Disclaimer:
Bažant voted against the statistical model comparisons in this guide and believes them to be misleading. His name appears since this was mandatory for committee members.

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Guide for Modeling and Calculating Shrinkage and Creep in Hardened Concrete

Reported by ACI Committee 209

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This guide is intended for the prediction of shrinkage and creep in compression in hardened concrete. It may be assumed that predictions apply to concrete under tension and shear. It outlines the problems and limitations in developing prediction equations for shrinkage and compressive creep of hardened concrete. It also presents and compares the prediction capabilities of four different numerical methods. The models presented are valid for hardened concrete moist cured for at least 1 day and loaded after curing or later. The models are intended for concretes with mean compressive cylindrical strengths at 28 days within a range of at least 20 to 70 MPa (3000 to 10,000 psi). This document is addressed to designers who wish to predict shrinkage and creep in concrete without testing. For structures that are sensitive to shrinkage and creep, the accuracy of an individual model's predictions can be improved and their applicable range expanded if the model is calibrated with test data of the actual concrete to be used in the project.

Keywords: creep; drying shrinkage; prediction models; statistical indicators.

CONTENTS

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CHAPTER 1—INTRODUCTION AND SCOPE

1.1—Background

To predict the strength and serviceability of reinforced and prestressed concrete structures, the structural engineer requires an appropriate description of the mechanical properties of the materials, including the prediction of the time-dependant strains of the hardened concrete. The prediction of shrinkage and creep is important to assess the risk of concrete cracking, and deflections due to stripping-reshoring. As discussed in ACI 209.1R, however, the mechanical properties of concrete are significantly affected by the temperature and availability of water during curing, the environmental humidity and temperature after curing, and the composition of the concrete, including the mechanical properties of the aggregates.

Among the time-dependant properties of concrete that are of interest to the structural engineer are the shrinkage due to cement hydration (self-desiccation), loss of moisture to the environment, and the creep under sustained loads. Drying before loading significantly reduces creep, and is a major complication in the prediction of creep, stress relaxation, and strain recovery after unloading. While there is a lot of data on shrinkage and compressive creep, not much data are available for creep recovery, and very limited data are available for relaxation and tensile creep.

Creep under variable stresses and the stress responses under constant or variable imposed strains are commonly determined adopting the principle of superposition. The limitations of this assumption are discussed in Section 1.3.

Further, the experimental results of Gamble and Parrott (1978) indicate that both drying and basic creep are only partially, not fully, recoverable. In general, provided that water migration does not occur as in sealed concrete or the interior of large concrete elements, superposition can be used to calculate both recovery and relaxation.

The use of the compressive creep to the tensile creep in calculation of beam's time-dependant deflections has been successfully applied in the work by Branson (1977), Bažant and Ho (1984), and Carreira and Chu (1986).

The variability of shrinkage and creep test measurements prevents models from closely matching experimental data. The within-batch coefficient of variation for laboratory-measured shrinkage on a single mixture of concrete was approximately 8% (Bažant et al. 1987). Hence, it would be unrealistic to expect results from prediction models to be within plus or minus 20% of the test data for shrinkage. Even larger differences occur for creep predictions. For structures where shrinkage and creep are deemed critical, material testing should be undertaken and long-term behavior extrapolated from the resulting data. For a discussion of testing for shrinkage and creep, refer to Acker (1993), Acker et al. (1998), and Carreira and Burg (2000).

1.2—Scope

This document was developed to address the issues related to the prediction of creep under compression and shrinkage-induced strains in hardened concrete. It may be assumed, however, that predictions apply to concrete under tension and shear. It outlines the problems and limitations in developing prediction equations, presents and compares the prediction capabilities of the ACI 209R-92 (ACI Committee 209 1992), Bažant-Baweja B3 (Bažant and Baweja 1995, 2000), CEB MC90-99 (Muller and Hillsdorf 1990; CEB 1991, 1993, 1999), and GL2000 (Gardner and Lockman 2001) models, and gives an extensive list of references. The models presented are valid for hardened concrete moist cured for at least 1 day and loaded at the end of 1 day of curing or later. The models apply to concretes with mean compressive cylindrical strengths at 28 days within a range of at least 20 to 70 MPa (3000 to 10,000 psi). The prediction models were calibrated with typical composition concretes, but not with concretes containing silica fume, fly ash contents larger than 30%, or natural pozzolans. Models should be calibrated by testing such concretes. This document does not provide information on the evaluation of the effects of creep and shrinkage on the structural performance of concrete structures.

1.3—Basic assumptions for development of prediction models

Various testing conditions have been established to standardize the measurements of shrinkage and creep. The following simplifying assumptions are normally adopted in the development of prediction models.

1.3.1 *Shrinkage and creep are additive*—Two nominally identical sets of specimens are made and subjected to the same curing and environment conditions. One set is not loaded and is used to determine shrinkage, while the other is generally loaded from 20 to 40% of the concrete compressive strength. Load-induced strains are determined by subtracting the measured shrinkage strains on the nonloaded specimens from the strains measured on the loaded specimens. Therefore, it is assumed that the shrinkage and creep are independent of each other.

Tests carried out on sealed specimens, with no moisture movement from or to the specimens, are used to determine autogenous shrinkage and basic creep.

1.3.2 *Linear aging model for creep*—Experimental research indicates that creep may be considered approximately proportional to stress (L'Hermite et al. 1958; Keeton 1965), provided that the applied stress is less than 40% of the concrete compressive strength.

The strain responses to stress increments applied at different times may be added using the superposition principle (McHenry 1943) for increasing and decreasing stresses, provided strain reversals are excluded (for example, as in relaxation) and temperature and moisture content are kept constant (Le Camus 1947; Hanson 1953; Davies 1957; Ross 1958; Neville and Dilger 1970; Neville 1973; Bažant 1975; Gamble and Parrot 1978; RILEM Technical Committee TC-69 1988). Major deviations from the principle of superposition are caused by the neglect of the random scatter of the creep properties, by hygrothermal effects, including water diffusion and time evolution of the distributions of pore moisture content and temperature, and by material damage, including distributed cracking and fracture, and also frictional microslips. A comprehensive summary of the debate on the applicability of the principle of superposition when dealing with the evaluation of creep structural effects can be found in the references (Bažant 1975, 1999, 2000; CEB 1984; RILEM Technical Committee TC-107 1995; Al Manaseer et al. 1999; Jirasek and Bažant 2002; Gardner and Tsuruta 2004; Bažant 2007).

1.3.3 *Separation of creep into basic creep and drying creep*—Basic creep is measured on specimens that are sealed to prevent the ingress or egress of moisture from or to its environment. It is considered a material constitutive property and independent of the specimen size and shape. Drying creep is the strain remaining after subtracting shrinkage, elastic, and basic creep strains from the total measured strain on nominally identical specimens in a drying environment. The measured average creep of a cross section at drying is strongly size-dependant. Any effects of thermal strains have to be removed in all cases or are avoided by testing at constant temperature.

In sealed concrete specimens, there is no moisture movement into or out of the specimens. Low-water-cement-ratio concretes self-desiccate, however, leading to autogenous shrinkage. Normal-strength concretes do not change volume at relative humidity in the range 95 to 99%, whereas samples stored in water swell (L'Hermite et al. 1958).

1.3.4 *Differential shrinkage and creep or shrinkage and creep gradients are neglected*—The shrinkage strains determined according to ASTM C157/C157M are measured along the longitudinal axis of prismatic specimens; however, the majority of reported creep and shrinkage data are based on surface measurements of cylindrical specimens (ASTM C512). Unless finite element analysis (Bažant et al. 1975) or equivalent linear gradients (Carreira and Walser 1980) are used, it is generally assumed that shrinkage and creep strains in a specimen occur uniformly through the specimen cross section. Kristek et al. (2006) concluded that for box girder bridges, the classical creep analysis that assumes the shrinkage and creep properties to be uniform throughout the cross section is inadequate. As concrete ages, differences in strain gradients reduce (Carreira and Walser 1980; Aguilar 2005).

1.3.5 *Stresses induced during curing phase are negligible*—Most test programs consider the measurement of strains from the start of drying. It is assumed that the restrained stresses due to swelling and autogenous shrinkage are negligible because of the large creep strains and stress relaxation of the concrete at early ages. For restrained swelling, this assumption leads to an overestimation of the tensile stresses and, therefore, it may be an appropriate basis for design when predicting deflections or prestress losses. For predicting the effects of restrained autogenous shrinkage or relaxation, however, the opposite occurs. Limited testing information exists for tensile creep.

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

a, b = constants used to describe the strength gain development of the concrete, ACI 209R-92 and GL2000 models

a = aggregate content of concrete, kg/m^3 or lb/yd^3, B3 model

$C_o(t,t_o)$ = compliance function for basic creep at concrete age t when loading starts at age t_o, B3 model

$C_d(t,t_o,t_c)$ = compliance function for drying creep at concrete age t when loading and drying starts at ages t_o and t_c, respectively, B3 model

c = cement content of concrete, kg/m^3 or lb/yd^3, ACI 209R-92 and B3 models

$d = 4V/S$ = average thickness of a member, mm or in., ACI 209R-92 model

E = modulus of elasticity, MPa or psi

E_{cm} = mean modulus of elasticity of concrete, MPa or psi

E_{cm28} = mean modulus of elasticity of concrete at 28 days, MPa or psi

E_{cmt} = mean modulus of elasticity of concrete at age t, MPa or psi

E_{cmto} = mean modulus of elasticity of concrete when loading starts at age t_o, MPa or psi

$e = 2V/S$ = effective cross section thickness of member or notional size of member according to B3 or CEB MC90 and CEB MC90-99 models, respectively, in mm or in.; defined as the cross-section divided by the semi-perimeter of the member in contact with the atmosphere, which coincides with the actual thickness in the case of a slab

f_{cm} = concrete mean compressive cylinder strength, MPa or psi

f_{cm28} = concrete mean compressive cylinder strength at 28 days, MPa or psi

f_{cmt} = concrete mean compressive cylinder strength at age t, MPa or psi

f_{cmtc} = concrete mean compressive cylinder strength when drying starts at age t_c, MPa or psi

f_{cmto} = concrete mean compressive cylinder strength when loading starts at age t_o, MPa or psi

f'_c = concrete specified cylinder strength at 28 days, MPa or psi

$H(t)$ = spatial average of pore relative humidity at concrete age t, B3 model

h = relative humidity expressed as a decimal

$J(t,t_o)$ = compliance at concrete age t when loading starts at age t_o, 1/MPa or 1/psi

$J(t_o,t_o)$ = elastic compliance at concrete age t_o when loading starts at age t_o, 1/MPa or 1/psi

k_h, $\beta_{RH}(h)$ or $\beta(h)$ = correction term for effect of humidity on shrinkage according to B3, CEB MC90 and CEB MC90-99, or GL2000 models, respectively

k_s = cross-section shape factor, B3 model

q_1 = inverse of asymptotic elastic modulus, 1/MPa or 1/psi, B3 model

$S(t-t_c)$, $\beta_s(t-t_c)$ or $\beta(t-t_c)$ = correction term for effect of time on shrinkage according to B3, CEB MC90, or GL2000 models, respectively

s = slump, mm or in., ACI 209R-92 model. Also, strength development parameter, CEB MC90, CEB MC90-99, and GL2000 models

T = temperature, °C, °F, or °K

t = age of concrete, days

$t-t_c$ = duration of drying, days

t_c = age of concrete when drying starts at end of moist curing, days

t_o = age of concrete at loading, days

V/S = volume-surface ratio, mm or in.

w = water content of concrete, kg/m^3 or lb/yd^3, B3 model

α = air content expressed as percentage, ACI 209R-92 model

α_1 or k = shrinkage constant as function of cement type, according to B3 or GL2000 models, respectively

α_2 = shrinkage constant related to curing conditions, B3 model

α_{as}, α_{ds1} and α_{ds2} = correction coefficients for effect of cement type on autogenous and drying shrinkage, CEB MC90-99 model

$\beta_{as}(t)$ = function describing time development of autogenous shrinkage, CEB MC90-99 model

$\beta_c(t-t_o)$ = correction term for effect of time on creep coefficient according to CEB MC90 and CEB MC90-99 models

$\beta_{ds}(t-t_c)$ = function describing time development of drying shrinkage, CEB MC90-99 model

β_e = factor relating strength development to cement type, GL2000

$\beta_{RH,T}$ = correction coefficient to account for effect of temperature on notional shrinkage, CEB MC90 model

β_{sc} = correction coefficient that depends on type of cement, CEB MC90 model

$\beta_{s,T}(t-t_c)$ = correction coefficient to account for effect of temperature on time development of shrinkage, CEB MC90 model

$\varepsilon_{cas}(t)$ = autogenous shrinkage strain at concrete age t, mm/mm or in./in., CEB MC90-99

$\varepsilon_{cds}(t,t_c)$ = drying shrinkage strain at concrete age t since the start of drying at age t_c, mm/mm or in./in., CEB MC90-99 model

ε_{cso} = notional shrinkage coefficient, mm/mm or in./in., CEB MC90 model

$\varepsilon_{caso}(f_{cm28})$ = notional autogenous shrinkage coefficient, mm/mm or in./in., CEB MC90-99 model

$\varepsilon_{cdso}(f_{cm28})$ = notional drying shrinkage coefficient, mm/mm or in./in., CEB MC90-99 model

$\varepsilon_{sh}(t,t_c)$ = shrinkage strain at concrete age t since the start of drying at age t_c, mm/mm or in./in.

ε_{shu} or $\varepsilon_{sh\infty}$ = notional ultimate shrinkage strain, mm/mm or in./in., ACI 209R-92 and GL2000 models and B3 model, respectively

$\phi(t,t_o)$ = creep coefficient (dimensionless)

$\phi_{28}(t,t_o)$ = 28-day creep coefficient (dimensionless), CEB MC90, CEB MC90-99, and GL2000 models

ϕ_o = notional creep coefficient (dimensionless), CEB MC90 and CEB MC90-99 models

$\phi_{RH}(h)$ = correction term for effect of relative humidity on notional creep coefficient, CEB MC90 and CEB M90-99 models

$\Phi(t_c)$ = correction term for effect of drying before loading when drying starts at age t_c, GL2000 model

ϕ_u = ultimate (in time) creep coefficient, ACI 209R-92 model

γ_c = unit weight of concrete, kg/m^3 or lb/ft^3

γ_{sh} and γ_c = shrinkage and creep correction factor, respectively; also used as product of all applicable corrections factors, ACI 209R-92 model

τ_{sh} = shrinkage half-time, days, ACI 209R-92 and B3 models

ψ = ratio of fine aggregate to total aggregate by weight expressed as percentage, ACI 209R-92 model

2.2—Definitions

autogenous shrinkage—the shrinkage occurring in the absence of moisture exchange (as in a sealed concrete specimen) due to the hydration reactions taking place in the cement matrix. Less commonly, it is termed basic shrinkage or chemical shrinkage.

basic creep—the time-dependent increase in strain under sustained constant load of a concrete specimen in which moisture losses or gains are prevented (sealed specimen).

compliance $J(t,t_o)$—the total load induced strain (elastic strain plus creep strain) at age t per unit stress caused by a unit uniaxial sustained load applied since loading age t_o.

creep coefficient—the ratio of the creep strain to the initial strain or, identically, the ratio of the creep compliance to the compliance obtained at early ages, such as after 2 minutes.

28-day creep coefficient—the ratio of the creep strain to the elastic strain due to the load applied at the age of 28 days ($\phi_{28}(t,t_o) = \phi(t,t_o) \cdot E_{cm28}/E_{cmto}$).

creep strain—the time-dependent increase in strain under constant load taking place after the initial strain at loading.

drying creep—the additional creep to the basic creep in a loaded specimen exposed to a drying environment and allowed to dry.

drying shrinkage—shrinkage occurring in a specimen that is allowed to dry.

elastic compliance or the nominal elastic strain per unit stress $J(t_o,t_o)$—the initial strain at loading age t_o per unit stress applied. It is the inverse of the mean modulus of elasticity of concrete when loading starts at age t_o.

initial strain at loading or nominal elastic strain—the short-term strain at the moment of loading and is frequently considered as a nominal elastic strain as it contains creep that occurs during the time taken to measure the strain.

load-induced strain—the time-dependent strain due to a constant sustained load applied at age t_o.

shrinkage—the strain measured on a load-free concrete specimen.

specific creep—the creep strain per unit stress.

total strain—the total change in length per unit length measured on a concrete specimen under a sustained constant load at uniform temperature.

CHAPTER 3—PREDICTION MODELS

3.1—Data used for evaluation of models

In 1978, Bažant and Panula started collecting shrinkage and creep data from around the world and created a computerized databank, which was extended by Muller and Panula as part of collaboration between the ACI and the CEB established after the ACI-CEB Hubert Rusch workshop on concrete creep (Hillsdorf and Carreira 1980). The databank, now known as the RILEM databank, has been extended and refined under the sponsorship of RILEM TC 107-CSP, Subcommittee 5 (Kuttner 1997; Muller et al. 1999).

