$

$$d\epsilon = \frac{d\sigma'}{3K} + d\lambda \quad (5b)$$

where $s_{ij} = \sigma'_{ij} - (1/3)\,\delta_{ij}\sigma'_{kk}$ is the deviator of the effective stress tensor; σ'_{kk} is the effective volumetric stress; and G and K are the elastic shear and bulk moduli, respectively. For cohesionless soils the approach described in References 4 and 11 is adopted to formulate the specific internal functions, while for cohesive soils the relationships suggested in References 2 and 15 are used.

TWO-PHASE MEDIUM APPROACH

Biot (6, 7) proposed a set of relationships to model the two-phase medium response of a soil element under quasi-static and dynamic loading conditions. This approach was based on the assumptions that (a) soils are isotropic, (b) the stress-strain relations are linear and elastic under final equilibrium conditions, (c) the pore fluid is compressible and has negligible shear resistance, (d) Darcy's law is valid, and (e) the strains are small. Since soils exhibit nonlinear behavior in nearly all stages of loading, the assumption of linear elasticity is not actually valid. However, the assumption that the pore fluid is compressible introduces a significant improvement over Terzaghi's classical consolidation theory and provides a more realistic explanation for pore pressure development, as well as some explanation for the dynamic behavior of soils. The assumption of isotropy is useful to achieve mathematical simplicity in expressing the constitutive relationships in deviatoric and volumetric forms. Based on these assumptions, the incremental stress-strain relations for a nonlinear elastic two-phase system may be expressed as (9)

$$d\sigma_S = P\, d\epsilon_S^{e\ell} + Q\, d\epsilon_F^{e\ell} \tag{6a}$$

$$d\sigma_F = Q\, d\epsilon_S^{e\ell} + R\, d\epsilon_F^{e\ell} \tag{6b}$$

$$d\tau_S = G d\gamma_S^{e\ell} \tag{6c}$$

where subscripts S and F denote solid and fluid phases, respectively; σ designates volumetric stresses; $\epsilon^{e\ell} = (\epsilon^{e\ell})_{kk}$ are the elastic volumetric strains; and τ_S and γ_S are the shear stress and strain in the solid phase. Equations (6) are valid under any circumstances; however, the elastic coefficients (P, Q, R, and G) are of paramount importance in the practical application of the theory, and they require a more detailed analysis.

Based on the ideas advanced by Biot (7) and Ishihara (13), a numerical procedure for calculating the values of the coefficients P, Q, and R has been developed (5). Two sets of formulas (one "exact" and the other "approximate") were proposed; this study will utilize the "approximate" relations, which can be expressed as

$$Q = \frac{1-n}{C_w};\quad R = \frac{n}{C_w};\quad P = \frac{1}{C_b} + \frac{Q^2}{R} \tag{7}$$

where n is the porosity of the soil; C_w is the compressibility of the water; and C_b is the drained bulk compressibility of the solid skeleton. For all practical purposes, Equations (7) constitute a realistic approximation for the case where soil particles are considered incompressible.

Once values for the four elastic coefficients (P, Q, R, and G) are determined, the elastic response can be obtained by merely applying Equations (6) in time, together with the equations of motion and the initial and boundary conditions. Inelastic deformations can be taken into account by modifying Equations (6) to read

$$d\sigma_S = P(d\epsilon_S - d\epsilon_S'') + Q(d\epsilon_F - d\epsilon_F'') \tag{8a}$$

$$d\sigma_F = Q(d\epsilon_S - d\epsilon_S'') + R(d\epsilon_F - d\epsilon_F'') \tag{8b}$$

$$d\tau_S = G(d\gamma_S - d\gamma_S'') \tag{8c}$$

where ϵ_S, ϵ_F, and γ_S are the total volumetric and deviatoric strains and ϵ_S'', ϵ_F'', and γ_S'' are the inelastic volumetric and deviatoric strains of the two-phase medium. The term "inelastic strain" is used in a broad sense to include all types of irrecoverable deformations, such as dilatancy, densification, and plastic creep. The inelastic volumetric strain of the fluid phase, ϵ_F'', is defined in terms of the volume of water that flows out of the system in a consolidation process and not as the plastic deformation of the pore fluid, which is assumed to be perfectly elastic.

The incorporation of the inelastic strain increments, $d\epsilon_S''$ and $d\epsilon_F''$, in Equations (8) constitutes a crucial step in the extension of Biot's theory to soil dynamics problems (7, 8). In order to accomplish this extension, two conditions must be satisfied. First, in the case of dry soils Equations (8) should reduce to

$$d\sigma_S = 3K(d\epsilon_S - d\epsilon_S'')\ ; \tag{9a}$$

$$d\tau_S = G(d\gamma_S - d\gamma_S'') \tag{9b}$$

and secondly, we must have

$$d\epsilon_F'' = -\frac{1-n}{n}\, d\epsilon_S'' \tag{10}$$

in order to maintain compatibility with regard to the volume of matter that flows in and out of the system during deformation. The coefficient Q, which is the coupling parameter, appears in Equations (8) as a consequence of the existence of an incremental strain energy, dW, which is a function of the elastic strain increments, $d\epsilon_S^{e\ell}$ and $d\epsilon_F^{e\ell}$. Furthermore, since P and R are always positive, Equations (8) guarantee that $PR - Q^2 > 0$; hence, this matrix of elastic constants is positive definite, and this guarantees the local stability of the material.

