Bažant, Z.P. (1991). "Fracture mechanics of quasi-brittle structures: Recent advances." (Proc., *11th International Conference on Structural Mechanics in Reactor Technology*, Tokyo, August), ed. by H. Shibata, Vol. H, Paper H04/1, pp. 97–107.

Fracture Mechanics of Quasibrittle Structures: Recent Advances

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ABSTRACT

Because of the need for correctly predicting the size effect in structural failures as well as limitations on ductility, realistic modeling of the failure of quasibrittle structures, distinguised by propagation of damage zones of distributed cracking, has considerable importance for the safety of structural designs in general, and nuclear structures in particular. The lecture reviews several recent advances made principally at the author's home institution, focused on fracturing damage in concrete structures. The nonlocal damage concept, which has recently been advanced as a general method to deal with distributed damage in a consistent and realistic manner, is examined from the viewpoint of micromechanics. It is shown that nonlocality of the macroscopic continuum that smears microcracking is a necessary consequence of two basic characteristics of fracture mechanics as applied to the microcracks: (1) The fact that pressure applied on one microcrack causes openings of adjacent microcracks, and (2) the fact that the formation of a microcrack depends not on the strain existing at the microcrack location before its formation but on the energy stored in a certain finite neighborhood of the microcrack. The transitional size effect between elasticity and plasticity, which is a consequence of nonlocality, is shown to be manifested not only in the nominal strength of structures but also in the value of the material fracture energy determined by the 1985 RILEM work-of-fracture method proposed by Hillerborg. The previously proposed size effect law is shown to imply a strong dependence of the value of fracture energy obtained in this manner on the size of the test specimen, as well as its shape. Furthermore, examining the modeling of quasibrittle failures by discrete cracks with softening crack-bridging stress-displacement relation, the bridging stress should depend not only on the displacement at the same point but on the distribution of the opening displacements along the fictitious crack in the neighborhood of the given point. Thus the law for crack bridging stresses on a fictitious crack requires, at least in principle, a nonlocal generalization. This generalization could explain the size and shape dependence of test results. In closing, several further advances in numerical modeling are briefly mentioned and documented by numerical results.

1 Introduction

Quasibrittle materials are materials made of brittle constituents which lack the capability of plastic yield but achieve a certain degree of macroscopic ductility by developing finite size zones of diffuse damage. The heterogeneity, aggregate or fiber microstructure, stiffness difference between the constituents of the composite, and weakness of inter-constituent interfaces generally promote the quasibrittle behavior. One important and the longest exploited example of quasibrittle materials is concrete, which is of principal interest in the present lecture. Other examples include many types of rocks and stiff soils (stiff clays, cemented sands), modern toughened ceramics, various types of aggregate or fiber composites, and ice (especially sea ice).

It must be emphasized, however, that the quasibrittle properties depend on the ratio of the structure size to the size of the zone of diffuse damage. For very large structure sizes, even a structure made of quasibrittle material becomes nearly perfectly brittle, which means it follows linear elastic fracture mechanics. Likewise, a structure from a certain material which in normal dimensions behaves in a brittle manner will generally become quasibrittle if its size is reduced to sufficiently small dimensions.

The principal characteristics that distinguish quasibrittle structures from perfectly brittle structures and ductile structures are the size effect and the necessity of some type of a nonlocal concept in the macroscopic continuum modeling of the material. There has been considerable research activity in these problems in many laboratories throughout the world. Their comprehensive review would require a lengthy report. The aim of the present lecture is to selectively review only several recent advances made at the author's home institution and concerned with the microscopic physical causes of nonlocality of damage in a continuum describing a quasibrittle material, the consequences of the nonlocality for the meaning of fracture energy and its determination by the work-of- fracture method (Hillerborg's 1985 RILEM method), and a method to effectively calculate the response of a structure with damage characterized by a stress-displacement relation for a fictitious crack replacing the damaged band.

2 Microscopic Physical Cause of Nonlocality of Damage Due to Microcracking

The concept of nonlocal continuum, which was introduced into continuum mechanics by Eringen [1,2], Kröner [3] and others, has recently been proven to be an effective approach to endow a continuum model for strain-softening damage with a localization limiter which prevents spurious localization instabilities [4–6]. Physical micromechanical justification of this concept, however, has been almost nonexistent. In a recent article [7], it was shown that a highly idealized model of a uniaxially stressed solid with a quasiperiodic array of small noninteracting cracks arranged on a cubic lattice leads to precisely the aforementioned type of nonlocal continuum. But this kind of damage was rather simplistic, particularly due to the neglect of crack interactions.