Problems encountered in the development of the databank have been discussed by Muller (1993) and others (Al-Manaseer and Lakshmikantan 1999; Gardner 2000). One problem involves which data sets should be included. For example, some investigators do not include the low-modulus sandstone concrete data of Hansen and Mattock (1966), but do include the Elgin gravel concrete data from the same researchers. A further problem is the data of some researchers are not internally consistent. For example, the results from the 150 mm. (6 in.) diameter specimens of Hansen and Mattock are not consistent with the results from the 100 and 200 mm (4 and 8 in.) diameter specimens. Finally, it is necessary to define the relative humidity for sealed and immersed concrete specimens.

A major problem for all models is the description of the concrete. Most models are sensitive to the type of cement and the related strength development characteristics of the material. Simple descriptions, such as ASTM C150 Type I, used in the databank are becoming increasingly difficult to interpret. For example, many cements meet the requirements of Types I, II, and III simultaneously; also, the multiple additions to the clinker allowed in ASTM C595 or in other standards are unknown to the researcher and designer. Nominally identical concretes stored in different environments, such as those tested by Keeton (1965), have different strength development rates. If this information exists, it should be taken into account in model development.

In addition, cement descriptions differ from country to country. The data obtained from European cement concretes may not be directly compared with that of United States cement concretes. Some researchers have suggested that correlation should only be done with recent and relevant data and that different shrinkage and creep curves should be developed for European, Japanese, North American, and South Pacific concretes (McDonald 1990; McDonald and Roper 1993; Sakata 1993; Sakata et al. 2001; Videla et al. 2004; Videla and Aguilar 2005a). While shrinkage and creep may vary with local conditions, research has shown that short-term shrinkage and creep measurements improve the predictions regardless of location (Bažant 1987; Bažant and Baweja 2000; Aguilar 2005). For this reason, the committee recommends short-term testing to determine the shrinkage, creep, and elastic modulus of the concrete to improve the predictions of the long-term deformations of the concrete.

Other issues include:

- The databank does not include sufficient data to validate modeling that includes drying before loading or loading before drying, which are common occurrences in practice;
- Many of the data sets in the databank were measured over relatively short durations, which reduces the usefulness of the data to predict long-term effects; and
- Most of the experiments were performed using small specimens compared with structural elements. It is debatable if the curing environment and consequent mechanical properties of concrete in the interior of large elements are well represented by small specimen experiments (Bažant et al. 1975; Kristek et al. 2006).

Despite these limitations, it is imperative that databanks such as the RILEM databank are maintained and updated as they provide an indispensable source of data in addition to a basis for comparing prediction models.

3.2—Statistical methods for comparing models

Several methods have been used for the evaluation of the accuracy of models to predict experimental data. Just as a single set of data may be described by its mean, mode, median, standard deviation, and maximum and minimum, a model for shrinkage or creep data may have several methods to describe its deviation from the data. The committee could not agree on a single method for comparison of test data with predictions from models for shrinkage and creep. Reducing the comparison between a large number of experimental results and a prediction method to a single number is fraught with uncertainty. Therefore, the committee strongly recommends designers to perform sensitivity analysis of the response of the structure using the models in this report and to carry out short-term testing to calibrate the models to improve their predictions. The summary of the statistical indicators given in Section 4 provides the user with basis for comparison without endorsing any method.

One of the problems with the comparison of shrinkage and creep data with a model's prediction is the increasing divergence and spread of data with time, as shown in the figures of Section 4. Thus, when techniques such as linear regression are used, the weighting of the later data is greater than that of the earlier data (Bažant 1987; Bažant et al. 1987). On the contrary, comparison of the percent deviation of the model from the data tends to weight early-age data more than later-age data. The divergence and spread are a measure of the limitation of the model's capabilities and variability in the experimental data.

Commonly used methods for determining the deviation of a model from the data include:

- Comparison of individual prediction curves to individual sets of test data, which requires a case-by-case evaluation;
- Comparison of the test data and calculated values using linear regression;
- Evaluation of the residuals (measured-predicted value) (McDonald 1990; McDonald and Roper 1993; Al-Manaseer and Lakshmikantan 1999). This method does not represent least-square regression and, if there is a trend in the data, it may be biased; and
- Calculation of a coefficient of variation or standard error of regression normalized by the data centroid.

In the committee's opinion, the statistical indicators available are not adequate to uniquely distinguish between models.

3.3—Criteria for prediction models

Over the past 30 years, several models have been proposed for the prediction of drying shrinkage, creep, and total strains under load. These models are compromises between accuracy and convenience. The committee concludes that one of the primary needs is a model or models accessible to engineers with little specialized knowledge of shrinkage and creep. Major issues include, but are not restricted to:

- How simple or complex a model would be appropriate, and what input information should be required;
- What data should be used for evaluation of the model;
- How closely the model should represent physical phenomena/behavior;
- What statistical methods are appropriate for evaluating a model.

There is no agreement upon which information should be required to calculate the time-dependent properties of concrete; whether the mechanical properties of the concrete specified at the time of design should be sufficient or if the mixture proportions are also required.

At a minimum, the committee believes that shrinkage and creep models should include the following information:

- Description of the concrete either as mixture proportions or mechanical properties such as strength or modulus of elasticity;
- Ambient relative humidity;
- Age at loading;
- Duration of drying;
- Duration of loading; and
- Specimen size.

Models should also:

- Allow for the substitution of test values of concrete strength and modulus of elasticity;
- Allow the extrapolation of measured shrinkage and creep compliance results to get long-term values; and
- Contain mathematical expressions that are not highly sensitive to small changes in input parameters and are easy to use.

As described in ACI 209.1R, it has long been recognized that the stiffness of the aggregate significantly affects the shrinkage and creep of concrete. Some models account for the effect of aggregate type by assuming that the effects of aggregate are related to its density or the concrete elastic modulus. Models that use concrete strength can be adjusted to use a measured modulus of elasticity to account for aggregate properties. Models that do not use the mechanical characteristics of the concrete and rely on mixture proportion information alone may not account for variations in behavior due to aggregate properties.

Until recently, autogenous shrinkage was not considered significant because, in most cases, it did not exceed 150 microstrains. For concretes with water-cement ratios (*w/c*) less than 0.4, mean compressive strengths greater than 60 MPa (8700 psi), or both, however, autogenous shrinkage may be a major component of the shrinkage strain.

Some models consider that basic creep and drying creep are independent and thus additive, while other models have shrinkage and creep as dependent, and thus use multiplicative factors. The physical phenomenon occurring in the concrete may be neither.

3.4—Identification of strains

Equations (3-1) and (3-2) describe the additive simplification discussed in 1.3.1

$$\text{total strain} = \text{shrinkage strain} + \text{compliance} \times \text{stress} \quad (3\text{-}1)$$

$$\text{compliance} = \frac{(\text{elastic strain} + \text{basic creep} + \text{drying creep})}{\text{stress}} \quad (3\text{-}2)$$

The total and shrinkage strains are measured in a creep and shrinkage test program from which the compliance is determined. Errors in the measured data result in errors in the compliance. The elastic strain is determined from early-age measurement, but as discussed previously, it is difficult to separate early-age creep from the elastic strain. Thus, the assumed elastic strain is dependent on the time at which the strain measurement is made and, therefore, on the ignored early creep.

Basic creep and drying creep are determined from the compliance by subtracting the elastic strain, which may have implicit errors, from the strains measured on drying and nondrying specimens. Errors in the measured elastic strain used to determine the modulus of elasticity (ASTM C469), in the measured total strain, or in the measured shrinkage strain, are all reflected in the calculated creep strain, the compliance, and creep coefficient.

For sealed specimens, the equations for compliance and total strain simplify significantly if autogenous shrinkage is ignored as in Eq. (3-3) and (3-4)

$$\text{total strain} = \text{compliance} \times \text{stress} \tag{3-3}$$

$$\text{compliance} = \frac{(\text{elastic strain} + \text{basic creep})}{\text{stress}} \tag{3-4}$$

3.5—Evaluation criteria for creep and shrinkage models

In 1995, RILEM Committee TC 107 published a list of criteria for the evaluation of shrinkage and creep models (RILEM 1995; Bažant 2000). In November 1999, ACI Committee 209, which has a number of members in common with RILEM TC 107, discussed the RILEM guidelines and agreed on the following:

- Drying shrinkage and drying creep should be bounded. That is, they do not increase indefinitely with time;
- Shrinkage and creep equations should be capable of extrapolation in both time and size;
- Shrinkage and creep models should be compared with the data in the databank limited by the conditions of applicability of the model(s). That is, some experimental data, such as those with high water-cement ratios or low-modulus concrete, may not be appropriate to evaluate a model;
- Equations should be easy to use and not highly sensitive to changes in input parameters;
- The shape of the individual shrinkage and creep curves over a broad range of time (minutes to years) should agree with individual test results;
- Creep values should be compared as compliance or specific creep rather than as the creep coefficient. The immediate strain/unit stress and the modulus of elasticity are dependent on the rate of loading; however, for developing the creep equations to determine long-term deformations, this effect should not play a major role;
- Creep expressions should accommodate drying before loading. Results by Abiar reported by Acker (1993) show that predried concrete experiences very little creep. Similarly, the very late-age loaded (2500 to 3000 days) results of Wesche et al. (1978) show reduced creep compared with similar concrete loaded at early ages. The effect of predrying may also be significantly influenced by the size of the specimen;
- Shrinkage and creep expressions should be able to accommodate concretes containing fly ash, slag (Videla and Gaedicke 2004), natural pozzolans (Videla et al. 2004; Videla and Aguilar 2005a), silica fume and chemical admixtures (Videla and Aguilar 2005b);
- The models should allow for the effect of specimen size; and
- The models should allow for changes in relative humidity.

Success in achieving the following guidelines is consequent to the method of calculation; that is, if the principle of superposition is valid and if the model includes drying before loading, and how they are considered under unloading:

- Recovery of creep strains under complete unloading should not exceed the creep strain from loading, and should asymptotically approach a constant value; and
- Stress relaxation should not exceed the initially applied stress.

Yue and Taerwe (1992, 1993) published two related papers on creep recovery. Yue and Taerwe (1992) commented, "It is well known that the application of the principle of superposition in the service stress range yields an inaccurate prediction of concrete creep when unloading takes place." In their proposed two-function method, Yue and Taerwe (1993) used a linear creep function to model the time-dependent deformations due to increased stress on concrete, and a separate nonlinear creep recovery function to represent concrete behavior under decreasing stress.

CHAPTER 4—MODEL SELECTION

There are two practical considerations in the models for prediction of shrinkage and creep, namely:

- Mathematical form of their time dependency; and
- Fitting of the parameters and the resulting expressions.

If the mathematical form of the model does not accurately describe the phenomena, extrapolations of shrinkage and creep results will deviate from reality. After the mathematical form has been justified, the fit of the prediction to measured results should be compared for individual data sets.

The models selected for comparison are the ACI 209R-92 (ACI Committee 209 1992), the Bažant-Baweja B3 developed by Bažant and Baweja (1995, 2000), the CEB Model Code 1990-99 (CEB MC90-99) (Muller and Hillsdorf 1990; CEB 1991, 1993, 1999), and the GL2000 developed by Gardner and Lockman (2001). Table 4.1 lists the individual model's applicable range for different input variables (adapted from Al-Manaseer and Lam 2005). Comparison of models with experimental data is complicated by the lack of agreement on selection of appropriate data and on the methods used to compare the correlation. Descriptions of the ACI 209R-92, Bažant-Baweja B3, CEB MC90-99, and GL2000 models are given in Appendix A. Kristek et al. (2001) and Sassone and Chiorino (2005) developed design aids for determination of shrinkage, compliance, and relaxation for ACI 209R-92, Bažant-Baweja B3, CEB MC90-99, and GL2000 models.

Figures 4.1 through 4.8 (Gardner 2004) compare the predicted values for two sets of input information for RILEM data sets extending longer than 500 days, concrete 28-day mean cylinder strengths f_{cm28} between 16 and 82 MPa (2320 and 11,890 psi), water-cement ratios between 0.4 and 0.6, duration of moist curing longer than 1 day (possibly biased against ACI 209R-92 because this model was developed for standard conditions considering 7 days of moist curing and 7 days of age at loading), age of loading greater than the duration of moist curing, and volume-surface ratios V/S greater than 19 mm (3/4 in.). The humidity range for compliance was 20 to 100%, and below 80% for

Table 4.1—Parameter ranges of each model

Input variables	ACI 209R-92	Bažant-Baweja B3	CEB MC90	CEB MC90-99	GL2000
	Model				
f_{cm28}, MPa (psi)	—	17 to 70 (2500 to 10,000)	20 to 90 (2900 to 13,000)	15 to 120 (2175 to 17,400)	16 to 82 (2320 to 11,900)
a/c	—	2.5 to 13.5	—	—	—
Cement content, kg/m^3 (lb/yd^3)	279 to 446 (470 to 752)	160 to 720 (270 to 1215)	—	—	—
w/c	—	0.35 to 0.85	—	—	0.40 to 0.60
Relative humidity, %	40 to 100	40 to 100	40 to 100	40 to 100	20 to 100
Type of cement, European (U.S.)	R or RS (I or III)	R, SL, RS (I, II, III)	R, SL, RS (I, II, III)	R, SL, RS (I, II, III)	R, SL, RS (I, II, III)
t_c (moist cured)	≥ 1 day	≥ 1 day	< 14 days	< 14 days	≥ 1 day
t_c (steam cured)	1 to 3 days	—	—	—	—
t_o	≥ 7 days	$t_o \geq t_c$	> 1 day	> 1 day	$t_o \geq t_c \geq 1$ day

shrinkage. Consequently, swelling was not included even if some specimens were initially moist cured.

Two sets of comparisons are shown in each figure. One set, identified as "f_{cm} only," assumes that only the measured 28-day strength f_{cm} is known. The second set, identified as "all data," uses the f_{cm} calculated as the average of the measured f_{cm}, and that back-calculated from the measured E_{cm} using the elastic modulus formula of the method and mixture proportions if required by the model. Calculated compliance is the calculated specific creep plus calculated elastic compliance for the f_{cm} graphs and the calculated specific creep plus measured elastic compliance for the all data graphs. The reported mixture composition was used for ACI 209R-92 and Bažant-Baweja B3. It was assumed that if mixture data were available, the strength development data and elastic modulus would also be available. Cement type was determined by comparison of measured strength gain data with the GL2000 strength gain equations. The same cement type was used for predictions in all methods. For CEB MC90-99, ASTM C150 Type I was taken as CEB Type N cement, Type III as CEB Type R, and Type II as CEB Type SL.

It should be noted that each model should use an appropriate value of elastic modulus for which the model was calibrated. Therefore, for CEB, the elastic modulus was taken as $E_{cm} = 9500(f_{cm})^{1/3}$ in MPa ($262{,}250[f_{cm}]^{1/3}$ in psi). For Bažant-Baweja B3, using the shape factor $k_s = 1.00$ in τ_s (the shrinkage time function) improved the results of the statistical analysis, and all concretes were assumed moist cured; that is, $\alpha_2 = 1.20$ for calculations using the Bažant-Baweja B3 model.

To calculate a coefficient of variation (Gardner 2004), the durations after drying or application of load were divided into seven half-log decade intervals: 3 to 9.9 days, 10 to 31 days, 32 to 99 days, 100 to 315 days, 316 to 999 days, 1000 to 3159 days, and greater than 3160 days. That is, each duration is 3.16 times the previous half-log decade; these are similar to the CEB ranges. The root mean square (RMS) (calculated-observed) was calculated for all comparisons in each half-log decade. The coefficient of variation was the average RMS/average experimental value for the same half-log decade.

4.1—ACI 209R-92 model

The model recommended by ACI Committee 209 (1971) was developed by Branson and Christiason (1971), with minor modifications introduced in ACI 209R-82 (ACI Committee 209 1982). ACI Committee 209 incorporated the developed model in ACI 209R-92 (ACI Committee 209 1992). Since then, it has not been revised or updated to the RILEM databank, and it is compared with very recent models. This model, initially developed for the precast-prestressing industry (Branson and Ozell 1961; Branson 1963, 1964, 1968; Branson et al. 1970; Meyers et al. 1970; Branson and Kripanayanan 1971; Branson and Chen 1972), has been used in the design of structures for many years.

Advantages of this model include:

- It is simple to use with minimal background knowledge; and
- It is relatively easy to adjust to match short-term test data simply by modifying ultimate shrinkage or creep to produce the best fit to the data.

Its disadvantages include:

- It is limited in its accuracy, particularly in the method of accommodating member size when its simplest form is used. This disadvantage, however, can be overriden if the methods provided for accommodating the shape and size effect on the time-ratio are applied; and
- It is empirically based, thus it does not model shrinkage or creep phenomena.

At its most basic level, the ACI 209R-92 method only requires:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing;
- Age of concrete at loading;
- Curing method;
- Relative humidity expressed as a decimal;
- Volume-surface ratio or average thickness; and
- Cement type.