The key to applying Equations (8) properly is the interpretation of inelastic deformations. In the proposed approach the endochronic constitutive law is used to obtain the inelastic strains. This approach is entirely different from the one originally proposed by Biot (8), where it was suggested that the elastic moduli be replaced by certain time operators, which are functions of frequency, and then the resulting equations be inverted to the time domain by applying Laplace transformations. This procedure is based on the Correspondence (Onsager's) Principle of irreversible thermodynamics and yields acceptable results for quasi-static loads, but it becomes awkward and impractical when dealing with more complex situations, such as random vibrations.

FINITE ELEMENT ANALYSIS

Based on the endochronic constitutive law and the two-phase medium relationships that have been described, a two-dimensional finite element scheme has been developed to conduct a dynamic analysis of Lopez Dam, which is located in California. The discretization of the central cross-section of Lopez Dam was accomplished by constant strain triangular elements for which (a) compatibility conditions are satisfied everywhere and (b) equilibrium conditions are satisfied within each element. Since Lopez Dam is zoned, smaller size elements were used in the core area to accommodate the possibility of stress concentrations. The difference in the material properties of the different zones was taken into account by defining the material properties of each element separately. Based on information provided (Personal Communication, A. G. Franklin, 1977) regarding the average grain size distributions of the component soils, the shell and foundation materials were treated as cohesionless soils and the core and debris materials were treated as cohesive soils because of the higher percentages of particles in the silt and clay size range (40% in the debris and 20% in the core).

Although the inclusion of the water table in the analysis facilitates a more realistic formulation, some assumptions had to be made with respect to drainage conditions. As shown in Figure 1, there is a difference of approximately 38 feet in the water table between the upstream and downstream portions of Lopez Dam. However, the seepage forces due to this head difference can be neglected if it is assumed that their magnitude is insignificant compared to the prevailing static and dynamic stresses. The soil in the debris section is assumed to be fully saturated, even for the 5-foot portion above the water table, and completely undrained during an earthquake loading due to the high capillarity and low permeability of the silty soils. The section of the core that is below the water table is treated similarly (that is, saturated and undrained during earthquake loading), but the soil above the water table is treated as a one-phase medium (the effect of the fluid is neglected). The soils above the water table in the shell and foundation are assumed dry, and the regions under the water table are assumed saturated; due to a probable high permeability of these latter soils, partial drainage is considered, as described in two-phase medium formulation. Displacements are specified at the boundary between the foundation and the bedrock and the boundaries at both ends of the finite element mesh (designated as S_u), and stresses are specified at the free surfaces (designated as S_σ).

The equilibrium equations for a planar differential element of a two-phase medium (inelastic soil skeleton and elastic fluid) may be expressed as

$$\sigma_{S,1} + \tau_{S_{1i,i}} = (\rho_S(1-n) + \rho_a)\ddot{u}_{S_1} - \rho_a \ddot{u}_{F_1} + b(\dot{u}_{S_1} - \dot{u}_{F_1}) - \rho_S(1-n)g \tag{11a}$$

$$\sigma_{S,2} + \tau_{S_{2i,i}} = (\rho_S(1-n) + \rho_a)\ddot{u}_{S_2} - \rho_a \ddot{u}_{F_2} + b(\dot{u}_{S_2} - \dot{u}_{F_2}) \tag{11b}$$

$$\sigma_{F,1} + \tau_{F_{1i,i}} = (n\rho_F + \rho_a)\ddot{u}_{F_1} - \rho_a \ddot{u}_{S_1} - b(\dot{u}_{S_1} - \dot{u}_{F_1}) - n\rho_F g \tag{11c}$$

$$\sigma_{F,2} + \tau_{F_{2i,i}} = (n\rho_F + \rho_a)\ddot{u}_{F_2} - \rho_a \ddot{u}_{S_2} - b(\dot{u}_{S_2} - \dot{u}_{F_2}) \tag{11d}$$

where ρ_S and ρ_F are the mass densities of the solid and fluid phases; respectively;

ρ_a is the coupling mass density (added mass); b is the damping parameter (which is related to the Darcian flow of the fluid); $\dot{u}_{S_1}$, $\dot{u}_{S_2}$, $\ddot{u}_{S_1}$, and $\ddot{u}_{S_2}$ are the velocities and accelerations of the solid skeleton; $\dot{u}_{F_1}$, $\dot{u}_{F_2}$, $\ddot{u}_{F_1}$, and $\ddot{u}_{F_2}$ are the velocities and accelerations of the fluid; subscripts 1 and 2 refer to the vertical and horizontal directions; $\sigma_{S_{ij}}$ and $\sigma_{F_{ij}}$ are Biot type stresses in the solid and fluid phases; and σ_S and $\tau_{S_{ij}}$ are the volumetric and deviatoric parts of $\sigma_{S_{ij}}$ (τ_S in Equation 9 being $\sigma_{S_{12}}$). The governing equations for a two-phase medium, considering the volumetric coupling between the solid and fluid phases, are obtained from Biot's two-phase medium concept (7) and are given by Equations (8).