We now briefly outline an argument showing that homogenization by a continuum with nonlocal damage is a necessary consequence of the existence of interactions between individual microcracks (in full detail this argument will appear in the ASCE Journal of Engineering Mechanics, Vol. 117, 1991, No. 5). Let us consider an elastic body intersected by an arbitrary array of mode I cracks (microcracks), which we number as $i = 1, 2, 3, ...n$ (Fig. 1a). As is usual in fracture mechanics, we may obtain the solution as a superposition of $n + 1$ elasticity problems (Fig. 1). The load is first imagined to be applied on the body in which the cracks are kept closed, say glued (Fig. 1b). The resultants of the stresses transmitted across the glued cracks are approximately espressed as:

$$R_i = A_i \, \boldsymbol{n}_i \cdot \boldsymbol{\sigma}_i \cdot \boldsymbol{n}_i \tag{1}$$

in which A_i is the area of the i-th microcrack, $\boldsymbol{n}_i$ is its unit normal vector, and $\boldsymbol{\sigma}_i$ is the stress tensor in the uncracked body at the crack center. In terms of the geometrically defined (local) damage ω, as introduced by Kachanov [8], $A_i = \omega_i A_M$ where A_M is the maximum possible crack area at which the adjacent cracks coalesce.

The releasing (ungluing) of the cracks is equivalent to applying on the surfaces of the cracks the pairs of forces $F_i = R_i$ in the opposite sense than R_i; see Fig. 1c–e.

The damage in the mechanical sense is not characterized by ω but by the opening displacements δ_i of the microcracks i, or the fracturing strain representing the smearing of these displacements. The opening displacement of the i crack is the sum of the opening displacements of this crack caused by the loads F_i applied on each other crack j (Fig. 1c–d). This may in general be expressed as

$$\delta_i^{fr} = \sum_{j=1}^{n} \lambda_{ij} \, F_j \tag{2}$$

where λ_{ij} are the crack interaction coefficients, each of which represents the opening displacement of crack i caused by a unit pair of forces $F_j = 1$ applied at crack j. Coefficients λ_{ij} may be expected to diminish rapidly with the distance between the cracks and become negligibly small for more distant cracks.

Note that λ_{ij} are similar but not identical to the crack influence coefficients of Kachanov [9], which define the stress intensity factor of one crack caused by the load on any other crack. A noteworthy finding of Kachanov was that the neighboring cracks are influenced primarily by the value of the resultant of the stresses applied at the crack faces and depend little on the precise distribution of these stresses over the crack faces, except when the tip of one crack is extremely close to the other crack (this property may also be expected on the basis of St.Venant's principle). It is for this reason that we take into acount only the stress resultants over the crack faces, and not the detailed stress distributions.

The fracturing strain tensor due to the smeared (homogenized) opening displacements of the i-th crack is

$$\epsilon_i^{fr} = \boldsymbol{n}_i \otimes \boldsymbol{n}_i \, \frac{\delta_i^{fr}}{s_i} \tag{3}$$

where s_i is the effective spacing of the adjacent cracks (s_i as well as A_M is difficult to define precisely, but for our purposes it suffices that it is a constant).

As a consequence of the foregoing relations, we get for the fracturing strain tensor the following general result:

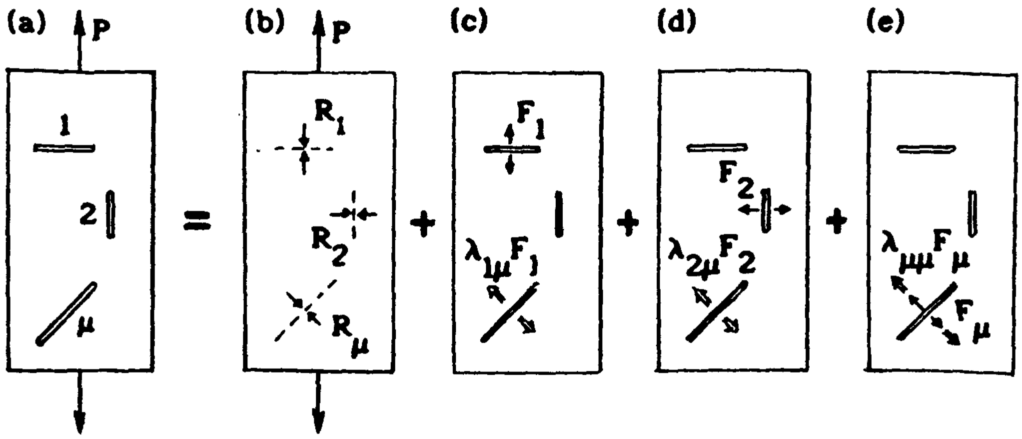

Figure 1: Elastic solid with interacting microcracks and analysis of deformations by superposition of solutions.