This model calculates the creep coefficient rather than the compliance, which may introduce problems due to the assumed value of elastic modulus. Figures 4.1 and 4.2 show the calculated and measured shrinkages and compliances, respectively. The comparison of shrinkage data in Fig. 4.1 clearly shows that the ACI 209R-92 model overestimates measured shrinkage at low shrinkage values (equivalent to short drying times) and underestimates at high shrinkage values (typical of long drying times). This result indicates the limitation of the model's equation used to predict shrinkage. The ACI 209R-92 compliance comparison is rather insensitive to using all of the available data, including mixture proportions, compared with just using the measured concrete strength.

4.2—Bažant-Baweja B3 model

The Bažant-Baweja B3 model (Bažant and Baweja 1995, 2000) is the culmination of work started in the 1970s (Bažant et al. 1976, 1991; Bažant and Panula 1978, 1984; Jirasek and Bažant 2002), and is based on a mathematical description of over 10 physical phenomena affecting creep and shrinkage (Bažant 2000), including known fundamental asymptotic properties that ought to be satisfied by a creep and shrinkage model (Bažant and Baweja 2000, RILEM Technical Committee TC 107 1995). This model has been found to be useful for those dealing with simple as well as complex structures. The Bažant-Baweja B3 model uses the compliance function. The compliance function reduces the risk of errors due to inaccurate values of the elastic modulus. The model clearly separates basic and drying creep.

The factors considered include:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing;
- Age of concrete at loading;
- Aggregate content in concrete;
- Cement content in concrete;
- Cement type;
- Concrete mean compressive strength at 28 days;
- Curing method;
- Relative humidity;
- Shape of specimen;
- Volume-surface ratio; and
- Water content in concrete.

Both Bažant-Baweja B3 shrinkage and creep models may require input data that are not generally available at time of design, such as the specific concrete proportions and concrete mean compressive strength. Default values of the input parameters can be automatically considered if the user lacks information on some of them. The authors suggest when only f_{cm28} is known, the water-cement ratio can be determined using Eq. (4-1), and typical values of cement content and aggregate cement ratio should be assumed

$$w/c = [(f_{cm28}/22.8) + 0.535]^{-1} \quad \text{in SI units}$$
$$w/c = [(f_{cm28}/3300) + 0.535]^{-1} \quad \text{in in.-lb units} \qquad (4\text{-}1)$$

Equation (4-1) represents the best-fit linear regression equation to the values reported in Tables A1.5.3.4(a) and A6.3.4(a) of ACI 211.1-91 (ACI Committee 211 1991) for non-air-entrained concretes made with Type 1 portland cement; for air-entrained concretes, similar equations can be derived by regression analysis of the reported values on ACI 211.1-91. For other cement types and cementitious materials, ACI 211.1-91 suggests that the relationship between water-cement or water-cementitious material ratio and compressive strength of concrete be developed for the materials actually to be used.

Figures 4.3 and 4.4 show the comparison between the calculated and measured shrinkages and compliances, respectively. The shrinkage equation is sensitive to the water content.

The model allows for extrapolation from short-term test data using short-term test data and a test of short-term moisture-content loss.

4.3—CEB MC90-99 model

In 1990, CEB presented a model for the prediction of shrinkage and creep in concrete developed by Muller and

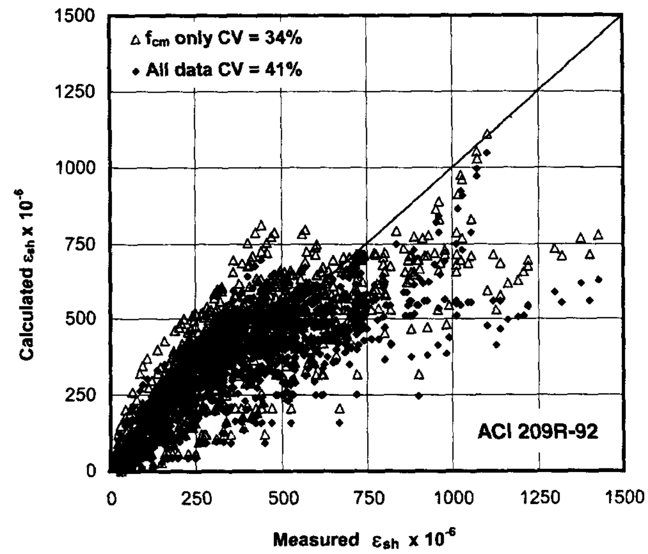

Fig. 4.1—ACI 209R-92 versus RILEM shrinkage databank (Gardner 2004).

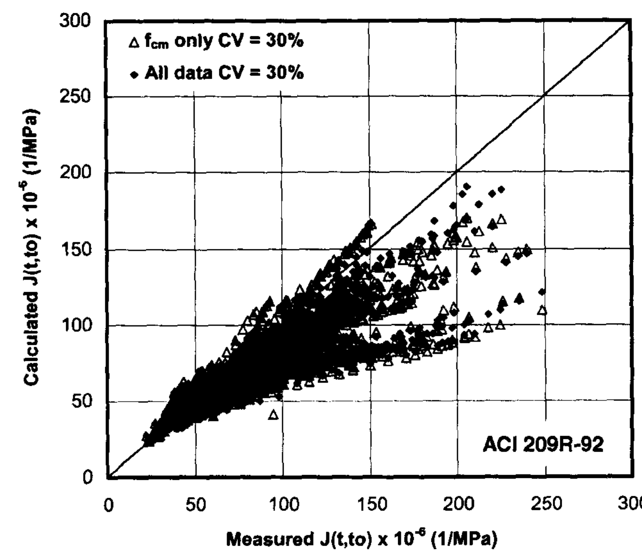

Fig. 4.2—ACI 209R-92 versus RILEM compliance databank (Gardner 2004).

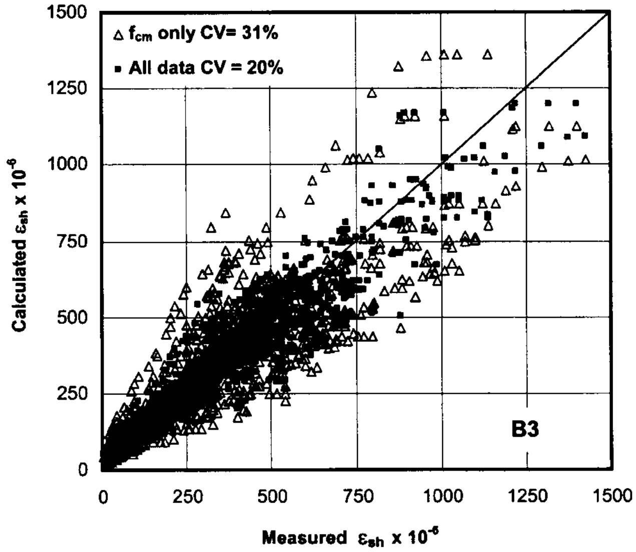

Fig. 4.3—Bažant-Baweja B3 versus RILEM shrinkage databank (Gardner 2004).

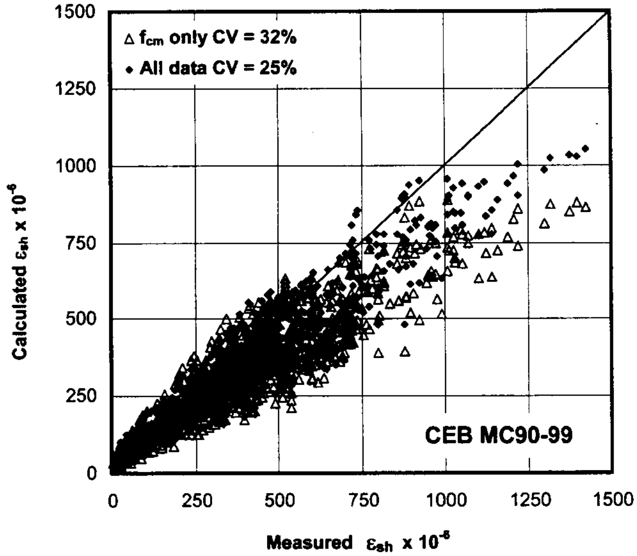

Fig. 4.5—CEB MC90-99 versus RILEM shrinkage databank (Gardner 2004).

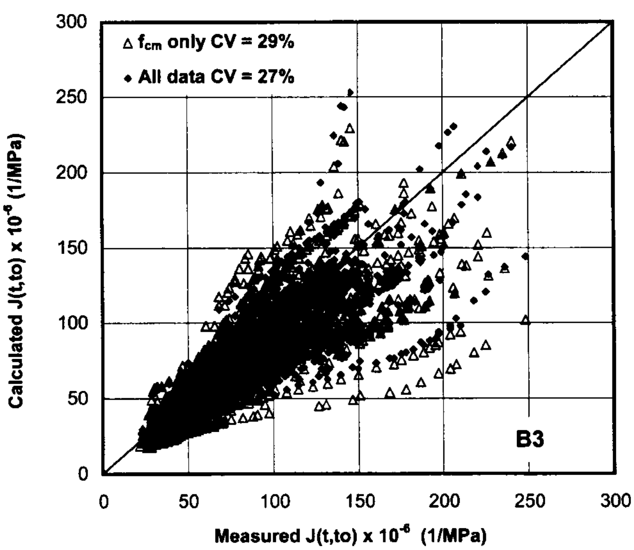

Fig. 4.4—Bažant-Baweja B3 versus RILEM compliance databank (Gardner 2004).

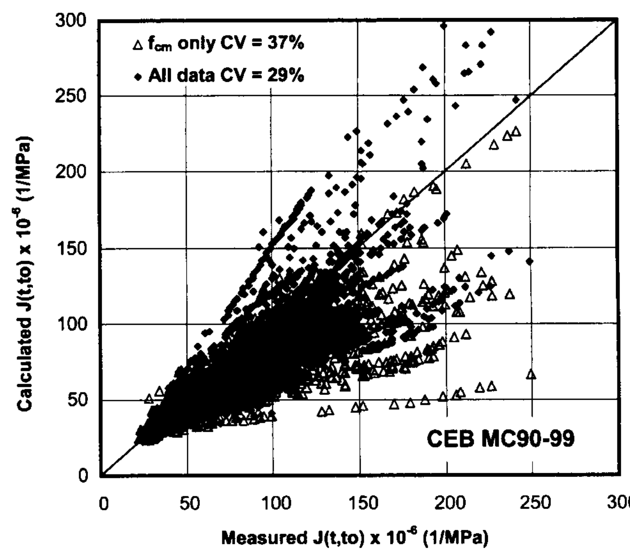

Fig. 4.6—CEB MC90-99 versus RILEM compliance databank (Gardner 2004).

Hilsdorf (1990). The model was revised in 1999 (CEB 1999) to include normal- and high-strength concretes and to separate the total shrinkage into its autogenous and drying shrinkage components, and it is called CEB MC90-99. While the revised models for the drying shrinkage component and for the compliance are closely related to the approach in CEB MC90 (Müller and Hilsdorf 1990, CEB 1993), for autogenous shrinkage, new relations were derived, and some adjustments were included for both normal- and high-strength concrete. For these reasons, the CEB 1990 and the revised CEB 1999 models are described in Appendix A. Some engineers working on creep and shrinkage-sensitive structures have accepted this model as preferable to the ACI 209R-92 model (based on the 1971 Branson and Christiason model). The CEB models do not require any information regarding the duration of curing or curing condition. The duration of drying might have a direct impact on the shrinkage and creep of concrete, and should not be ignored when predicting the shrinkage and compliance. The correction term used for relative humidity in the creep equation is extremely sensitive to any variation in relative humidity. Figures 4.5 and 4.6 compare the calculated and measured shrinkages and compliances, respectively.

The method requires:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing;
- Age of concrete at loading;
- Concrete mean compressive strength at 28 days;
- Relative humidity expressed as a decimal;
- Volume-surface ratio; and
- Cement type.

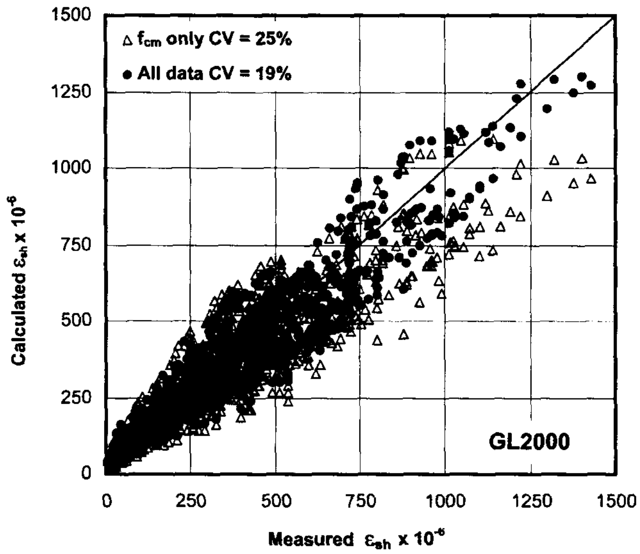

Fig. 4.7—GL2000 versus RILEM shrinkage databank (Gardner 2004).

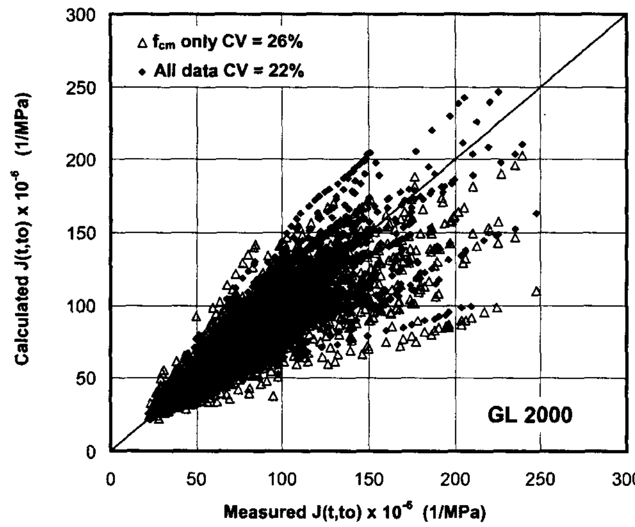

Fig. 4.8—GL2000 versus RILEM compliance databank (Gardner 2004).

Using only the data with reported concrete strength, the model generally underestimates the shrinkage of North American concretes, and substantially underestimates the shrinkage of concretes containing basalt aggregates found in Hawaii, Australia, and New Zealand (McDonald 1990; McDonald and Roper 1993; Robertson 2000). The main reason is that primarily European concretes (lower cement content and other types of cement) were considered when optimizing the model. The shrinkage model does not respond well to early-age extrapolation using the simple linear regression method suggested by Bažant (1987); however, the creep model does (Robertson 2000).

4.4—GL2000 model

The GL2000 model was developed by Gardner and Lockman (2001), with minor modifications introduced by Gardner (2004). The model is a modification of the GZ Atlanta 97 model (Gardner 2000) made to conform to the ACI 209 model guidelines given in Section 3.5. Except for the concrete compressive strength, the model only requires input data that are available to engineer at time of design. Figure 4.7 and 4.8 compare the calculated and measured shrinkages and compliances, respectively.

The method requires:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing;
- Age of concrete at loading;
- Relative humidity expressed as a decimal;
- Volume-surface ratio;
- Cement type; and
- Concrete mean compressive strength at 28 days.

4.5—Statistical comparisons

As stated previously, there is no agreement as to which statistical indicator(s) should be used, which data sets should be used, or what input data should be considered. To avoid revising any investigator's results, the statistical comparisons of Bažant and Baweja (2000), Al-Manaseer and Lam (2005), and Gardner (2004) are summarized in Table 4.2 for shrinkage and in Table 4.3 for compliance. As the statistical indicators represent different quantities and the investigators used different experimental results, comparisons can only be made across a row, but cannot be made between lines in the tables. Descriptions of the statistical indicators are given in Appendix B.

Al-Manaseer and Lam (2005) noted that careful selection and interpretation of concrete data and the statistical methods can influence the conclusions on the performance of model prediction on creep and shrinkage.

Brooks (2005) also reported the accuracy of five prediction models, including ACI 209R-92, Bažant-Baweja B3, CEB MC90, and GL2000 models, in estimating 30-year deformation, concluding that most methods fail to recognize the influence of strength of concrete and type of aggregate on creep coefficient, which ranged from 1.2 to 9.2. Brooks (2005) also reported that shrinkage ranged from 280 to 1460 x 10^{-6}, and swelling varied from 25 to 35% of shrinkage after 30 years.

4.6—Notes about models

The prediction capabilities of the four shrinkage and compliance models were evaluated by comparing calculated results with the RILEM databank. For shrinkage strain prediction, Bažant-Baweja B3 and GL2000 provide the best results. The CEB MC90-99 underestimates the shrinkage. For compliance, GL2000, CEB MC90-99, and Bažant-Baweja B3 give acceptable predictions. The ACI 209R-92 method underestimates compliance for the most of the RILEM databank. It should be noted that for shrinkage predictions, Bažant-Baweja B3 using Eq. (4-1) instead of experimental values for water, cement, and aggregate masses provides less accurate, but still acceptable, results.