Due to the nonlinear nature of this problem, it is necessary to use the virtual work equation in its incremental form. The specific formulation of the governing equation is based on the type of element used and the nature of the displacement functions selected, and it can be given as

$$[K_m]\left\{\Delta u_{i_m}\right\} + \left\{F''_{i_m}\right\} + [M_n]\left\{\Delta\ddot{u}_{i_m}\right\}$$
$$+ [C_m]\left\{\Delta\dot{u}_{i_m}\right\} - \left\{T_m\right\} = 0 \qquad (12)$$

where $[K_m]$ is the stiffness matrix; $[M_m]$ is the mass matrix; $[C_m]$ is the damping matrix; $\left\{F''_{i_m}\right\}$ is the inelastic stress vector; and $\left\{T_m\right\}$ is the load vector. In defining the damping characteristics of a soil-water system, the permeability and porosity of a soil must be taken into consideration, as suggested by Biot (7); the damping coefficient, b, is given as $\mu n^2/k$, where n is the porosity, μ is the fluid viscosity, and k is the permeability. In the case of dry or relatively dry soils, the damping due to coupling between the two phases is not present and no damping term is necessary in the governing equation, because material damping is already included in the constitutive equations. The mass matrix used is a consistent mass matrix, which again accounts for material coupling due to the relative motion of the solid and fluid phases, as suggested by Biot (7).

The final step, following integrations of the stiffness, damping, and mass matrices and the inelastic and surface force vectors, is the solution of Equation (11) for a given earthquake acceleration-time history. The step-by-step solution of the simultaneous differential equations of motion is accomplished by introducing a simple relationship between the displacement, velocity, and acceleration; this relationship is assumed to be valid for a short increment of time, such that Equation (11) can be treated as a set of simultaneous algebraic equations. The assumption of linear acceleration provides an efficient step-by-step integration procedure as long as a sufficiently short time increment is used. The method used in this study is known as the Wilson θ method. It is a modified version of a standard linear acceleration approach in which it is assumed that the acceleration varies linearly over an extended computation interval $\tau = \theta\Delta t$, where $\theta > 1.37$. Based on this assumption, the velocity and displacement can be obtained in terms of the acceleration increment. However, it is more convenient to use the displacement increment as the basic variable that can be obtained directly from the solution of the pseudostatic relationship.

DYNAMIC RESPONSE OF DAM

The response of earth dams under earthquake loadings has been studied by many investigators; however, most of the proposed approaches are based on pseudostatic or one-dimensional analyses (1, 16, 17, 18, 22, 23, 25) or two-dimensional finite element models with elastic stress-strain relationships (10, 12, 20, 21), where inelasticity of the soil and pore pressure response are determined by use of empirical explicit schemes. Only Ghaboussi and Wilson (14) have applied the concept of a two-phase medium using linear elastic stress-strain relationships; however, a concern was expressed for the necessity to incorporate nonlinear inelastic constitutive laws in order to establish realistic field models.

In the first part of this investigation a two-dimensional finite element program using the grid shown in Figure 1 was developed based on linear elastic stress-strain relationships and a two-phase medium concept; subsequently, the program was extended to include nonlinear, inelastic, endochronic constitutive relationships. The dynamic analysis was carried out using a modified version of the N-S component of the El Centro accelerogram from the 1940 California earthquake, shown in Figure 2, with a peak acceleration of 587.8 cm/sec^2 for a period of 10 seconds. In the second