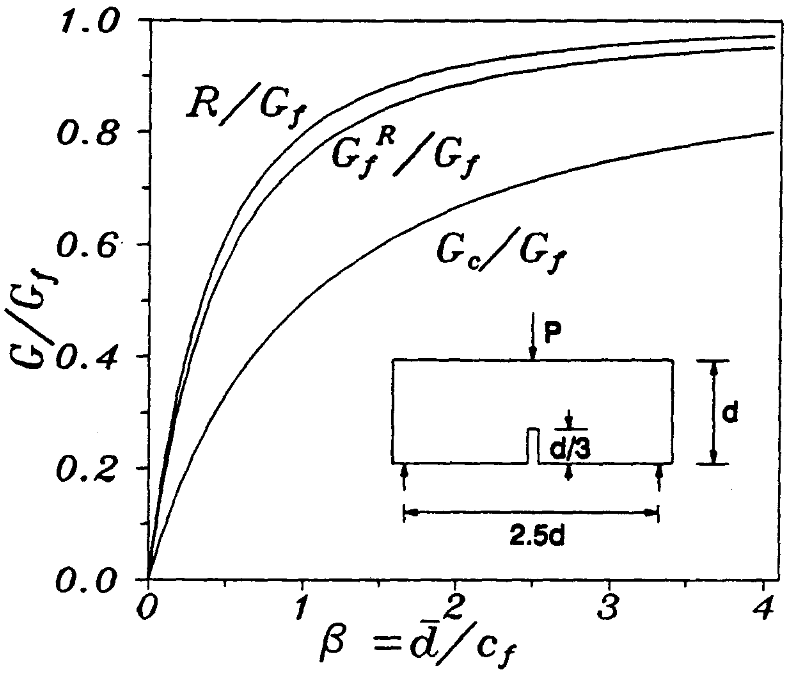

Figure 2: Variation of fracture energy obtained from work-of-fracture method (RILEM 1985).

$$\epsilon_i^{fr} = \boldsymbol{n}_i \otimes \boldsymbol{n}_i \frac{\delta_i^{fr}}{s_i} \sum_{i=1}^{n} \lambda_{ij} \omega_j (\boldsymbol{n}_i \cdot \boldsymbol{\sigma}_i \cdot \boldsymbol{n}_i) \tag{4}$$

The stress-strain relation for the macroscopic continuum with damage has the form:

$$\boldsymbol{\sigma}_i = E : (\epsilon_i - \epsilon_i^{fr}) \tag{5}$$

which may be also written as

$$\boldsymbol{\sigma}_i = (\boldsymbol{I} - \Omega_i) : \boldsymbol{E} : \epsilon_i \tag{6}$$

where Ω_i is the fourth-order damage tensor at the location of crack i, $\boldsymbol{I}$ is a fourth order unit tensor, and $\boldsymbol{E}$ is the fourth-order tensor of elastic moduli of the uncracked material.

The damage tensor Ω that enters the constitutive equations is defined by the comparison of Eq. 6 with Eqs. 4 and 5. There are, however, infinitely many possible such damage tensors, and it turns out that this damage tensor can be defined uniquely only if the process of damage evolution is followed incrementally from the start of loading. It is nevertheless clear that Ω is in any case defined by spatial summation or, in the homogenized case, spatial integration that corresponds to the summation in Eq. 4. This proves that continuum damage is a nonlocal variable. The classical theories of damage, in which the damage is calculated solely from the histories of the stress and strain tensors at the same point of the material, are in conflict with this result.