Except for ACI 209R-92, using more information improved the prediction for all other methods. The predictions from the CEB, GL2000, and Bažant-Baweja B3 models were significantly improved by using measured strength development

Table 4.2—Statistical indicators for shrinkage

Investigator	Indicator	Model: ACI 209R-92	Bažant-Baweja B3	CEB MC90	CEB MC90-99	GL2000
Bažant and Baweja (2000)	$\varpi_{BP}{}^*$	55%	34%	46%	—	—
Al-Manaseer and Lam (2005)	$V_{CEB}{}^*$	46%	41%	52%	37%	37%
	$F_{CEB}{}^*$	83%	84%	60%	65%	84%
	$M_{CEB}{}^\dagger$	1.22	1.07	0.75	0.99	1.26
	$\varpi_{BP}{}^*$	102%	55%	90%	48%	46%
Gardner (2004), f_{cm} only	$\omega_G{}^*$	34%	31%	—	32%	25%
Gardner (2004), all data	$\omega_G{}^*$	41%	20%	—	25%	19%

*Perfect correlation = 0%.
†Perfect correlation = 1.00.

Table 4.3—Statistical indicators for compliance

Investigator	Indicator	Model: ACI 209R-92	Bažant-Baweja B3	CEB MC90	CEB MC90-99	GL2000
Bažant and Baweja (2000), basic creep	$\varpi_{BP}{}^*$	58%	24%	35%	—	—
Bažant and Baweja (2000), drying creep	$\varpi_{BP}{}^*$	45%	23%	32%	—	—
Al-Manaseer and Lam (2005)	$V_{CEB}{}^*$	48%	36%	36%	38%	35%
	$F_{CEB}{}^*$	32%	35%	31%	32%	34%
	$M_{CEB}{}^\dagger$	0.86	0.93	0.92	0.89	0.92
	$\varpi_{BP}{}^*$	87%	61%	75%	80%	47%
Gardner (2004), f_{cm} only	$\omega_G{}^*$	30%	29%	—	37%	26%
Gardner (2004), all data	$\omega_G{}^*$	30%	27%	—	29%	22%

*Perfect correlation = 0%.
†Perfect correlation = 1.00.

and measured elastic modulus of the concrete to modify the concrete strength used in creep and shrinkage equations.

It should be noted that the accuracy of the models is limited by the many variables outlined previously and measurement variability. For design purposes, the accuracy of the prediction of shrinkage calculated using GL2000 and Bažant-Baweja B3 models may be within ±20%, and the prediction of compliance ±30%. Parametric studies should be made by the designer to ensure that expected production variations in concrete composition, strength, or the environment do not cause significant changes in structural response.

The coefficients of variation for shrinkage measured by Bažant et al. (1987) in a statistically significant investigation were 10% at 7 days and 7% at 1100 days, and can be used as a benchmark for variations between batches. A model that could predict the shrinkage within 15% would be excellent, and 20% would be adequate. For compliance, the range of expected agreement would be wider because, experimentally, compliance is determined by subtracting two measured quantities of similar magnitude.

There is not an accepted sign convention for stress and strain. In this document, shortening strains and compressive stresses are positive. For all models, it is necessary to estimate the environmental humidity. The Precast/Prestressed Concrete Institute's *PCI Design Handbook* (2005) gives values of the annual average ambient relative humidity throughout the United States and Canada that may be used as a guide. Care should be taken when considering structures, such as swimming pools or structures near water. Although the models are not sensitive to minor changes in input values, the effect of air conditioning in moist climates and exposure to enclosed pool in dry climates can be significant. Therefore, the effects of air conditioning and heating on the local environment around the concrete element should be considered.

Relaxation, the gradual reduction of stress with time under sustained strain, calculated using ACI 209R-92, Bažant-Baweja B3, CEB MC90-99, and GL2000, agreed with Rostasy et al.'s (1972) experimental results indicating that the principle of superposition can be used to calculate relaxation provided that calculations are done keeping any drying before loading term constant at the initial value (Lockman 2000).

Lockman (2000) did a parametric comparison of models based upon the work of Chiorino and Lacidogna (1998a,b); see also Chiorino (2005). CEB MC90 and ACI 209R-92 underestimate the compliance compared with the GL2000 and Bažant-Baweja B3 models using the same input parameters. Relaxations calculated by Bažant-Baweja B3 are significantly different than those calculated for the three other models. The elastic strains, calculated at 30 seconds after loading, for the Bažant-Baweja B3 model are very different from those calculated by the other three models. The method of calculating the elastic strain is unique to this model, and the initial stresses of relaxation differ radically from other models.

For all ages of loading, especially in a drying environment, Bažant-Baweja B3 predicts more relaxation than the other models. Unlike the other models, Bažant-Baweja B3 uses an asymptotic elastic modulus (fast rate of loading), and not the conventional elastic modulus, which typically includes a significant early-age creep portion. The use of a larger asymptotic elastic modulus explains the comments about relaxation curves obtained from the Bažant-Baweja B3 model. For early ages of loading, the relaxations calculated using CEB MC90-99 and ACI 209R-92 are nearly 100% of the initial stress, with residual stresses close to zero.

For creep recovery, GL2000 and Bažant-Baweja B3 are the only models that predict realistic recoveries by superposition. For partial creep recovery, that is, superposition not assumed, with complete removal of the load, no model provides realistic results. Calculating recovery by superposition is subject to more problems than calculating relaxation by superposition. If recovery is to be calculated by superposition, both basic and drying creep compliance functions have to be parallel in time to give a constant compliance after unloading. As drying before loading reduces both basic and drying creep, it is not yet possible to determine a formulation that permits calculating recovery by superposition in a drying environment. Experimental evidence (Neville 1960) is inconclusive on whether either drying creep or basic creep is completely recoverable.

High-strength concretes with water-cement ratios less than 0.40 and mean concrete strengths greater than 80 MPa (11,600 psi) experience significant autogenous shrinkage. The magnitude of the autogenous shrinkage also depends on the availability of moisture during early-age curing. Concretes containing silica fume appear to behave differently from conventional concretes. Few data on such concretes are held in the databank and hence, caution should be exercised using equations justified by the databank for such concretes. The models, however, can be used in such circumstances if they are calibrated with test data.

CHAPTER 5—REFERENCES

5.1—Referenced standards and reports

The latest editions of the standards and reports listed below were used when this document was prepared. Because these documents are revised frequently, the reader is advised to review the latest editions for any changes.

American Concrete Institute

116R	Cement and Concrete Terminology
209.1R	Report on Factors Affecting Shrinkage and Creep of Hardened Concrete

ASTM International

C150	Specification for Portland Cement
C595	Specification for Blended Hydraulic Cements
C157	Test Method for Length Change of Hardened Hydraulic Cement, Mortar, and Concrete
C512	Test Method for Creep of Concrete in Compression
C469	Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression

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APPENDIX A—MODELS

A.1—ACI 209R-92 model

This is an empirical model developed by Branson and Christiason (1971), with minor modifications introduced in ACI 209R-82 (ACI Committee 209 1982). ACI Committee 209 incorporated the developed model in ACI 209R-92 (ACI Committee 209 1992).

The models for predicting creep and shrinkage strains as a function of time have the same principle: a hyperbolic curve that tends to an asymptotic value called the ultimate value. The form of these equations is thought to be convenient for design purposes, in which the concept of the ultimate (in time) value is modified by the time-ratio (time-dependent development) to yield the desired result. The shape of the curve and ultimate value depend on several factors, such as curing conditions, age at application of load, mixture proportioning, ambient temperature, and humidity.

The design approach presented for predicting creep and shrinkage refers to standard conditions and correction factors for other-than-standard conditions. The correction factors are applied to ultimate values. Because creep and shrinkage equations for any period are linear functions of the ultimate values, however, the correction factors in this procedure may be applied to short-term creep and shrinkage as well.

The recommended equations for predicting a creep coefficient and an unrestrained shrinkage strain at any time, including ultimate values, apply to normalweight, sand lightweight, and all lightweight concrete (using both moist and steam curing, and Types I and III cement) under the standard conditions summarized in Table A.1.

Required parameters:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing (days);
- Age of concrete at loading (days);
- Curing method;
- Ambient relative humidity expressed as a decimal;
- Volume-surface ratio or average thickness (mm or in.);
- Concrete slump (mm or in.);
- Fine aggregate percentage (%);
- Cement content (kg/m^3 or lb/yd^3);
- Air content of the concrete expressed in percent (%); and
- Cement type

A.1.1 *Shrinkage*—The shrinkage strain $\varepsilon_{sh}(t,t_c)$ at age of concrete t (days), measured from the start of drying at t_c (days), is calculated by Eq. (A-1)

$$\varepsilon_{sh}(t,t_c) = \frac{(t-t_c)^\alpha}{f+(t-t_c)^\alpha}\cdot\varepsilon_{shu} \quad \text{(A-1)}$$

where f (in days) and α are considered constants for a given member shape and size that define the time-ratio part, ε_{shu} is the ultimate shrinkage strain, and $(t-t_c)$ is the time from the end of the initial curing.

For the standard conditions, in the absence of specific shrinkage data for local aggregates and conditions and at ambient relative humidity of 40%, the average value suggested for the ultimate shrinkage strain ε_{shu}, is

$$\varepsilon_{shu} = 780\times10^{-6} \text{ mm/mm (in./in.)} \quad \text{(A-2)}$$

For the time-ratio in Eq. (A-1), ACI 209R-92 recommends an average value for f of 35 and 55 for 7 days of moist curing and 1 to 3 days of steam curing, respectively, while an average value of 1.0 is suggested for α (flatter hyperbolic form). It should be noted that the time-ratio does not differentiate between drying, autogenous, and carbonation shrinkage. Also, it is independent of member shape and size, because f and α are considered as constant.

The shape and size effect can be totally considered on the time-ratio by replacing $\alpha = 1.0$, and f as given by Eq. (A-3), in Eq. (A-1), where V/S is the volume-surface ratio in mm or in.

Table A.1—Factors affecting concrete creep and shrinkage and variables considered in recommended prediction method

Factors			Variables considered	Standard conditions
Concrete (creep and shrinkage)	Concrete composition	Cement paste content	Type of cement	Type I and III
		Water-cement ratio	Slump	70 mm (2.7 in.)
		Mixture proportions	Air content	≤ 6%
			Fine aggregate percentage	50%
		Aggregate characteristics Degrees of compaction	Cement content	279 to 446 kg/m^3 (470 to 752 lb/yd^3)
	Initial curing	Length of initial curing	Moist cured	7 days
			Steam cured	1 to 3 days
		Curing temperature	Moist cured	23.2 ± 2 °C (73.4 ± 4 °F)
			Steam cured	≤ 100 °C (≤ 212 °F)
		Curing humidity	Relative humidity	≥ 95%
Member geometry and environment (creep and shrinkage)	Environment	Concrete temperature	Concrete temperature	23.2 ± 2 °C (73.4 ± 4 °F)
		Concrete water content	Ambient relative humidity	40%
	Geometry	Size and shape	Volume-surface ratio	V/S = 38 mm (1.5 in.)
			or minimum thickness	150 mm (6 in.)
Loading (creep only)	Loading history	Concrete age at load application	Moist cured	7 days
			Steam cured	1 to 3 days
		During of loading period	Sustained load	Sustained load
		Duration of unloading period	—	—
		Number of load cycles	—	—
	Stress conditions	Type of stress and distribution across the section	Compressive stress	Axial compression
		Stress/strength ratio	Stress/strength ratio	≤ 0.50

$$f = 26.0e^{\{1.42 \times 10^{-2}(V/S)\}} \quad \text{in SI units}$$
$$f = 26.0e^{\{0.36(V/S)\}} \quad \text{in in.-lb units} \tag{A-3}$$

For conditions other than the standard conditions, the average value of the ultimate shrinkage ε_{shu} (Eq. (A-2)) needs to be modified by correction factors. As shown in Eq. (A-4) and (A-5), ACI 209R-92 (ACI Committee 209 1992) suggests multiplying ε_{shu} by seven factors, depending on particular conditions

$$\varepsilon_{shu} = 780\gamma_{sh} \times 10^{-6} \text{ mm/mm (in./in.)} \tag{A-4}$$

with

$$\gamma_{sh} = \gamma_{sh,tc}\gamma_{sh,RH}\gamma_{sh,vs}\gamma_{sh,s}\gamma_{sh,\psi}\gamma_{sh,c}\gamma_{sh,\alpha} \tag{A-5}$$

where γ_{sh} represents the cumulative product of the applicable correction factors as defined as follows.

The initial moist curing coefficient $\gamma_{sh,tc}$ for curing times different from 7 days for moist-cured concrete, is given in Table A.2 or Eq. (A-6); for steam curing with a period of 1 to 3 days, $\gamma_{sh,tc} = 1$.

The $\gamma_{sh,cp}$ correction factors shown in Table A.2 for the initial moist curing duration variable can be obtained by linear regression analysis as given in Eq. (A-6)

$$\gamma_{sh,tc} = 1.202 - 0.2337\log(t_c) \quad R^2 = 0.9987 \tag{A-6}$$

The ambient relative humidity coefficient $\gamma_{sh,RH}$ is

$$\gamma_{sh,RH} = \begin{cases} 1.40 - 1.02h & \text{for } 0.40 \le h \le 0.80 \\ 3.00 - 3.0h & \text{for } 0.80 \le h \le 1 \end{cases} \tag{A-7}$$

where the relative humidity h is in decimals.

For lower than 40% ambient relative humidity, values higher than 1.0 should be used for shrinkage $\gamma_{sh,RH}$. Because $\gamma_{sh,RH} = 0$ when $h = 100\%$, the ACI method does not predict swelling.

Coefficient $\gamma_{sh,vs}$ allows for the size of the member in terms of the volume-surface ratio, for members with volume-surface ratio other than 38 mm (1.5 in.), or average thickness other than 150 mm (6 in.). The average thickness d of a member is defined as four times the volume-surface ratio; that is $d = 4V/S$, which coincides with twice the actual thickness in the case of a slab

$$\gamma_{sh,vs} = 1.2e^{\{-0.00472(V/S)\}} \quad \text{in SI units}$$
$$\gamma_{sh,vs} = 1.2e^{\{-0.12(V/S)\}} \quad \text{in in.-lb units} \tag{A-8}$$

where V is the specimen volume in mm^3 or in.^{3}, and S the specimen surface area in mm^2 or in^2.

Alternatively, the method also allows the use of the average-thickness method to account for the effect of member size on ε_{shu}. The average-thickness method tends to compute

Table A.2—Shrinkage correction factors for initial moist curing, $\gamma_{sh,tc}$, for use in Eq. (A-5), ACI 209R-92 model

Moist curing duration t_c, days	$\gamma_{sh,tc}$
1	1.2
3	1.1
7	1.0
14	0.93
28	0.86
90	0.75

Table A.3—Shrinkage correction factors for average thickness of members, $\gamma_{sh,d}$, for use in Eq. (A-5), ACI 209R-92 model

Average thickness of member d, mm (in.)	Volume/surface ratio V/S, mm (in.)	Shrinkage factor $\gamma_{sh,d}$
51 (2)	12.5 (0.50)	1.35
76 (3)	19 (0.75)	1.25
102 (4)	25 (1.00)	1.17
127 (5)	31 (1.25)	1.08
152 (6)	37.5 (1.50)	1.00

correction factor values that are higher, as compared with the volume-surface ratio method.

For average thickness of member less than 150 mm (6 in.) or volume-surface ratio less than 37.5 mm (1.5 in.), use the factors given in Table A.3.

For average thickness of members greater than 150 mm (6 in.) and up to about 300 to 380 mm (12 to 15 in.), use Eq. (A-9) and (A-10).

During the first year drying, $(t - t_c) \leq 1$ year

$$\begin{aligned}\gamma_{sh,d} &= 1.23 - 0.0015d \\ \gamma_{sh,d} &= 1.23 - 0.006(V/S)\end{aligned} \quad \text{in SI units}$$

$$\begin{aligned}\gamma_{sh,d} &= 1.23 - 0.038d \\ \gamma_{sh,d} &= 1.23 - 0.152(V/S)\end{aligned} \quad \text{in in.-lb units} \tag{A-9}$$

For ultimate values, $(t - t_c) > 1$ year

$$\begin{aligned}\gamma_{sh,d} &= 1.17 - 0.00114d \\ \gamma_{sh,d} &= 1.17 - 0.00456(V/S)\end{aligned} \quad \text{in SI units}$$

$$\begin{aligned}\gamma_{sh,d} &= 1.17 - 0.029d \\ \gamma_{sh,d} &= 1.23 - 0.116(V/S)\end{aligned} \quad \text{in in.-lb units} \tag{A-10}$$

where $d = 4V/S$ is the average thickness (in mm or in.) of the part of the member under consideration.