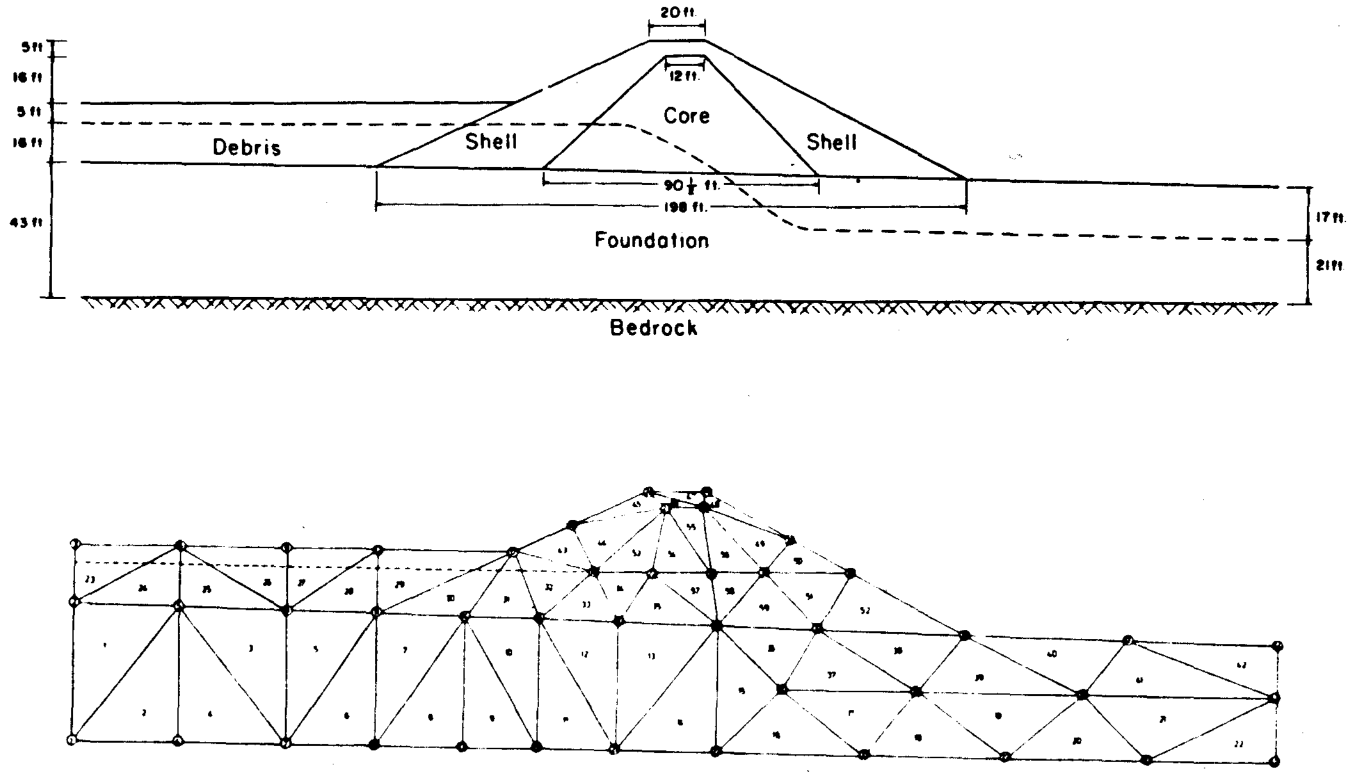


Figure 1. Cross-section and Finite Element Mesh of Lopez Dam

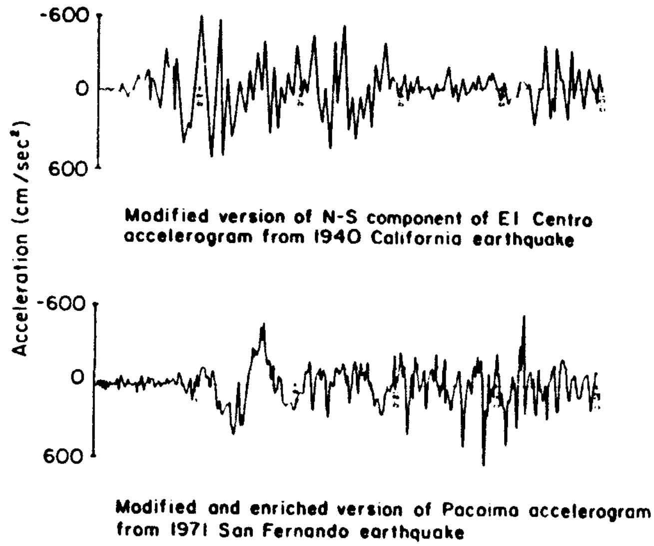


Figure 2. Accelerograms Used in Dynamic Analysis

part of the study an inelastic nonlinear dynamic analysis is performed by using a modified and enriched version of the Pacoima acceleration record, shown in Figure 2, with a peak acceleration of 638.0 cm/sec^2 for a period of 20 seconds.

The results obtained to compare elastic and endochronic stress-strain relations are shown in Figures 3 through 7 in terms of relative accelerations, velocities, displacements, shear stresses, and shear strains for nodes located on the crest and at the midheight of the embankment. As can be seen, there is very little difference between the elastic and inelastic solutions for the acceleration and velocity records. In addition, the computed maximum acceleration at the crest is similar to that at the base; this same situation was also observed in the dynamic analysis proposed by Seed et al (20), and it was possible to justify this type of response based on the seismoscope records obtained by Scott (19) in the Lower San Fernando dam during the February 1971 earthquake. On the other hand, there are significant differences between the displacement, shear stress, and shear strain solutions for the same nodal points. The inelastic analysis yields larger inelastic displacements and shear strains and larger residual shear stresses.

In the second part of this study the dynamic analysis was performed for similar conditions, except that an enriched version of the Pacoima accelerogram was used. The variations of the horizontal, vertical, and shear stresses and strains for the element located at the downstream side of the crest of the dam are shown in Figure 8. The shear strains obtained from the Pacoima record show a similar trend to those obtained from the El Centro record, but they have a lower inelastic component. The difference is more apparent in the comparisons of shear stresses; in particular, the peak shear stress obtained from the Pacoima record is approximately one-half of that obtained with the El Centro record. The displacement histories at various points of the dam are shown in Figure 9. The calculated displacement for the point located at the crest is smaller than the displacement calculated previously.