To sum up, the existence of crack interactions, that is, the fact that a loading of the surfaces of one microcrack causes crack opening displacements not only at the same microcrack but also at the adjacent microcracks, implies that, in the homogenized continuum model for damage due to microcracking, the damage variable that enters the stress-strain relation must be *nonlocal.*

3 Size Effect in Work-of-Fracture Determination of Fracture Energy (RILEM 1985 Method)

In another development[10], the size dependence of the fracture energy of concrete determined according to the 1985 RILEM recommendation [12-16] has been examined in the light of what is known about the size effect in general and the nonlocal features of fracture in particular. In this method, which was originated by Nakayama, Tattersal and Tappin [17-18], the energy dissipated at the fracture front is evaluated from the measured load-displacement curve. Recent experience showed however that the value of fracture energy thus obtained, G_f^R, is strongly size dependent. A theoretical argument to this effect has been advanced by Planas and Elices [19], based on their asymptotic analysis of fracture according to a stress-displacement relation for a fictitious crack. The size dependence of the fracture energy G_f^R has been analyzed at Northwestern University on the basis of the size effect law proposed by Bažant—a rather general law which has been shown to apply to the size range up to about 1:20 and to have approximately the same form for all specimen geometries. The analysis used the previously developed method for calculating the R-curve from the size effect and the load-deflection curve from the R-curve [10]. In this definition, the R-curve is dependent on the geometry of the specimen.

The results of this analysis have shown that if the size effect law proposed by Bažant is valid, the fracture energy determined according to the 1985 RILEM recommendation cannot be size-independent, as originally intended. Rather, it must strongly depend on the specimen size. The law for this dependence has been calculated. In the limit case, when the specimen size is extrapolated to infinity, the fracture energy according to the 1985 RILEM recommendation coincides, at least in theory, with the fracture energy obtained by the size effect method (1990 RILEM recommendation [11]). The typical results are exhibited in Fig. 2, which shows the variation of the fracture energy required for crack growth (the R-curve) with the relative specimen size, as well as the variation of the fracture energy according to the 1985 RILEM recommendation, G_f^R. The figure also shows the variation of the fracture energy G_c obtained when the tests are evaluated according to linear elastic fracture mechanics (LEFM). G_f represents the value of fracture energy in the size effect definition, that is the fracture energy for extrapolation to an infinite specimen size, for which the value of fracture energy must be both size and shape independent. It may be noted from the figure that the variation of G_f^R in fact occurs over a broader range of sizes than the variation of R, which is a very strong size dependence indeed ($\overline{d}$ is the intrinsic specimen size and c_f in the effective length of the fracture process zone for extrapolation to infinity).

An important point in the foregoing analysis has been a previous result [23] regarding the shape of the R-curve after the peak load is reached and softening response sets in. It has been found that after the peak load the R-curve must be assumed to be approximately horizontal, i.e., R = constant, equal to the value that R has reached on the master R-curve at the peak load. This behavior is explained by the fact that the fracture process zone apparently ceases to grow as soon as the softening response begins and moves ahead approximately as a rigid body.

4 Nonlocality of Bridging Stress in Fictitious Crack Model

The most general and realistic, yet feasible, method for simulating arbitrary propagation of damage zones and fracture is the nonlocal damage model (see Ref. 12, Chapter 13). In this model, which was developed and applied for concrete by Bažant, Pijaudier-Cabot, Lin and Ožbolt [6, 21], the damage or other variables characterizing damage, such as the fracturing strain, is considered to be a function of a weighted average of the strains within a certain neighborhood of a given point, rather than a function of the strain at that point. This concept prevents spurious localization of continuum damage and thus enables a continuum modeling of fracture in a smeared manner.

An alternative approach, which is particularly effective when the cracking is known to localize into a discrete crack and the crack path is known in advance, is the fictitious crack model, which was pioneered for concrete by Hillerborg and co-workers and was embodied in the first RILEM recommendation on the measurement of fracture energy [13]. The fictitious crack is used to model the fracture process zone ahead of the actual crack tip, in which the material exhibits strain softening, with the stresses increasing at decreasing overall strain, due to progressive microcracking as well as other inelastic phenomena such as aggregate pullout and slip. The basic material characteristic in this method is the relationship between the crack bridging stess $\sigma(x)$ versus the opening displacement $v(x)$ of the fictitious crack, i.e.,

$$\sigma(x) = \mathcal{F}[v(x)] \tag{7}$$

in which F represents a certain function or functional characterizing the material properties, and x is a coordinate measured along the crack length. The value of v represents the accumulated transverse displacement due to all the microcracks in the crack band forming the fracture process zone. In comparison to the nonlocal continuum model with smeared cracking, v represents the integral of the cracking strain, ϵ_{ij}^{fr}, across the width of the crack band.