For either method, however, γ_{sh} should not be taken less than 0.2. Also, use $\gamma_{sh}\varepsilon_{shu} \geq 100 \times 10^{-6}$ mm/mm (in./in.) if concrete is under seasonal wetting and drying cycles and $\gamma_{sh}\varepsilon_{shu} \geq 150 \times 10^{-6}$ mm/mm (in./in.) if concrete is under sustained drying conditions.

The correction factors that allow for the composition of the concrete are:

- Slump factor $\gamma_{sh,s}$, where s is the slump of fresh concrete (mm or in.)

$$\begin{aligned}\gamma_{sh,s} &= 0.89 + 0.00161s \quad \text{in SI units} \\ \gamma_{sh,s} &= 0.89 + 0.041s \quad \text{in in.-lb units}\end{aligned} \tag{A-11}$$

- Fine aggregate factor $\gamma_{sh,\psi}$, where ψ is the ratio of fine aggregate to total aggregate by weight expressed as percentage

$$\begin{aligned}\gamma_{sh,\psi} &= 0.30 + 0.014\psi \quad \text{for } \psi \leq 50\% \\ \gamma_{sh,\psi} &= 0.90 + 0.002\psi \quad \text{for } \psi > 50\%\end{aligned} \tag{A-12}$$

- Cement content factor $\gamma_{sh,c}$, where c is the cement content in kg/m^3 or lb/yd^3

$$\begin{aligned}\gamma_{sh,c} &= 0.75 + 0.00061c \quad \text{in SI units} \\ \gamma_{sh,c} &= 0.75 + 0.00036c \quad \text{in in.-lb units}\end{aligned} \tag{A-13}$$

- Air content factor $\gamma_{sh,\alpha}$, where α is the air content in percent

$$\gamma_{sh,\alpha} = 0.95 + 0.008\alpha \geq 1 \tag{A-14}$$

These correction factors for concrete composition should be used only in connection with the average values suggested for $\varepsilon_{shu} = 780 \times 10^{-6}$ mm/mm (in./in.). This average value for ε_{shu} should be used only in the absence of specific shrinkage data for local aggregates and conditions determined in accordance with ASTM C512.

A.1.2 *Compliance*—The compliance function $J(t,t_o)$ that represents the total stress-dependent strain by unit stress is given by

$$J(t, t_o) = \frac{1 + \phi(t, t_o)}{E_{cmto}} \tag{A-15}$$

where E_{cmto} is the modulus of elasticity at the time of loading t_o (MPa or psi), and $\phi(t,t_o)$ is the creep coefficient as the ratio of the creep strain to the elastic strain at the start of loading at the age t_o (days).

a) *Modulus of elasticity*—The secant modulus of elasticity of concrete E_{cmto} at any time t_o of loading is given by

$$\begin{aligned}E_{mcto} &= 0.043\gamma_c^{1.5}\sqrt{f_{cmto}} \text{ (MPa) in SI units} \\ E_{mcto} &= 33\gamma_c^{1.5}\sqrt{f_{cmto}} \text{ (psi) in in.-lb units}\end{aligned} \tag{A-16}$$

where γ_c is the unit weight of concrete (kg/m^3 or lb/ft^3), and f_{cmto} is the mean concrete compressive strength at the time of loading (MPa or psi).

The general equation for predicting compressive strength at any time t is given by

$$f_{cmt} = \left[\frac{t}{a + bt}\right] f_{cm28} \quad \text{(A-17)}$$

where f_{cm28} is the concrete mean compressive strength at 28 days in MPa or psi, a (in days) and b are constants, and t is the age of the concrete. The ratio a/b is the age of concrete in days at which one half of the ultimate (in time) compressive strength of concrete is reached.

The constants a and b are functions of both the type of cement used and the type of curing employed. The ranges of a and b for the normalweight, sand lightweight, and all lightweight concretes (using both moist and steam curing, and Types I and III cement) are: a = 0.05 to 9.25, and b = 0.67 to 0.98. Typical recommended values are given in Table A.4.

The concrete required mean compressive strength f_{cm28} should exceed the specified compressive strength f_c' as required in Section 5.3.2 of ACI 318 (ACI Committee 318 2005).

b) *Creep coefficient*—The creep model proposed by ACI 209R-92 has two components that determine the asymptotic value and the time development of creep. The predicted parameter is not creep strain, but creep coefficient $\phi(t,t_o)$ (defined as the ratio of creep strain to initial strain). The latter allows for the calculation of a creep value independent from the applied load. Equation (A-18) presents the general model

$$\phi(t, t_o) = \frac{(t - t_o)^{\psi}}{d + (t - t_o)^{\psi}} \phi_u \quad \text{(A-18)}$$

where $\phi(t,t_o)$ is the creep coefficient at concrete age t due to a load applied at the age t_o; d (in days) and ψ are considered constants for a given member shape and size that define the time-ratio part; $(t - t_o)$ is the time since application of load, and ϕ_u is the ultimate creep coefficient.

For the standard conditions, in the absence of specific creep data for local aggregates and conditions, the average value proposed for the ultimate creep coefficient ϕ_u is

$$\phi_u = 2.35 \quad \text{(A-19)}$$

For the time-ratio in Eq. (A-18), ACI-209R-92 recommends an average value of 10 and 0.6 for d and ψ (steeper curve for larger values of $(t - t_o)$), respectively.

The shape and size effect can be totally considered on the time-ratio by replacing ψ = 1.0 and $d = f$ as given by Eq. (A-3), in Eq. (A-18), where V/S is the volume-surface ratio in mm or in.

For conditions other than the standard conditions, the value of the ultimate creep coefficient ϕ_u (Eq. (A-19)) needs to be modified by correction factors. As shown in Eq. (A-20) and (A-21), ACI 209R-92 suggests multiplying ϕ_u by six factors, depending on particular conditions.

$$\phi_u = 2.35\gamma_c \quad \text{(A-20)}$$

$$\gamma_c = \gamma_{c,to}\gamma_{c,RH}\gamma_{c,vs}\gamma_{c,s}\gamma_{c,\psi}\gamma_{sh,\alpha} \quad \text{(A-21)}$$

Table A.4—Values of the constant *a* and *b* for use in Eq. (A-17), ACI 209R-92 model

Type of cement	Moist-cured concrete		Steam-cured concrete	
	a	b	a	b
I	4.0	0.85	1.0	0.95
III	2.3	0.92	0.70	0.98

where γ_c represent the cumulative product of the applicable correction factors as defined as follows.

For ages at application of load greater than 7 days for moist-cured concrete or greater than 1 to 3 days for steam-cured concrete, the age of loading factor for creep $\gamma_{c,to}$ is estimated from

$$\gamma_{c,to} = 1.25t_o^{-0.118} \quad \text{for moist curing} \quad \text{(A-22)}$$

$$\gamma_{c,to} = 1.13t_o^{-0.094} \quad \text{for steam curing} \quad \text{(A-23)}$$

where t_o is the age of concrete at loading (days).

The ambient relative humidity factor $\gamma_{c,RH}$ is

$$\gamma_{c,RH} = 1.27 - 0.67h \quad \text{for } h \geq 0.40 \quad \text{(A-24)}$$

where the relative humidity h is in decimals.

For lower than 40% ambient relative humidity, values higher than 1.0 should be used for creep γ_h.

Coefficient $\gamma_{c,vs}$ allows for the size of the member in terms of the volume-surface ratio, for members with a volume-surface ratio other than 38 mm (1.5 in.), or an average thickness other than 150 mm (6 in.)

$$\begin{aligned} \gamma_{c,vs} &= \frac{2}{3}\left(1 + 1.13e^{\{-0.0213(V/S)\}}\right) \quad \text{in SI units} \\ \gamma_{c,vs} &= \frac{2}{3}\left(1 + 1.13e^{\{-0.54(V/S)\}}\right) \quad \text{in in.-lb units} \end{aligned} \quad \text{(A-25)}$$

where V is the specimen volume in mm^3 or in^3, and S the specimen surface area in mm^2 or in^2.

Alternatively, the method also allows the use of the average-thickness method to account for the effect of member size on ϕ_u. The average-thickness method tends to compute correction factor values that are higher, as compared with the volume-surface ratio method.

For the average thickness of a member less than 150 mm (6 in.) or volume-surface ratio less than 37.5 mm (1.5 in.), use the factors given in Table A.5.

For the average thickness of members greater than 150 mm (6 in.) and up to about 300 to 380 mm (12 to 15 in.), use Eq. (A-26) and (A-27).

During the first year after loading, $(t - t_o) \leq 1$year

$$\left.\begin{aligned} \gamma_{c,d} &= 1.14 - 0.00092d \\ \gamma_{c,d} &= 1.14 - 0.00363(V/S) \end{aligned}\right\} \text{in SI units}$$

$$\text{(A-26)}$$

$$\left.\begin{aligned} \gamma_{c,d} &= 1.14 - 0.023d \\ \gamma_{c,d} &= 1.14 - 0.092(V/S) \end{aligned}\right\} \text{in in.-lb units}$$

Table A.5—Creep correction factors for average thickness of members, $\gamma_{c,d}$, for use in Eq. (A-21), ACI 209R-92 model

Average thickness of member d, mm (in.)	Volume/surface ratio V/S, mm (in.)	Creep factor $\gamma_{c,d}$
51 (2)	12.5 (0.50)	1.30
76 (3)	19 (0.75)	1.17
102 (4)	25 (1.00)	1.11
127 (5)	31 (1.25)	1.04
152 (6)	37.5 (1.50)	1.00

For ultimate values, $(t - t_o) > 1$ year

$$\begin{aligned}\gamma_{c,d} &= 1.10 - 0.00067d \\ \gamma_{c,d} &= 1.10 - 0.00268(V/S)\end{aligned} \quad \text{in SI units}$$

$$(A\text{-}27)$$

$$\begin{aligned}\gamma_{c,d} &= 1.10 - 0.017d \\ \gamma_{c,d} &= 1.10 - 0.068(V/S)\end{aligned} \quad \text{in in.-lb units}$$

where $d = 4(V/S)$ is the average thickness in mm or inches of the part of the member under consideration.

The correction factors to allow for the composition of the concrete are:

- Slump factor $\gamma_{c,s}$, where s is the slump of fresh concrete (mm or in.)

$$\begin{aligned}\gamma_{c,s} &= 0.82 + 0.00264s \quad \text{in SI units} \\ \gamma_{c,s} &= 0.82 + 0.067s \quad \text{in in.-lb units}\end{aligned} \qquad (A\text{-}28)$$

- Fine aggregate factor $\gamma_{c,\psi}$, where ψ is the ratio of fine aggregate to total aggregate by weight expressed as percentage

$$\gamma_{c,\psi} = 0.88 + 0.0024\psi \qquad (A\text{-}29)$$

- Air content factor $\gamma_{c,\alpha}$, where α is the air content in percent

$$\gamma_{c,\alpha} = 0.46 + 0.09\alpha \geq 1 \qquad (A\text{-}30)$$

These correction factors for concrete composition should be used only in connection with the average values suggested for $\phi_u = 2.35$. This average value for ϕ_u should be used only in the absence of specific creep data for local aggregates and conditions determined in accordance with ASTM C512.

A.2—Bažant-Baweja B3 model

The Bažant-Baweja (1995) B3 model is the latest variant in a number of shrinkage and creep prediction methods developed by Bažant and his coworkers at Northwestern University. According to Bažant and Baweja (2000), the B3 model is simpler and is better theoretically justified than the previous models. The effect of concrete composition and design strength on the model parameters is the main source of error of the model.

The prediction of the material parameters of the B3 model from strength and composition is restricted to portland cement concrete with the following parameter ranges:

- $0.35 \leq w/c \leq 0.85$;
- $2.5 \leq a/c \leq 13.5$;
- 17 MPa $\leq f_{cm28} \leq$ 70 MPa (2500 psi $\leq f_{cm28} \leq$ 10,000 psi); and
- 160 kg/m$^3 \leq c \leq$ 720 kg/m^3 (270 lb/yd$^3 \leq c \leq$ 1215 lb/yd^3)

where f_{cm28} is the 28-day standard cylinder compression strength of concrete (in MPa or psi), w/c is the water-cement ratio by weight, c is the cement content (in kg/m^3 or lb/yd^3), and a/c is the aggregate-cement ratio by weight. If only design strength is known, then $f_{cm28} = f'_c + 8.3$ MPa ($f_{cm28} = f'_c + 1200$ psi).

The Bažant-Baweja B3 model is restricted to the service stress range (or up to about $0.45f_{cm28}$). The formulas are valid for concretes cured for at least 1 day.

Required parameters:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing, (days);
- Age of concrete at loading (days);
- Aggregate content in concrete (kg/m^3 or lb/yd^3);
- Cement content in concrete (kg/m^3 or lb/yd^3);
- Water content in concrete (kg/m^3 or lb/yd^3);
- Cement type;
- Concrete mean compressive strength at 28 days (MPa or psi);
- Modulus of elasticity of concrete at 28 days (MPa or psi);
- Curing condition;
- Relative humidity expressed as a decimal;
- Shape of specimen; and
- Volume-surface ratio or effective cross-section thickness (mm or in.).

A.2.1 *Shrinkage*—The mean shrinkage strain $\varepsilon_{sh}(t,t_c)$ in the cross section at age of concrete t (days), measured from the start of drying at t_c (days), is calculated by Eq. (A-31)

$$\varepsilon_{sh}(t,t_c) = -\varepsilon_{sh\infty}k_h S(t - t_c) \qquad (A\text{-}31)$$

where $\varepsilon_{sh\infty}$ is the ultimate shrinkage strain, k_h is the humidity dependence factor (Table A.6), $S(t - t_c)$ is the time curve, and $(t - t_c)$ is the time from the end of the initial curing.

The ultimate shrinkage $\varepsilon_{sh\infty}$ is given by Eq. (A-32)

$$\varepsilon_{sh\infty} = -\varepsilon_{s\infty}\frac{E_{cm607}}{E_{cm(t_c+\tau_{sh})}} \qquad (A\text{-}32)$$

where $\varepsilon_{s\infty}$ is a constant given by Eq. (A-33), and $E_{cm607}/E_{cm(t_c+\tau_{sh})}$ is a factor to account for the time dependence of ultimate shrinkage (Eq. (A-34))

$$\begin{aligned}\varepsilon_{s\infty} &= -\alpha_1\alpha_2[0.019w^{2.1}f_{cm28}^{-0.28} + 270] \times 10^{-6} \quad \text{in SI units} \\ \varepsilon_{s\infty} &= -\alpha_1\alpha_2[0.02565w^{2.1}f_{cm28}^{-0.28} + 270] \times 10^{-6} \quad \text{in in.-lb units}\end{aligned} \qquad (A\text{-}33)$$

and

$$E_{cmt} = E_{cm28}\left(\frac{t}{4+0.85t}\right) \quad \text{(A-34)}$$

where w is the water content in kg/m^3 or lb/yd^3, f_{cm28} is the concrete mean compressive strength at 28 days in MPa or psi, and α_1 and α_2 are constants related to the cement type and curing condition. (Note: The negative sign is the model authors' convention.) The values of α_1 and α_2 are given in Tables A.7 and A.8, respectively. This means that $\varepsilon_{sh\infty} = \varepsilon_{s\infty}$ for $t_c = 7$ days, and $\tau_{sh} = 600$ days.

The time function for shrinkage $S(t - t_c)$ is given by Eq. (A-35)

$$S(t-t_c) = \tanh\sqrt{\frac{(t-t_c)}{\tau_{sh}}} \quad \text{(A-35)}$$

where t and t_c are the age of concrete and the age drying commenced or end of moist curing in days, respectively, and τ_{sh} is the shrinkage half-time in days as given in Eq. (A-36).

The size dependence of shrinkage is given by

$$\begin{aligned} \tau_{sh} &= 0.085 t_c^{-0.08} f_{cm28}^{-0.25}[2k_s(V/S)]^2 \quad \text{in SI units} \\ \tau_{sh} &= 190.8 t_c^{-0.08} f_{cm28}^{-0.25}[2k_s(V/S)]^2 \quad \text{in in.-lb units} \end{aligned} \quad \text{(A-36)}$$

where k_s is the cross-section shape-correction factor (Table A.9), and V/S is the volume-surface ratio in mm or in.

A.2.2 *Compliance*—The average compliance function $J(t,t_o)$ at concrete age t caused by a unit uniaxial constant stress applied at age t_o, incorporating instantaneous deformation, basic and drying creep, is calculated from

$$J(t,t_o) = q_1 + C_o(t,t_o) + C_d(t,t_o,t_c) \quad \text{(A-37)}$$

where q_1 is the instantaneous strain due to unit stress (inverse of the asymptotic elastic modulus) that is, in theory, approached at a time of about 10^{-9} second; $C_o(t,t_o)$ is the compliance function for basic creep; $C_d(t,t_o,t_c)$ is the additional compliance function for drying creep; and t, t_c, and t_o are the age of concrete, the age drying began or end of moist curing, and the age of concrete loading in days, respectively.