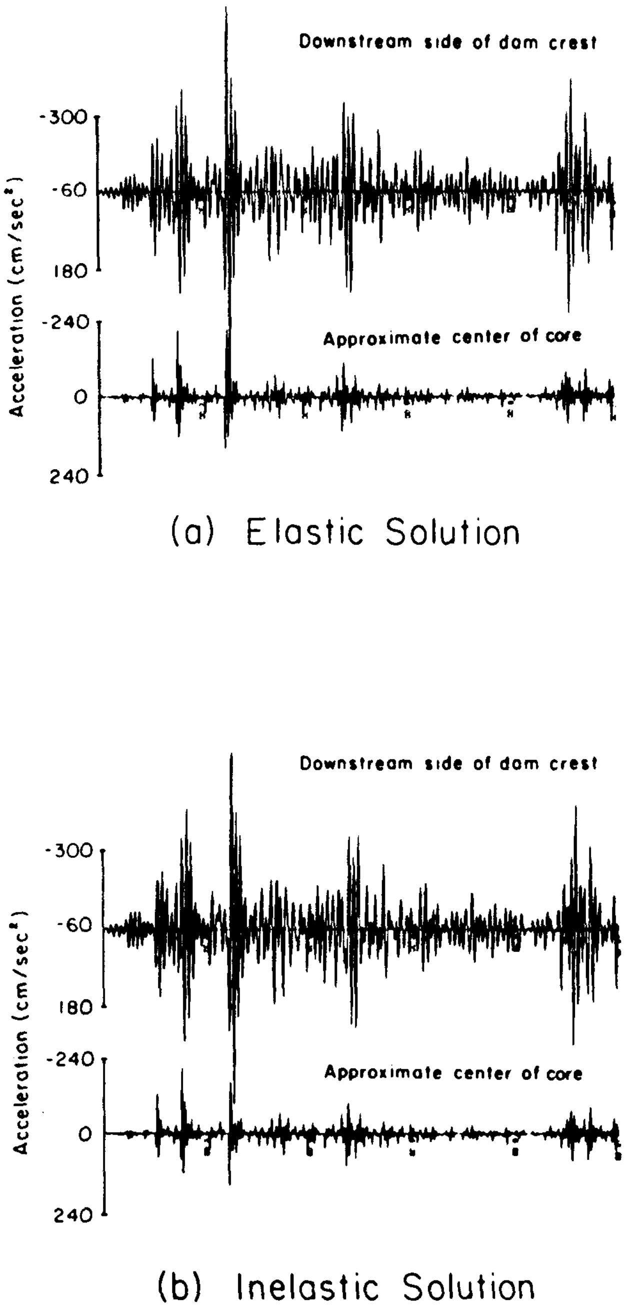


Figure 3. Time-dependent Variation of Acceleration at Select Locations

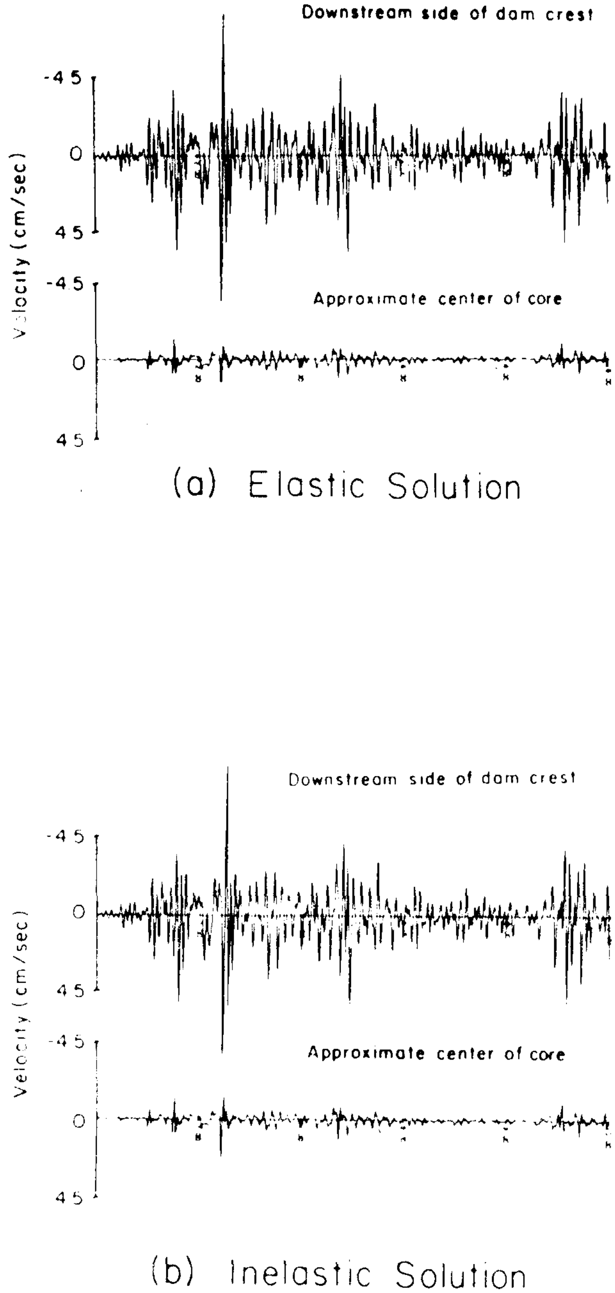


Figure 4. Time-dependent Variation of Velocity at Select Locations