The accumulation of cracking strains into the crack opening displacement v of the fictitious crack involves an integration in the transverse direction to the crack, and therefore represents a sort of nonlocal averaging. However, it represents only a part of the nonlocal averaging implied in the nonlocal continuum concept, in which the averaging needs to be done over a two-dimensional domain surrounding the point, i.e., a circle (rather than a one-dimensional domain, a line segment in the transverse direction).

Aside from averaging over coordinate y normal to the crack direction this also involves averaging over coordinate x in the crack direction. In this light, it appears proper to redefine the stress-displacement relation for the fictitious crack in the following form:

$$\sigma(x) = \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} \mathcal{F}[v(s)]\alpha(s-x)dx \tag{8}$$

in which α is a given weight function, typically with a bell shape, which introduces interactions along the crack. These interactions may be explained by the effect of a microcrack at one location along the crack on the stress or displacement at an adjacent location (Fig.3), as explained in the first section.

Now one obvious consequence of Eq. 8 would be that the actual curve of σ versus displacement v at the same point, which is followed at various points on the crack is nonunique. Different cracks can be obtained depending on the distribution of crack opening displacements. This could explain that for various types of tests and various geometries of test specimens, analyses in the past have indicated various shapes of the stress displacement curves. This would further mean that the area under the stress-displacement curve depends on the displacement distribution along the crack, and thus on the specimen geometry as well as size. It would be an explanation of the dependence of the apparent fracture energy, in the sense of the work of fracture, on the specimen shape and size. This explanation might of course be only partial, since there are other effects that could cause a nonunique shape of the curve of the bridging stress σ versus the opening displacement v.

5 Smeared-Tip Superposition Method

The numerical modeling of structure response with the fictitious crack model and a law for bridging stress necessitates a method to solve the interaction between the crack opening

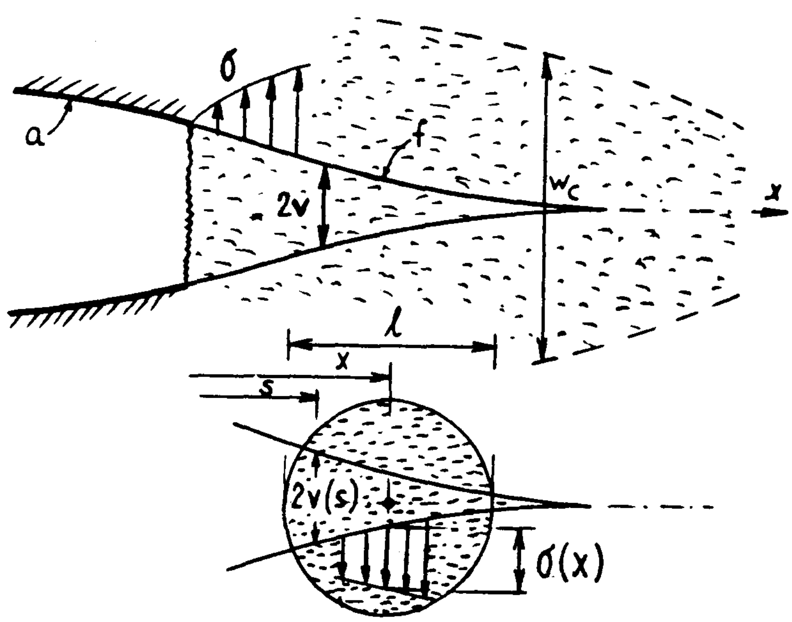

Figure 3: Fictitious crack and averaging zone of nonlocal crack-bridging stress.