The instantaneous strain may be written $q_1 = 1/E_o$, where E_o is the asymptotic elastic modulus. The use of E_o instead of the conventional static modulus E_{cm} is convenient because concrete exhibits pronounced creep, even for very short loads duration. E_o should not be regarded as a real elastic modulus, but merely an empirical parameter that can be considered age independent. Therefore, the instantaneous strain due to unit stress is expressed in Eq. (A-38)

$$q_1 = 0.6/E_{cm28} \quad \text{(A-38)}$$

where

Table A.6—Humidity dependence k_h, B3 model

Relative humidity	k_h
$h \le 0.98$	$1 - h^3$
$h = 1.00$	-0.2
$0.98 < h < 1.00$	Linear interpolation: $12.74 - 12.94h$

Table A.7—α_1 as function of cement type, B3 model

Type of cement	α_1
Type I	1.00
Type II	0.85
Type III	1.10

Table A.8—α_2 as function of curing condition, B3 model

Curing method	α_2
Steam cured	0.75
Cured in water or at 100% relative humidity	1.00
Sealed during curing or normal curing in air with initial protection against drying	1.20

$$\begin{aligned} E_{cm28} &= 4734\sqrt{f_{cm28}} \quad \text{in SI units} \\ E_{cm28} &= 57{,}000\sqrt{f_{cm28}} \quad \text{in in.-lb units} \end{aligned} \quad \text{(A-39)}$$

According to this model, the basic creep is composed of three terms: an aging viscoelastic term, a nonaging viscoelastic term, and an aging flow term

$$C_o(t,t_o) = q_2 Q(t,t_o) + q_3 \cdot \ln[1 + (t - t_o)^n] + q_4 \cdot \ln(t/t_o) \quad \text{(A-40)}$$

where $q_2Q(t,t_o)$ is the aging viscoelastic compliance term. The cement content c (in kg/m^3 or lb/yd^3) and the concrete mean compressive strength at 28 days f_{cm28} (in MPa or psi) are required to calculate the parameter q_2 in Eq. (A-41)

$$\begin{aligned} q_2 &= 185.4 \times 10^{-6} c^{0.5} f_{cm28}^{-0.9} \quad \text{in SI units} \\ q_2 &= 86.814 \times 10^{-6} c^{0.5} f_{cm28}^{-0.9} \quad \text{in in.-lb units} \end{aligned} \quad \text{(A-41)}$$

$Q(t,t_o)$ is an approximate binomial integral that must be multiplied by the parameter q_2 to obtain the aging viscoelastic term

$$Q(t,t_o) = Q_f(t_o)\left[1 + \left(\frac{Q_f(t_o)}{Z(t,t_o)}\right)^{r(t_o)}\right]^{-1/r(t_o)} \quad \text{(A-42)}$$

Equations (A-43) to (A-45) can be used to approximate the binomial integral

$$Q_f(t_o) = [0.086(t_o)^{2/9} + 1.21(t_o)^{4/9}]^{-1} \quad \text{(A-43)}$$

$$Z(t,t_o) = (t_o)^{-m} \cdot \ln[1 + (t - t_o)^n] \quad \text{(A-44)}$$

$$r(t_o) = 1.7(t_o)^{0.12} + 8 \quad \text{(A-45)}$$

Table A.9—k_s as function of cross section shape, B3 model

Cross section shape	k_s
Infinite slab	1.00
Infinite cylinder	1.15
Infinite square prism	1.25
Sphere	1.30
Cube	1.55

Note: The analyst needs to estimate which of these shapes best approximates the real shape of the member or structure. High accuracy in this respect is not needed, and $k_s \approx 1$ can be used for simplified analysis.

where m and n are empirical parameters whose value can be taken the same for all normal concretes ($m = 0.5$ and $n = 0.1$).

In Eq. (A-40), q_3 is the nonaging viscoelastic compliance parameter, and q_4 is the aging flow compliance parameter. These parameters are a function of the concrete mean compressive strength at 28 days f_{cm28} (in MPa or psi), the cement content c (in kg/m^3 or lb/yd^3), the water-cement ratio w/c, and the aggregate-cement ratio a/c

$$q_3 = 0.29(w/c)^4 q_2 \tag{A-46}$$

$$\begin{aligned} q_4 &= 20.3 \times 10^{-6}(a/c)^{-0.7} \quad \text{in SI units} \\ q_4 &= 0.14 \times 10^{-6}(a/c)^{-0.7} \quad \text{in in.-lb units} \end{aligned} \tag{A-47}$$

The compliance function for drying creep is defined by Eq. (A-48). This equation accounts for the drying before loading. Note that drying before loading is considered only for drying creep

$$C_d(t,t_o,t_c) = q_5[\exp\{-8H(t)\} - \exp\{8H(t_o)\}]^{1/2} \tag{A-48}$$

In Eq. (A-48), q_5 is the drying creep compliance parameter. This parameter is a function of the concrete mean compressive strength at 28 days f_{cm28} (in MPa or psi), and of $\varepsilon_{sh\infty}$, the ultimate shrinkage strain as given in Eq. (A-32)

$$q_5 = 0.757 f_{cm28}{}^{-1} |\varepsilon_{sh\infty} \times 10^6|^{-0.6} \tag{A-49}$$

$H(t)$ and $H(t_o)$ are spatial averages of pore relative humidity. Equations (A-50) to (A-53) and Eq. (A-36) are required to calculate $H(t)$ and $H(t_o)$.

$$H(t) = 1 - (1 - h)S(t - t_c) \tag{A-50}$$

$$H(t_o) = 1 - (1 - h)S(t_o - t_c) \tag{A-51}$$

where $S(t - t_c)$ and $S(t_o - t_c)$ are the time function for shrinkage calculated at the age of concrete t and the age of concrete at loading t_o in days, respectively, and τ_{sh} is the shrinkage half-time

$$S(t - t_c) = \tanh\left[\left(\frac{t - t_c}{\tau_{sh}}\right)^{1/2}\right] \tag{A-52}$$

$$S(t_o - t_c) = \tanh\left[\left(\frac{t_o - t_c}{\tau_{sh}}\right)^{1/2}\right] \tag{A-53}$$

A.3—CEB MC90-99 model

The CEB MC90 model (Muller and Hilsdorf 1990; CEB 1993) is intended to predict the time-dependent mean cross-section behavior of a concrete member. It has concept similar to that of ACI 209R-92 model in the sense that it gives a hyperbolic change with time for creep and shrinkage, and it also uses an ultimate value corrected according mixture proportioning and environment conditions. Unless special provisions are given, the models for shrinkage and creep predict the time-dependent behavior of ordinary-strength concrete (12 MPa [1740 psi] $\leq f_c' \leq$ 80 MPa [11,600 psi]) moist cured at normal temperatures not longer than 14 days and exposed to a mean ambient relative humidity in the range of 40 to 100% at mean ambient temperatures from 5 to 30 °C (41 to 86 °F). The models are valid for normalweight plain structural concrete having an average compressive strength in the range of 20 MPa (2900 psi) $\leq f_{cm28} \leq$ 90 MPa (13,000 psi). The age at loading t_o should be at least 1 day, and the sustained stress should not exceed 40% of the mean concrete strength f_{cmto} at the time of loading t_o. Special provisions are given for elevated or reduced temperatures and for high stress levels.

The CEB MC90-99 model (CEB 1999) includes the latest improvements to the CEB MC90 model. The model has been developed for normal- and high-strength concrete, and considers the separation of the total shrinkage into autogenous and drying shrinkage components. The models for shrinkage and creep are intended to predict the time-dependent mean cross-section behavior of a concrete member moist cured at normal temperatures not longer than 14 days and exposed to a mean ambient relative humidity in the range of 40 to 100% at mean ambient temperatures from 10 to 30 °C (50 to 86 °F). It is valid for normalweight plain structural concrete having an average compressive strength in the range of 15 MPa (2175 psi) $\leq f_{cm28} \leq$ 120 MPa (17,400 psi). The age at loading should be at least 1 day, and the creep-induced stress should not exceed 40% of the concrete strength at the time of loading.

The CEB model does not require any information regarding the duration of curing or curing condition, but takes into account the average relative humidity and member size.

Required parameters:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing (days);
- Age of concrete at loading (days);
- Concrete mean compressive strength at 28 days (MPa or psi);
- Relative humidity expressed as a decimal;
- Volume-surface ratio or effective cross-section thickness of the member (mm or in.); and
- Cement type.

A.3.1 *Shrinkage CEB MC90*—The total shrinkage strains of concrete $\varepsilon_{sh}(t,t_c)$ may be calculated from

$$\varepsilon_{sh}(t,t_c) = \varepsilon_{cso}\beta_s(t - t_c) \tag{A-54}$$

where ε_{cso} is the notional shrinkage coefficient, $\beta_s(t-t_c)$ is the coefficient describing the development of shrinkage with time of drying, t is the age of concrete (days) at the moment considered, t_c is the age of concrete at the beginning of drying (days), and $(t-t_c)$ is the duration of drying (days).

The notional shrinkage coefficient may be obtained from

$$\varepsilon_{cso} = \varepsilon_s(f_{cm28})\beta_{RH}(h) \tag{A-55}$$

with

$$\varepsilon_s(f_{cm28}) = [160 + 10\beta_{sc}(9 - f_{cm28}/f_{cm0})] \times 10^{-6} \tag{A-56}$$

$$\begin{aligned}\beta_{RH}(h) &= -1.55\left[1-\left(\frac{h}{h_o}\right)^3\right] \quad \text{for } 0.4 \le h < 0.99\\ \beta_{RH}(h) &= 0.25 \quad \text{for } h \ge 0.99\end{aligned} \tag{A-57}$$

where f_{cm28} is the mean compressive cylinder strength of concrete at the age of 28 days (MPa or psi), f_{cmo} is equal to 10 MPa (1450 psi), β_{sc} is a coefficient that depends on the type of cement (Table A.10), h is the ambient relative humidity as a decimal, and h_o is equal to 1.

The development of shrinkage with time is given by

$$\beta_s(t-t_c) = \left[\frac{(t-t_c)/t_1}{350[(V/S)/(V/S)_o]^2 + (t-t_c)/t_1}\right]^{0.5} \tag{A-58}$$

where $(t-t_c)$ is the duration of drying (days), t_1 is equal to 1 day, V/S is the volume-surface ratio (mm or in.), and $(V/S)_o$ is equal to 50 mm (2 in.).

The method assumes that, for curing periods of concrete members not longer than 14 days at normal ambient temperature, the duration of moist curing does not significantly affect shrinkage. Hence, this parameter, as well as the effect of curing temperature, is not taken into account. Therefore, in Eq. (A-54) and (A-58), the actual duration of drying $(t-t_c)$ has to be used.

When constant temperatures above 30 °C (86 °F) are applied while the concrete is drying, CEB MC90 recommends using an elevated temperature correction for $\beta_{RH}(h)$ and $\beta_s(t-t_c)$, shown as follows.

The effect of temperature on the notional shrinkage coefficient is taken into account by

In SI units:

$$\beta_{RH,T} = \beta_{RH}(h)\left[1 + \left(\frac{0.08}{1.03 - h/h_o}\right)\left(\frac{T/T_o - 20}{40}\right)\right] \tag{A-59}$$

In in.-lb units:

$$\beta_{RH,T} = \beta_{RH}(h)\left[1 + \left(\frac{0.08}{1.03 - h/h_o}\right)\left(\frac{18.778 \cdot T/T_o - 37.778}{40}\right)\right]$$

Table A.10—Coefficient β_{sc} according to Eq. (A-56), CEB MC90 model

Type of cement according to EC2	β_{sc}
SL (slowly-hardening cements)	4
N and R (normal or rapid hardening cements)	5
RS (rapid hardening high-strength cements)	8

The effect of temperature on the time development of shrinkage is taken into account by

In SI units:

$$\beta_{s,T}(t-t_c) = \left[\frac{(t-t_c)/t_1}{350\left[\left(\frac{V}{S}\right)/\left(\frac{V}{S}\right)_o\right]^2 \exp\left[-0.06\left(\frac{T}{T_o}-20\right)\right] + \frac{(t-t_c)}{t_1}}\right]^{0.5} \tag{A-60}$$

In in.-lb units:

$$\beta_{s,T}(t-t_c) = \left[\frac{(t-t_c)/t_1}{350\left[\left(\frac{V}{S}\right)/\left(\frac{V}{S}\right)\right]^2 \exp\left[-0.06\left(18.778\frac{T}{T_o}-37.778\right)\right] + \frac{(t-t_c)}{t_1}}\right]^{0.5}$$

where $\beta_{RH,T}$ is the relative humidity factor corrected by temperature that replaces β_{RH} in Eq. (A-55), $\beta_{s,T}(t-t_c)$ is the temperature-dependent coefficient replacing $\beta_s(t-t_c)$ in Eq. (A-54), h is the relative humidity in decimals, h_o is equal to 1, V/S is the volume-surface ratio (mm or in.); (V/S) is equal to 50 mm (2 in.), T is the ambient temperature (°C or °F), and T_o is equal to 1 °C (33.8 °F).

A.3.2 *Shrinkage CEB MC90-99*—With respect to the shrinkage characteristics of high-performance concrete, the new approach for shrinkage subdivides the total shrinkage into the components of autogenous shrinkage and drying shrinkage. While the model for the drying shrinkage component is closely related to the approach given in CEB MC90 (CEB 1993), for autogenous shrinkage, new relations had to be derived. Some adjustments, however, should also be carried out for the drying shrinkage component, as the new model should cover both the shrinkage of normal- and high-performance concrete; consequently, the autogenous shrinkage also needs to be modeled for normal-strength concrete.

The total shrinkage of concrete $\varepsilon_{sh}(t,t_c)$ can be calculated from Eq. (A-61)

$$\varepsilon_{sh}(t,t_c) = \varepsilon_{cas}(t) + \varepsilon_{cds}(t,t_c) \tag{A-61}$$

where $\varepsilon_{sh}(t,t_c)$ is the total shrinkage, $\varepsilon_{cas}(t)$ the autogenous shrinkage, and $\varepsilon_{cds}(t,t_c)$ is the drying shrinkage at concrete age t (days) after the beginning of drying at t_c (days).

The autogenous shrinkage component $\varepsilon_{cas}(t)$ is calculated from Eq. (A-62)

$$\varepsilon_{cas}(t) = \varepsilon_{caso}(f_{cm28})\beta_{as}(t) \tag{A-62}$$

where $\varepsilon_{caso}(f_{cm28})$ is the notional autogenous shrinkage coefficient from Eq. (A-63), and $\beta_{as}(t)$ is the function describing the time development of autogenous shrinkage from Eq. (A-64)

$$\varepsilon_{caso}(f_{cm28}) = -\alpha_{as}\left(\frac{f_{cm28}/f_{cmo}}{6 + f_{cm28}/f_{cmo}}\right)^{2.5} \times 10^{-6} \quad \text{(A-63)}$$

$$\beta_{as}(t) = 1 - \exp\left[-0.2\left(\frac{t}{t_1}\right)^{0.5}\right] \quad \text{(A-64)}$$

where f_{cm28} is the mean compressive strength of concrete at an age of 28 days (MPa or psi), f_{cmo} = 10 MPa (1450 psi), t is the concrete age (days), t_1 = 1 day, and α_{as} is a coefficient that depends on the type of cement (Table A.11).

The autogenous shrinkage component is independent of the ambient humidity and of the member size, and develops more rapidly than drying shrinkage.

The drying shrinkage $\varepsilon_{cds}(t,t_c)$ is calculated from Eq. (A-65)

$$\varepsilon_{cds}(t,t_c) = \varepsilon_{cdso}(f_{cm28})\beta_{RH}(h)\beta_{ds}(t - t_c) \quad \text{(A-65)}$$

where $\varepsilon_{cdso}(f_{cm28})$ is the notional drying shrinkage coefficient from Eq. (A-66), $\beta_{RH}(h)$ is the coefficient that takes into account the effect of relative humidity on drying shrinkage from Eq. (A-67), and $\beta_{ds}(t - t_c)$ is the function describing the time development of drying shrinkage from Eq. (A-68)

$$\varepsilon_{cdso}(f_{cm28}) = [(220 + 110\alpha_{ds1})\exp(-\alpha_{ds2}f_{cm28}/f_{cmo})] \times 10^{-6} \quad \text{(A-66)}$$

$$\beta_{RH}(h) = -1.55\left[1 - \left(\frac{h}{h_o}\right)^3\right] \text{ for } 0.4 \le h < 0.99\beta_{s1} \quad \text{(A-67)}$$

$$\beta_{RH}(h) = 0.25 \text{ for } h \ge 0.99\beta_{s1}$$

$$\beta_{ds}(t - t_c) = \left[\frac{(t - t_c)/t_1}{350[(V/S)/(V/S)_o]^2 + (t - t_c)/t_1}\right]^{0.5} \quad \text{(A-68)}$$

$$\beta_{s1} = \left(\frac{3.5f_{cmo}}{f_{cm28}}\right)^{0.1} \le 1.0 \quad \text{(A-69)}$$

where α_{ds1} and α_{ds2} are coefficients that depend on the type of cement (Table A.11), β_{s1} is a coefficient that takes into account the self-desiccation in high-performance concrete, h is the ambient relative humidity as a decimal, h_o = 1, V/S is the volume-surface ratio (mm or in.), $(V/S)_o$ = 50 mm (2 in.), f_{cmo} = 10 MPa (1450 psi), t_c is the concrete age at the beginning of drying (days), and $(t - t_c)$ is the duration of drying (days).