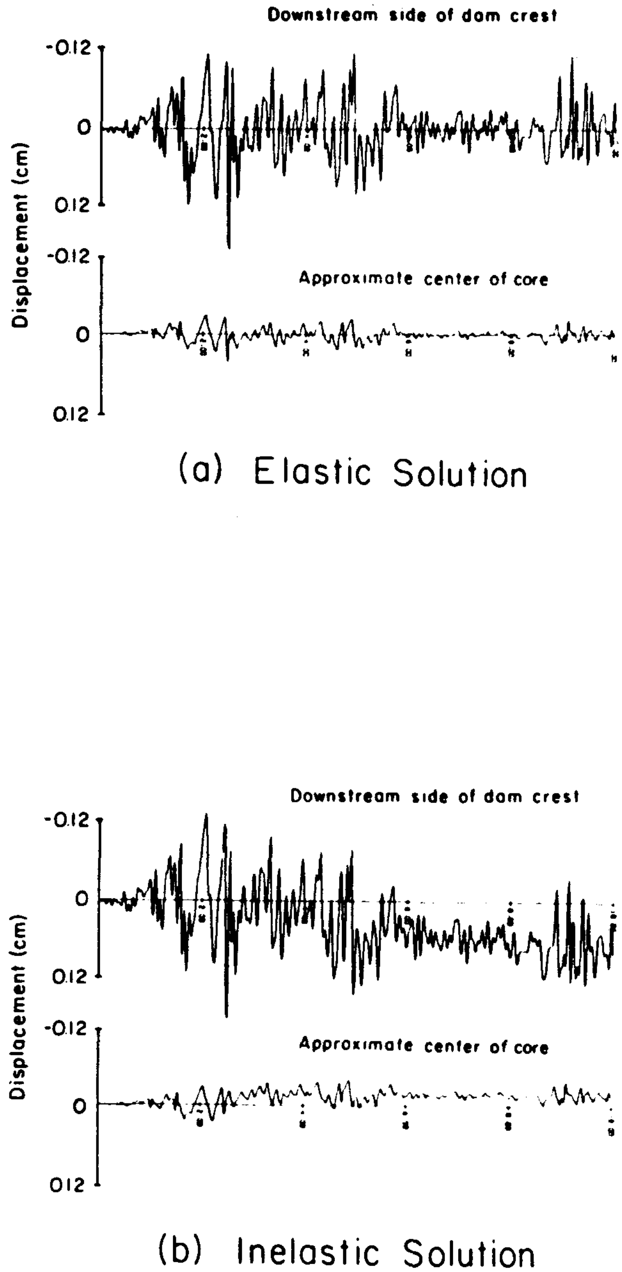


Figure 5. Time-dependent Variation of Displacement at Select Locations

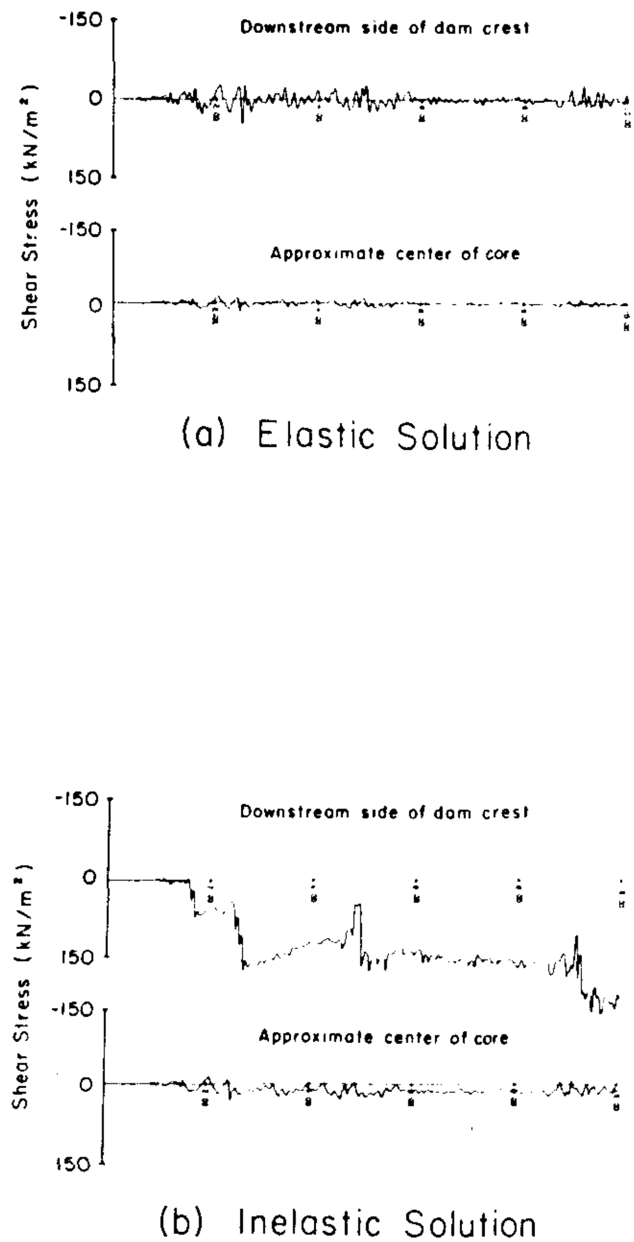
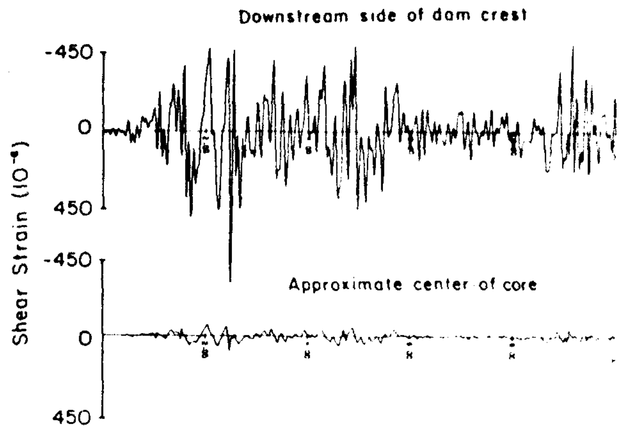
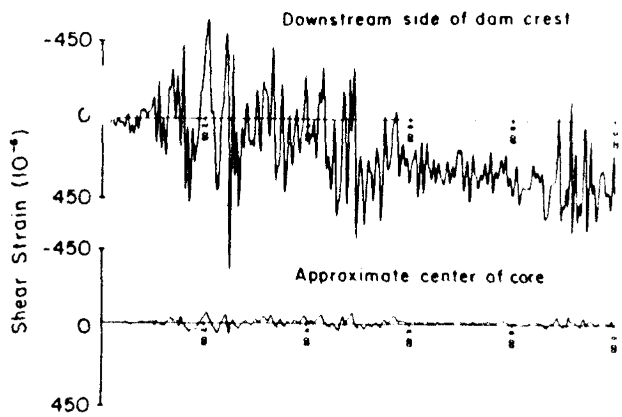