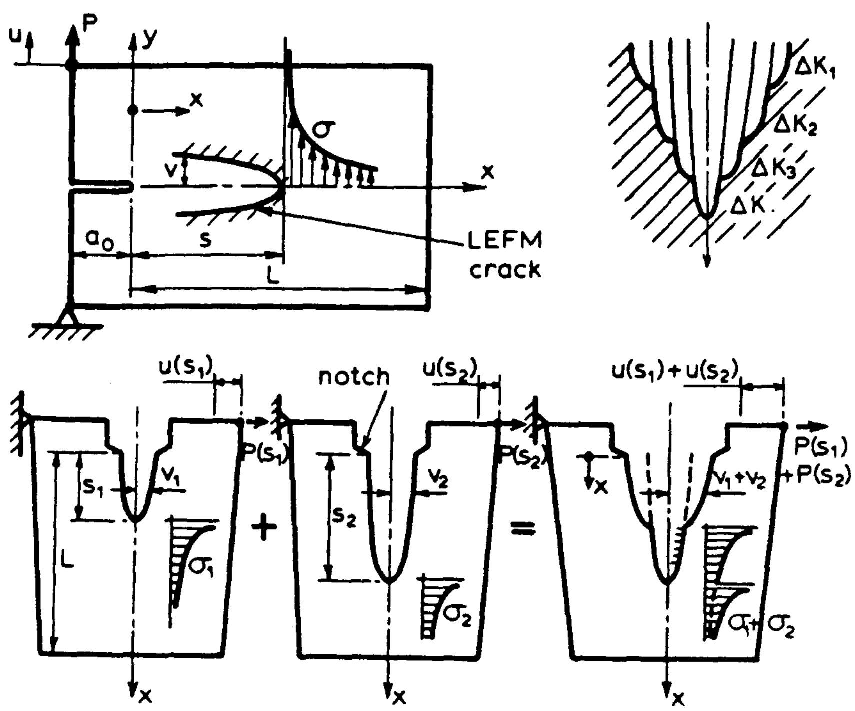

Figure 4: Superposition of LEFM cracks whose limit in the smeared-tip method represents a smooth crack of arbitrary opening profile, with bridging stresses.

profile and the bridging stress distribution for an arbitrary specimen geometry. A general solution according to the standard approach of elasticity requires using the Green's function which gives the displacement field due to a concentrated load applied at any point on the crack line. Such a function, unfortunately, is available only for an infinite space but not for typical specimen geometries.

Another approach, however, has recently been discovered by Planas and Elices and extended by Bažant [24-26]. In this approach, an arbitrary profile of a fictitious crack is modeled as the superposition of infinitely many linear elastic cracks whose tips are continuously distributed (or smeared) along the crack line. Superposition of several linear elastic (LEFM) cracks is illustrated in Fig. 4, and a smooth profile of a fictitious crack is obtained in the limit of infinitely many such cracks spaced infinitely closely. The superposition of the smeared crack tips can in general be worked out for any structure geometry for which the solution of the stress intensity factor is available for arbitrary location of the crack tip on the crack line. This comprises a large number of specimen geometries.

The mathematics of the smeared tip superposition has been worked out in general in Ref. 26, along with a general method of finite difference solution in which the singularities associated with each crack tip are taken properly into account (which is the main difficulty to be overcome in this type of approach). This method of solution can also be adapted to the nonlocal form based on Eq. 8.

6 Conclusion

Recent researches that have been reviewed in the present lecture indicate that there are solid micromechanics reasons for the nonlocal character of the macroscopic continuum that models damage due to microcracking. One reason for the nonlocality is the fact that a loading applied at the surfaces of one microcrack causes opening displacements at adjacent microcracks, so that there is interaction in space, which is not implied in the macroscopic smoothing continuum. Another reason is that the formation of a microcrack does not depend on the continuum stress at the location of the microcrack but on the energy contained within a certain neighborhood of the microcrack, which is not a local property but a nonlocal property depending on the strain distribution on the macroscale over a certain neighborhood of the microcrack. The basic consequence of the nonlocality is the size effect on the nominal strength of structures. This size effect is also manifested in the value of fracture energy experimentally determined in the conventional way, including the fracture energy according to the work of fracture definition used in the intitial 1985 RILEM recommendation based on Hillerborg's work. The consequence of nonlocality, or of the size effect law for the nominal strength of the structures, is that the fracture energy determined on the basis of the work-of-fracture method ought to depend on the specimen size as well as shape, in fact quite strongly. Another consequence of nonlocality is that the fictitious crack model with crack bridging stresses should use a stress displacement relation that is nonlocal along the crack line, in which the bridging stress at one point of the crack line depends not only on the displacement at the same point of the crack line but on the displacement distribution in the vicinity of that point.

There are of course various other noteworthy recent results in this hot field of research, but they are beyond the scope of a single lecture.

Acknowledgement

Financial support under AFOSR Grant No. 91-0140 to Northwestern University is gratefully acknowledged.

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