According to Eq. (A-67) for normal-strength concretes, swelling is to be expected if the concrete is exposed to an ambient relative humidity near 99%. For higher-strength grades, swelling will occur at lower relative humidities because of the preceding reduction of the internal relative humidity due to self-desiccation of the concrete.

Table A.11—Coefficients according to Eq. (A-63) and (A-66), CEB MC90-99 model

Type of cement according to EC2	α_{as}	α_{ds1}	α_{ds2}
SL (slowly-hardening cements)	800	3	0.13
N or R (normal or rapid hardening cements)	700	4	0.12
RS (rapid hardening high-strength cements)	600	6	0.12

A.3.3 *Compliance*—The compliance function $J(t,t_o)$ that represents the total stress-dependent strain by unit stress is given by

$$J(t, t_o) = \frac{1}{E_{cm28}}[\eta(t_o) + \phi_{28}(t, t_o)] = \frac{1}{E_{cmto}} + \frac{\phi_{28}(t, t_o)}{E_{cm28}} \quad \text{(A-70)}$$

where $\eta(t_o) = E_{cm28}/E_{cmto}$, E_{cm28} is the mean modulus of elasticity of concrete at 28 days (MPa or psi), E_{cmto} is the modulus of elasticity at the time of loading t_o (MPa or psi), and the dimensionless 28-day creep coefficient $\phi_{28}(t,t_o)$ gives the ratio of the creep strain since the start of loading at the age t_o to the elastic strain due to a constant stress applied at a concrete age of 28 days. Hence, $1/E_{cmto}$ represents the initial strain per unit stress at loading.

The CEB MC90-99 model is closely related to the CEB MC90 model; however, it has been adjusted to take into account the particular characteristics of high-strength concretes.

a) *Modulus of elasticity*—For the prediction of the creep function, the initial strain is based on the tangent modulus of elasticity at the time of loading as defined in Eq. (A-71) and (A-72).

The modulus of elasticity of concrete at a concrete age t different than 28 days may be estimated from

$$E_{cmt} = E_{cm28}\exp\left[\frac{s}{2}\left(1 - \sqrt{\frac{28}{t/t_1}}\right)\right] \quad \text{(A-71)}$$

where E_{cm28} is the mean modulus of elasticity of concrete at 28 days from Eq. (A-72); the coefficient s depends on the type of cement and the compressive strength of concrete and may be taken from Table A.12; and t_1 = 1 day.

The modulus of elasticity of concrete made of quartzitic aggregates at the age of 28 days E_{cm28} (MPa or psi) may be estimated from the mean compressive strength of concrete by Eq. (A-72)

$$E_{cm28} = 21{,}500\sqrt[3]{\frac{f_{cm28}}{f_{cmo}}} \text{ in SI units}$$

$$E_{cm28} = 3{,}118{,}310\sqrt[3]{\frac{f_{cm28}}{f_{cmo}}} \text{ in in.-lb units} \quad \text{(A-72)}$$

where f_{cm28} is the mean compressive cylinder strength of concrete at 28 days (MPa or psi), and f_{cmo} = 10 MPa (1450 psi).

For concrete made of basalt, dense limestone, limestone, or sandstone, CEB MC90 recommends calculating the modulus of elasticity of concrete by multiplying E_{cm28} (MPa or psi) according to Eq. (A-72) with the coefficients α_E from Table A.13.

The mean compressive cylinder strength of concrete (MPa or psi) is given by Eq. (A-73)

$$f_{cm28} = f'_c + 8.0 \quad \text{in SI units}$$
$$f_{cm28} = f'_c + 1160 \quad \text{in in.-lb units} \tag{A-73}$$

where f'_c is the specified/characteristic compressive cylinder strength (MPa or psi) defined as that strength below which 5% of all possible strength measurements for the specified concrete may be expected to fall.

b) *Creep coefficient*—Within the range of service stresses (not larger than 40% of the mean concrete strength f_{cmto} at the time of loading t_o), the 28-day creep coefficient $\phi_{28}(t,t_o)$ may be calculated from Eq. (A-74)

$$\phi_{28}(t,t_o) = \phi_o\beta_c(t-t_o) \tag{A-74}$$

where ϕ_o is the notional creep coefficient, $\beta_c(t-t_o)$ is the coefficient that describes the development of creep with time after loading, t is the age of concrete (days) at the moment considered, and t_o is the age of concrete at loading (days), adjusted according to Eq. (A-81) and (A-87).

The notional creep coefficient ϕ_o may be determined from Eq. (A-75) to (A-81)

$$\phi_o = \phi_{RH}(h)\beta(f_{cm28})\beta(t_o) \tag{A-75}$$

with

$$\phi_{RH}(h) = \left[1 + \frac{1-h/h_o}{\sqrt[3]{0.1[(V/S)/(V/S)_o]}}\alpha_1\right]\alpha_2 \tag{A-76}$$

$$\beta(f_{cm28}) = \frac{5.3}{\sqrt{f_{cm28}/f_{cmo}}} \tag{A-77}$$

$$\beta(t_o) = \frac{1}{0.1 + (t_o/t_1)^{0.2}} \tag{A-78}$$

$$\alpha_1 = \left[\frac{3.5f_{cmo}}{f_{cm28}}\right]^{0.7} \tag{A-79}$$

$$\alpha_2 = \left[\frac{3.5f_{cmo}}{f_{cm28}}\right]^{0.2} \tag{A-80}$$

where f_{cm28} is the mean compressive strength of concrete at the age of 28 days (MPa or psi), f_{cmo} = 10 MPa (1450 psi), h is the relative humidity of the ambient environment in decimals, $h_o = 1$, V/S is the volume-surface ratio (mm or in.), $(V/S)_o$ = 50 mm (2 in.), t_1 = 1 day, t_o is the age of concrete at loading (days) adjusted according to Eq. (A-81) and (A-87), and α_1 and α_2 are coefficients that depend on the mean compressive strength of concrete ($\alpha_1 = \alpha_2 = 1$ in CEB MC90).

Table A.12—Coefficient *s* according to Eq. (A-71), CEB MC90 and CEB MC90-99 models

f_{cm28}	Type of cement	s
	RS (rapid hardening high-strength cement)	0.20
≤60 MPa (8700 psi)	N or R (normal or rapid hardening cements)	0.25
	SL (slowly-hardening cement)	0.38
>60 MPa (8700 psi)*	All types	0.20

*Case not considered in CEB MC90.

Table A.13—Effect of type of aggregate on modulus of elasticity, CEB MC90 model

Aggregate type	α_E
Basalt, dense limestone aggregates	1.2
Quartzitic aggregates	1.0
Limestone aggregates	0.9
Sandstone aggregates	0.7

The effect of type of cement and curing temperature on the creep coefficient may be taken into account by modifying the age at loading t_o according to Eq. (A-81)

$$t_o = t_{o,T}\left[\frac{9}{2 + (t_{o,T}/t_{1,T})^{1.2}} + 1\right]^{\alpha} \geq 0.5 \text{ days} \tag{A-81}$$

where $t_{o,T}$ is the age of concrete at loading (days) adjusted to the concrete temperature according to Eq. (A-87) (for T = 20 °C [68 °F], $t_{o,T}$ corresponds to t_o) and $t_{1,T}$ = 1 day. α is a power that depends on the type of cement; $\alpha = -1$ for slowly hardening cement; $\alpha = 0$ for normal or rapidly hardening cement; and $\alpha = 1$ for rapid hardening high-strength cement. The value for t_o according to Eq. (A-81) has to be used in Eq. (A-78).

The coefficient $\beta_c(t-t_o)$ that describes the development of creep with time after loading may be determined from Eq. (A-82) to (A-84)

$$\beta_c(t-t_o) = \left[\frac{(t-t_o)/t_1}{\beta_H + (t-t_o)/t_1}\right]^{0.3} \tag{A-82}$$

with

$$\beta_H = 150[1 + (1.2 \cdot h/h_o)^{18}](V/S)/(V/S)_o + 250\alpha_3 \leq 1500\alpha_3 \tag{A-83}$$

$$\alpha_3 = \left[\frac{3.5f_{cmo}}{f_{cm28}}\right]^{0.5} \tag{A-84}$$

where t_1 = 1 day, h_o = 1, $(V/S)_o$ = 50 mm (2 in.), and α_3 is a coefficient that depends on the mean compressive strength of concrete (α_3 = 1 in CEB MC90).

The duration of loading $(t - t_o)$ used in Eq. (A-82) is the actual time under load.

Temperature effects—The effect of elevated or reduced temperatures at the time of testing on the modulus of elasticity of concrete, at an age of 28 days without exchange of moisture, for a temperature range 5 to 80 °C (41 to 176 °F), may be estimated from

$$E_{cm28}(T) = E_{cm28}(1.06 - 0.003T/T_o) \quad \text{in SI units} \tag{A-85}$$
$$E_{cm28}(T) = E_{cm28}[1.06 - 0.003(18.778T - 600.883)/T_o] \quad \text{in in.-lb units}$$

where T is the temperature (°C or °F), and T_o = 1 °C (33.8 °F). Equation (A-85) can also be used for a concrete age other than t = 28 days.

The 28-day creep coefficient at an elevated temperature may be calculated as

$$\phi_{28}(t,t_o,T) = \phi_o\beta_c(t - t_o) + \Delta\phi_{T,trans} \tag{A-86}$$

where ϕ_o is the notional creep coefficient according to Eq. (A-75) and temperature adjusted according to Eq. (A-90), $\beta_c(t - t_o)$ is a coefficient that describes the development of creep with time after loading according to Eq. (A-82) and temperature adjusted according to Eq. (A-88) and (A-89), and $\Delta\phi_{T,trans}$ is the transient thermal creep coefficient that occurs at the time of the temperature increase, and may be estimated from Eq. (A-92).

The effect of temperature to which concrete is exposed before loading may be taken into account by calculating an adjusted age at loading from Eq. (A-87)

$$t_{o,T} = \sum_{i=1}^{n}\Delta t_i \exp\left[13.65 - \frac{4000}{273 + \dfrac{T(\Delta t_i)}{T_o}}\right] \quad \text{in SI units}$$

$$t_{o,T} = \sum_{i=1}^{n}\Delta t_i \exp\left[13.65 - \frac{4000}{273 + \dfrac{(18.778T(\Delta t_i) - 600.883)}{T_o}}\right] \quad \text{in in.-lb units} \tag{A-87}$$

where $t_{o,T}$ is the temperature-adjusted age of concrete at loading, in days, from Eq. (A-81), $T(\Delta t_i)$ is the temperature (°C or °F) during the time period Δt_i, Δt_i is the number of days where a temperature T prevails, n is the number of time intervals considered, and T_o = 1 °C (33.8 °F).

The effect of temperature on the time development of creep is taken into consideration using $\beta_{H,T}$ (Eq. (A-88))

$$\beta_{H,T} = \beta_H\beta_T \tag{A-88}$$

with

$$\beta_T = \exp\left[\frac{1500}{(273 + T/T_o)} - 5.12\right] \quad \text{in SI units} \tag{A-89}$$
$$\beta_T = \exp\left[\frac{1500}{[273 + (18.778T - 600.883)/T_o]} - 5.12\right] \quad \text{in in.-lb units}$$

where $\beta_{H,T}$ is a temperature-dependent coefficient that replaces β_H in Eq. (A-82), β_H is a coefficient according to Eq. (A-83), T is the temperature (°C or °F), and T_o = 1 °C (33.8 °F).

The effect of temperature conditions on the magnitude of the creep coefficient ϕ_o in Eq. (A-74) and (A-75), respectively, may be calculated using Eq. (A-90)

$$\phi_{RH,T} = \phi_T + [\phi_{RH}(h) - 1]\phi_T^{1.2} \tag{A-90}$$

with

$$\phi_T = \exp[0.015(T/T_o - 20)] \quad \text{in SI units} \tag{A-91}$$
$$\phi_T = \exp[0.015[(18.778T - 600.883)/T_o - 20]] \quad \text{in in.-lb units}$$

where $\phi_{RH,T}$ is a temperature-dependent coefficient that replaces $\phi_{RH}(h)$ in Eq. (A-75), $\phi_{RH}(h)$ is a coefficient according to Eq. (A-76), and T_o = 1 °C (33.8 °F).

Transient temperature conditions, that is, an increase of temperature while the structural member is under load, leads to additional creep $\Delta\phi_{T,trans}$ that may be calculated from Eq. (A-92)

$$\Delta\phi_{T,trans} = 0.0004(T/T_o - 20)^2 \quad \text{in SI units} \tag{A-92}$$
$$\Delta\phi_{T,trans} = 0.0004[(18.778T - 600.883)/T_o - 20]^2 \quad \text{in in.-lb units}$$

Effect of high stresses—When stresses in the range of 40 to 60% of the compressive strength are applied, CEB MC90-99 (CEB 1993, 1999) recommends using a high stress correction to the notional creep ϕ_o as shown in Eq. (A-93)

$$\phi_{o,k} = \phi_o\exp\{1.5(k_\sigma - 0.4)\} \tag{A-93}$$

where $\phi_{o,k}$ is the notional creep coefficient that replaces ϕ_o in Eq. (A-74), and k_σ is the stress-strength ratio at the time of application of the load.

A.4—GL2000 model

The model presented herein corresponds to the last version of the GL2000 model (Gardner 2004), including minor modifications to some coefficients and to the strength development with time equation of the original model developed by Gardner and Lockman (2001). It is a modified Atlanta 97 model (Gardner and Zhao 1993), which itself was influenced by CEB MC90. It presents a design-office procedure for calculating the shrinkage and creep of normal-strength concretes, defined as concretes with mean compressive strengths less than 82 MPa (11,890 psi) that do not experience self-desiccation, using the information available at design, namely, the 28-day specified concrete strength, the concrete strength at loading, element size, and relative humidity. According to Gardner and Lockman (2001), the method can be used regardless of what chemical admixtures or mineral by-products are in the concrete, casting temperature, or curing regime. The predicted values can be improved by simply measuring concrete strength development with time and modulus of elasticity. Aggregate stiffness is taken into account by using the average of the measured cylinder strength and that back-calculated from the measured modulus of elasticity of the concrete. The compliance expression is based on the modulus of elasticity at 28 days instead of the modulus elasticity at the age of loading. This model includes a term for drying before loading, which applies to both basic and drying creep.

Required parameters:

- Age of concrete when drying starts, usually taken as the age at the end of moist curing (days);
- Age of concrete at loading (days);
- Concrete mean compressive strength at 28 days (MPa or psi);
- Concrete mean compressive strength at loading (MPa or psi);
- Modulus of elasticity of concrete at 28 days (MPa or psi);
- Modulus of elasticity of concrete at loading (MPa or psi);
- Relative humidity expressed as a decimal; and
- Volume-surface ratio (mm or in.).

A.4.1 *Relationship between specified and mean compressive strength of concrete*—If experimental values are not available, the relationship between the specified/characteristic compressive strength f_c' and the mean compressive strength of concrete f_{cm28} can be estimated from Eq. (A-94)

$$\begin{aligned} f_{cm28} &= 1.1 f_c' + 5.0 \quad \text{in SI units} \\ f_{cm28} &= 1.1 f_c' + 700 \quad \text{in in.-lb units} \end{aligned} \tag{A-94}$$

Equation (A-94) is a compromise between the recommended equations of ACI Committee 209 (1982) and ACI Committee 363 (1992). It can be noted that Eq. (A-94) does not include any effects for aggregate stiffness or concrete density. Instead of making an allowance for the density of the concrete, it is preferable to measure the modulus of elasticity.

If experimental values are not available, the modulus of elasticity E_{cmt} and the strength development with time f_{cmt} can be calculated from the compressive strength using Eq. (A-95) and (A-96).

A.4.2 *Modulus of elasticity*

$$\begin{aligned} E_{cmt} &= 3500 + 4300\sqrt{f_{cmt}} \quad \text{in SI units} \\ E_{cmt} &= 500{,}000 + 52{,}000\sqrt{f_{cmt}} \quad \text{in in.-lb units} \end{aligned} \tag{A-95}$$

A.4.3 *Aggregate stiffness*—Aggregate stiffness can be accommodated by using the average of the measured cylinder strength and that back-calculated from the measured modulus of elasticity using Eq. (A-95) in the shrinkage and specific creep equations. Effectively, Eq. (A-95) is used as an indicator of the divergence of the measured stiffness from standard values.

A.4.4 *Strength development with time*

$$f_{cmt} = \beta_e^2 f_{cm28} \tag{A-96}$$

where

$$\beta_e = \exp\left[\frac{s}{2}\left(1 - \sqrt{\frac{28}{t}}\right)\right] \tag{A-97}$$

where s is a CEB (1993) style strength-development parameter (Table A.14), and β_e relates strength development to cement type. Equation (A-96) is a modification of the CEB strength-development relationship.