Figure 6. Time-dependent Variation of Shear Stress at Select Locations



(a) Elastic Solution



(b) Inelastic Solution

Figure 7. Time-dependent Variation of Shear Strain at Select Locations

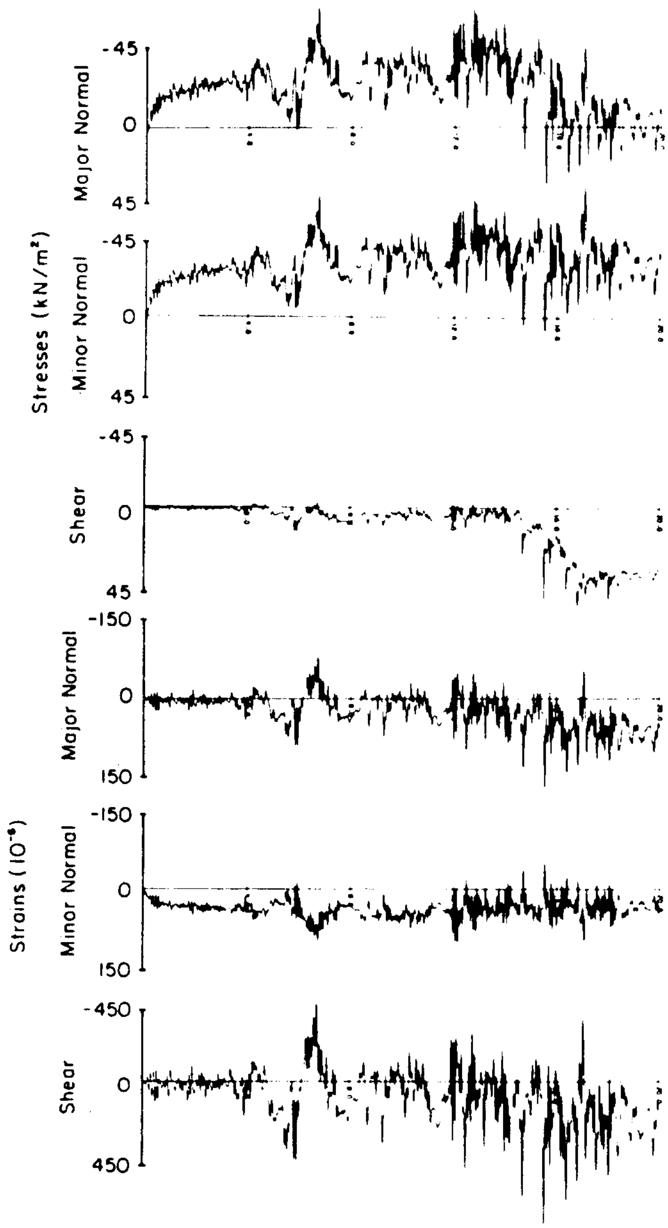


Figure 8. Time-dependent Variation of Principal and Shear Stresses and Strains at Crest

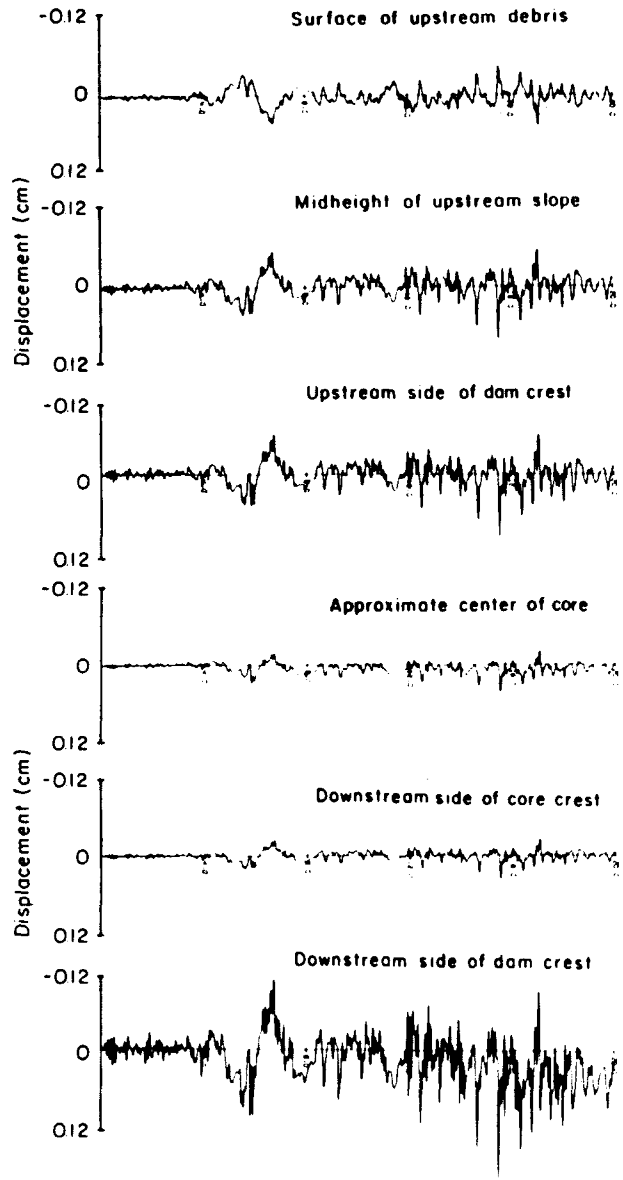


Figure 9. Time-dependent Variation of Horizontal Displacement at Select Locations as Calculated by Using the Pacoima Record

The relative accelerations and velocities obtained from the Pacoima record are shown in Figure 10. These peak accelerations and velocities are significantly smaller than those shown in Figures 4 and 5 for the El Centro earthquake. Accordingly, the effect of different earthquake inputs, even those with the same peak accelerations, cannot be disregarded. In the case of the El Centro record, the peaks occur in the first 6 seconds, whereas in the Pacoima record there are very few peaks in the first 6 seconds; this situation is probably responsible for the significant reduction in the inelastic displacements and stresses in the case of the Pacoima record.

The pore pressures in two elements located under the water table at the upstream side of the dam are shown in Figure 11. The variation of the pore pressure at the crest shows a sudden increase after 11 seconds of shaking, indicating a liquefaction type of failure, even though the debris is assumed to be partly cohesive. However, at a point it is possible for pore pressures to increase suddenly in silty soils due to the accumulated loss of shear strength.

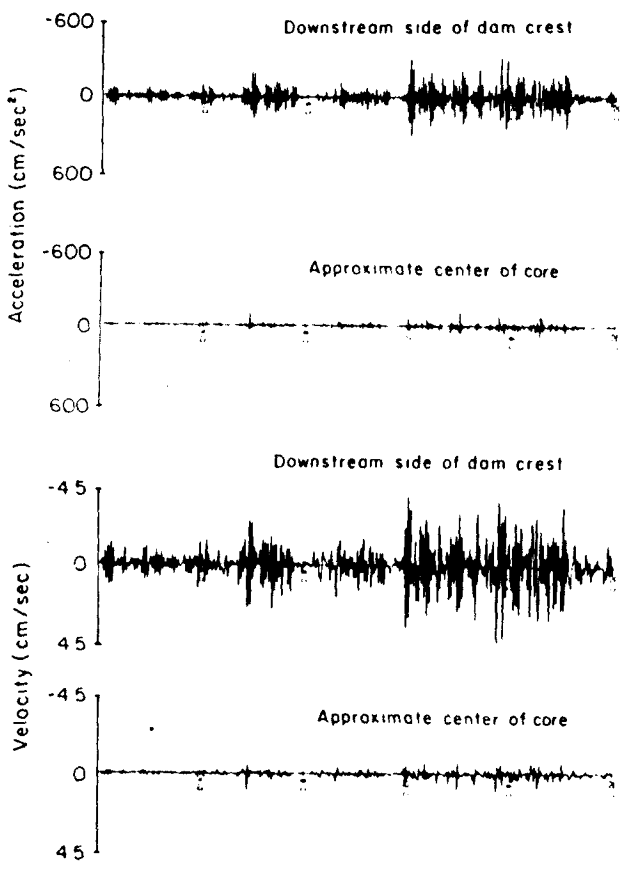


Figure 10. Time-dependent Variation of Acceleration and Velocity at Select Locations as Calculated by Using the Pacoima Record

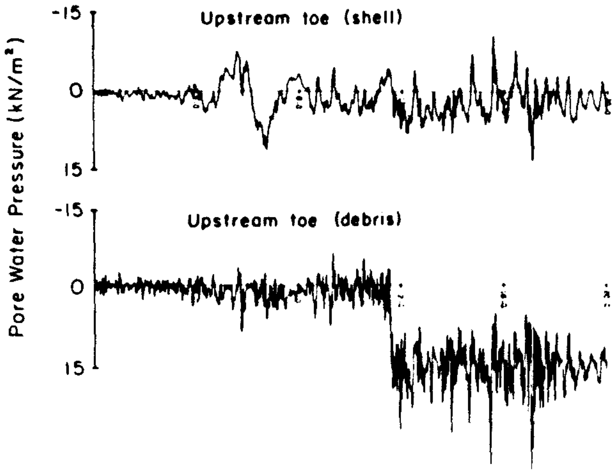


Figure 11. Time-dependent Variation of Pore Water Pressure at Select Locations as Calculated by Using the Pacoima Record

CONCLUSIONS

A two-dimensional finite element program incorporating inelastic, nonlinear constitutive relationships and a two-phase medium concept was developed to investigate and compare the dynamic response of an earth dam subjected to two different earthquake acceleration records with approximately similar peak acceleration values. Based on calculated response patterns utilizing both inelastic and linear elastic constitutive relationships, the following conclusions can be advanced:

1. The results are in general agreement with the observations and investigations reported in the literature.

2. A realistic assessment of the behavior of earth dams requires the use of inelastic nonlinear stress-strain relationships.

3. Variations in the acceleration and peak acceleration calculated by dynamic analyses appear to be unreliable parameters for use in evaluating the stability of earth dams.

4. The nature of the input earthquake record has a significant influence on the dynamic response of an earth dam.

5. The proposed finite element model incorporating endochronic constitutive relationships and a two-phase medium concept offers a significant improvement over previously proposed models in evaluating the dynamic response and stability of earth dams.

ACKNOWLEDGEMENT

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