A single measured value of s permits values of k in the shrinkage equation to be interpolated, where k is a correction term for the effect of cement type on shrinkage (Table A.14). If experimental results are available, the cement type is determined from the strength development characteristic of the concrete, regardless of the nominal designation of the cement. This enables the model to accommodate concretes incorporating any chemical or mineral admixtures.

A.4.5 *Shrinkage*—Calculate the shrinkage strain $\varepsilon_{sh}(t,t_c)$ from Eq. (A-98)

$$\varepsilon_{sh}(t,t_c) = \varepsilon_{shu}\beta(h)\beta(t - t_c) \tag{A-98}$$

where ε_{shu} is the ultimate shrinkage strain, $\beta(h)$ is a correction term for the effect of humidity, and $\beta(t - t_c)$ is a correction term for the effect of time of drying.

The ultimate shrinkage ε_{shu} is given by

$$\begin{aligned} \varepsilon_{shu} &= 900k\left(\frac{30}{f_{cm28}}\right)^{1/2} \times 10^{-6} \quad \text{in SI units} \\ \varepsilon_{shu} &= 900k\left(\frac{4350}{f_{cm28}}\right)^{1/2} \times 10^{-6} \quad \text{in in.-lb units} \end{aligned} \tag{A-99}$$

where f_{cm28} is the concrete mean compressive strength at 28 days in MPa or psi, and k is a shrinkage constant that depends on the cement type (Table A.14).

If test results for strength development are available, the shrinkage term can be improved by interpolating k from Table A.14 using the experimentally determined cement type/characteristic.

The correction term for effect of humidity $\beta(h)$ is given by

$$\beta(h) = (1 - 1.18h^4) \tag{A-100}$$

Note that for a relative humidity of 0.96, there is no shrinkage. At a higher relative humidity, swelling occurs.

The time function for shrinkage $\beta(t - t_c)$ is given by

$$\begin{aligned} \beta(t - t_c) &= \left[\frac{(t - t_c)}{(t - t_c) + 0.12(V/S)^2}\right]^{1/2} \quad \text{in SI units} \\ \beta(t - t_c) &= \left[\frac{(t - t_c)}{(t - t_c) + 77(V/S)^2}\right]^{1/2} \quad \text{in in.-lb units} \end{aligned} \tag{A-101}$$

where t and t_c are the age of concrete and the age drying starts or end of moist curing in days, respectively, and V/S is the volume-surface ratio in mm or in.

A.4.6 *Compliance equations*—The "compliance is composed of the elastic and the creep strains. The elastic strain is the reciprocal of the modulus of elasticity at the age

Table A.14—Parameters *s* and *k* as function of cement type, GL2000 model

Cement type	s	k
Type I	0.335	1.0
Type II	0.4	0.75
Type III	0.13	1.15

of loading E_{cmto}, and the creep strain is the 28-day creep coefficient $\phi_{28}(t,t_o)$ divided by the modulus of elasticity at 28 days E_{cm28} as in Eq. (A-102). The creep coefficient $\phi_{28}(t,t_o)$ is the ratio of the creep strain to the elastic strain due to the load applied at the age of 28 days

$$J(t,t_o) = \frac{1}{E_{cmto}} + \frac{\phi_{28}(t,t_o)}{E_{cm28}} \qquad \text{(A-102)}$$

The 28-day creep coefficient $\phi_{28}(t,t_o)$ is calculated using Eq. (A-103)

In SI units:

$$\phi_{28}(t,t_o) = \Phi(t_c)\left[2\frac{(t-t_o)^{0.3}}{(t-t_o)^{0.3}+14} + \left(\frac{7}{t_o}\right)^{0.5}\left(\frac{(t-t_o)}{(t-t_o)+7}\right)^{0.5} + 2.5(1-1.086h^2)\left(\frac{(t-t_o)}{(t-t_o)+0.12(V/S)^2}\right)^{0.5}\right] \qquad \text{(A-103)}$$

In in.-lb units:

$$\phi_{28}(t,t_o) = \Phi(t_c)\left[2\frac{(t-t_o)^{0.3}}{(t-t_o)^{0.3}+14} + \left(\frac{7}{t_o}\right)^{0.5}\left(\frac{(t-t_o)}{(t-t_o)+7}\right)^{0.5} + 2.5(1-1.086h^2)\left(\frac{(t-t_o)}{(t-t_o)+77(V/S)^2}\right)^{0.5}\right]$$

The creep coefficient includes three terms. The first two terms are required to calculate the basic creep, and the third term is for the drying creep. Similar to the shrinkage Eq. (A-100), at a relative humidity of 0.96, there is only basic creep (there is no drying creep). $\Phi(t_c)$ is the correction term for the effect of drying before loading.

If $t_o = t_c$

$$\Phi(t_c) = 1 \qquad \text{(A-104)}$$

When $t_o > t_c$

$$\Phi(t_c) = \left[1-\left(\frac{(t_o-t_c)}{(t_o-t_c)+0.12(V/S)^2}\right)^{0.5}\right]^{0.5} \text{ in SI units}$$

$$\Phi(t_c) = \left[1-\left(\frac{(t_o-t_c)}{(t_o-t_c)+77(V/S)^2}\right)^{0.5}\right]^{0.5} \text{ in in.-lb units} \qquad \text{(A-105)}$$

To calculate relaxation, $\Phi(t_c)$ remains constant at the initial value throughout the relaxation period. For creep recovery calculations, $\Phi(t_c)$ remains constant at the value at the age of loading.

APPENDIX B—STATISTICAL INDICATORS

B.1—BP coefficient of variation (ϖ_{BP}%) method

Developed by Bažant and Panula (1978), a coefficient of variation ϖ_{BP} is determined for each data set. Data points in each logarithmic decade, 0 to 9.9 days, 10 to 99.9 days, and so on, are considered as one group. Weight is assigned to each data point based on the decade in which it falls and number of data points in that particular decade. The overall coefficient of variation (ϖ_{B3}) for all data sets is the root mean square (RMS) of the data set values

$$\bar{O}_j = \frac{1}{n_w}\sum_{i=1}^{n}(\varpi_{ij}O_{ij}) \qquad \text{(B-1)}$$

$$\varpi_j = \frac{1}{\bar{O}_j}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\varpi_{ij}(C_{ij}-O_{ij})^2} \qquad \text{(B-2)}$$

$$\varpi_{ij} = \frac{n}{n_d n_k} \qquad \text{(B-3)}$$

$$\varpi_{BP} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\varpi_j^2} \qquad \text{(B-4)}$$

where

- n = number of data points in data set number j;
- n_w = sum of the weights of all data points in a data set;
- n_k = number of data points in the k-th decade;
- n_d = number of decades on the logarithmic scale spanned by measured data in data set j;
- N = number of data sets;
- O_{ij} = measured value of the shrinkage strain or creep compliance for the i-th data point in data set number j;
- C_{ij} = predicted value of the shrinkage strain or creep compliance for the i-th data point in data set number j;
- $C_{ij} - O_{ij}$ = deviation of the predicted shrinkage strain or creep compliance from the measured value for the i-th data point in data set number, j;
- ϖ_{ij} = weight assigned to the i-th data point in data set number j;
- ϖ_j = coefficient of variation for data set number j; and
- ϖ_{B3} = overall coefficient of variation.

B.2—CEB statistical indicators

The CEB statistical indicators: coefficient of variation V_{CEB}, the mean square error F_{CEB}, and the mean deviation M_{CEB} were suggested by Muller and Hilsdorf (1990). The indicators are calculated in six time ranges: 0 to 10 days, 11 to 100 days, 101 to 365 days, 366 to 730 days, 731 to 1095 days, and above 1095 days. The final values are the RMS of the six interval values.

B.2.1 *CEB coefficient of variation*

$$\overline{O}_i = \frac{1}{n}\sum_{j=1}^{n}(O_{ij}) \tag{B-5}$$

$$V_i = \frac{1}{\overline{O}_j}\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(C_{ij}-O_{ij})^2} \tag{B-6}$$

$$V_{CEB} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}V_i^2} \tag{B-7}$$

where
n = number of data points considered;
N = total number of data sets considered;
V_i = coefficient of variation in interval i; and
V_{CEB} = RMS coefficient of variation.

B.2.2 *CEB mean square error*—The mean square error uses the difference between the calculated and observed values relative to the observed value

$$f_j = \frac{(C_{ij}-O_{ij})}{O_{ij}} \times 100 \tag{B-8}$$

$$F_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}f_j^2} \tag{B-9}$$

$$F_{CEB} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}F_i^2} \tag{B-10}$$

where
f_j = percent difference between calculated and observed data point j; and
F_{CEB} = mean square error, %.

B.2.3 *CEB mean deviation*—The CEB mean deviation M_{CEB} indicates systematic overestimation or underestimation of a given model

$$M_i = \frac{1}{n}\sum_{j=1}^{n}\frac{C_{ij}}{O_{ij}} \tag{B-11}$$

$$M_{CEB} = \frac{\sum_{i=1}^{N}M_i}{N} \tag{B-12}$$

where
M_i = ratio of calculated to experimental values in time range i;
M_{CEB} = mean deviation;
n = number of values considered in time interval; and
N = total number of data sets considered.

B.3—The Gardner coefficient of variation (ω_G)

Developed by Gardner (2004), the mean observed value and RMS of the difference between calculated and observed values were calculated in half logarithmic time intervals: 3 to 9.9 days, 10 to 31.5 days, 31.6 to 99 days, 100 to 315 days, 316 to 999 days, 1000 to 3159 days, and above 3160 days. That is, the duration of each time interval is 3.16 times the previous value. To obtain a criterion of fit, the average values and RMSs were averaged without regard to the number of observations in each half-decade. A coefficient of variation is obtained by dividing the average RMS normalized by the average value. It is necessary to emphasize that this is not the conventional definition of the coefficient of variation

$$\overline{O}_j = \frac{1}{n}\sum_{i=1}^{n}(O_{ij}) \tag{B-13}$$

$$\mathrm{RMS}_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(C_{ij}-O_{ij})^2} \tag{B-14}$$

$$\overline{O} = \frac{1}{N}\sum_{j=1}^{n}(\overline{O}_j) \tag{B-15}$$

$$\overline{\mathrm{RMS}} = \frac{1}{N}\sum_{j=1}^{n}(\mathrm{RMS}_j) \tag{B-16}$$

$$\omega_G = \frac{\overline{\mathrm{RMS}}}{\overline{O}} \tag{B-17}$$

APPENDIX C—NUMERIC EXAMPLES

Find the creep coefficients and shrinkage strains of concrete at 14, 28, 60, 90, 180, and 365 days after casting, from the following information: specified concrete compressive strength of 25 MPa (3626 psi), 7 days of moist curing, age of loading t_o = 14 days, 70% ambient relative humidity, and volume-surface ratio of the member = 100 mm (4 in.).

Problem data			
Concrete data:		SI units	in.-lb units
Specified 28-day strength	f'_c =	25 MPa	3626 psi
Ambient conditions:			
Relative humidity	h =	0.7	
Temperature	T =	20 °C	68 °F
Specimen:			
Volume-surface ratio	V/S =	100 mm	4 in.
Shape		Infinite slab	
Initial curing:			
Curing time	t_c =	7 days	
Curing condition		Moist cured	
Concrete at loading:			
Age at loading	t_o =	14 days	
Applied stress range	k_s =	40%	

C.1—ACI 209R-92 model solution

C.1.1 *Estimated concrete properties*

		SI units	in.-lb units	
Mean 28-day strength	f_{cm28} =	33.3 MPa	4830 psi	Table 5.3.2.2 ACI 318-05
Mean 28-day elastic modulus	E_{cm28} =	28,178 MPa	4,062,346 psi	(A-16)

C.1.2 *Estimated concrete mixture*

		SI units	in.-lb units	
Cement type		I	I	
Maximum aggregate size		20 mm	3/4 in.	
Cement content	c =	409 kg/m^3	690 lb/yd^3	
Water content	w =	205 kg/m^3	345 lb/yd^3	Table 6.3.3 ACI 211.1-91
Water-cement ratio	w/c =	0.50	(4-1)	
Aggregate-cement ratio	a/c =	4.23		
Fine aggregate percentage	ψ =	40%		
Air content	α =	2%		Table 6.3.3 ACI 211.1-91
Slump	s =	75 mm	2.95 in.	
Unit weight of concrete	γ_c =	2345 kg/m^3	3953 lb/yd^3	146* lb/ft^3

*Table A1.5.3.7.1 and 6.3.7.1 of ACI 211.1-91.

C.1.3 *Shrinkage strains* $\varepsilon_{sh}(t,t_c)$

	SI units	in.-lb units
Nominal ultimate shrinkage strain	$\varepsilon_{shu} = 780 \times 10^{-6}$	(A-2)
Moist curing correction factor	$\gamma_{sh,tc} = 1.202 - 0.2337\log(t_c) = 1.005$	(A-6)

Ambient relative humidity factor	$\gamma_{sh,RH} = 1.40 - 1.02h$ if $0.4 \leq h \leq 0.8$			(A-7)
	$\gamma_{sh,RH} = 3.00 - 3h$ if $0.8 < h \leq 1$			(A-7)
	$\gamma_{sh,RH} = 0.686$			(A-7)
Volume-to-surface ratio factor	$\gamma_{sh,vs} = 1.2e^{[-0.00472(V/S)]}$	(A-8)	$\gamma_{sh,vs} = 1.2e^{[-0.12(V/S)]}$	(A-8)
	$\gamma_{sh,vs} = 0.749$	(A-8)	$\gamma_{sh,vs} = 0.743$	(A-8)
Slump of fresh concrete factor	$\gamma_{sh,s} = 0.89 + 0.00161s$	(A-11)	$\gamma_{sh,s} = 0.89 + 0.041s$	(A-11)
	$\gamma_{sh,s} = 1.011$	(A-11)	$\gamma_{sh,s} = 1.011$	(A-11)
Fine aggregate factor	$\gamma_{sh,\psi} = 0.30 + 0.014\psi$ if $\psi \leq 50\%$			(A-12)
	$\gamma_{sh,\psi} = 0.90 + 0.002\psi$ if $\psi > 50\%$			(A-12)
	$\gamma_{sh,\psi} = 0.860$			(A-12)
Cement content factor	$\gamma_{sh,c} = 0.75 + 0.00061c$	(A-13)	$\gamma_{sh,c} = 0.75 + 0.00036c$	(A-13)
	$\gamma_{sh,c} = 0.999$	(A-13)	$\gamma_{sh,c} = 0.998$	(A-13)
Air content factor	$\gamma_{sh,\alpha} = 0.95 + 0.008\alpha \geq 1$			(A-14)
	$\gamma_{sh,\alpha} = 1.000$			(A-14)
Cumulative correction factor	$\gamma_{sh} = \gamma_{sh,tc}\gamma_{sh,RH}\gamma_{sh,vs}\gamma_{sh,s}\gamma_{sh,\psi}\gamma_{sh,c}\gamma_{sh,\alpha}$			(A-5)
	$\gamma_{sh} = 0.448$	(A-5)	$\gamma_{sh} = 0.444$	(A-5)
Ultimate shrinkage strain	$\varepsilon_{shu} = 780\gamma_{sh} \times 10^{-6}$			(A-4)
	$\varepsilon_{shu} = 350 \times 10^{-6}$	(A-4)	$\varepsilon_{shu} = 347 \times 10^{-6}$	(A-4)
Shrinkage time function	$f(t,t_c) = [(t - t_c)^{\alpha}/(f + (t - t_c)^{\alpha})]$			
Shrinkage strains	$\varepsilon_{sh}(t,t_c) = [(t - t_c)^{\alpha}/(f + (t - t_c)^{\alpha})]\varepsilon_{shu}$			(A-1)

$\alpha = 1$	t, days	$f(t - t_c)$	$\varepsilon_{sh}(t,t_c)$, $\times 10^{-6}$	t, days	$f(t - t_c)$	$\varepsilon_{sh}(t,t_c)$, $\times 10^{-6}$
$f = 35$ days	7	0.000	0	7	0.000	0
	14	0.167	58	14	0.167	58
	28	0.375	131	28	0.375	130
	60	0.602	211	60	0.602	209
	90	0.703	246	90	0.703	244
	180	0.832	291	180	0.832	288
	365	0.911	318	365	0.911	316

Note that the 365-day shrinkage strain reduces to 268×10^{-6} when the effect of the volume-surface ratio on the shrinkage time function is considered, that is, if $f = 26e^{0.0142(V/S)} = 108$ days ($f = 26e^{0.36(V/S)} = 110$ days).

C.1.4 *Compliance* $J(t,t_o)$

a) *Elastic compliance* $J(t_o,t_o)$

	SI units		in.-lb units	
Cement type	I			
	$a = 4$			(Table A.4)
	$b = 0.85$			(Table A.4)
Mean strength at age t_o	$f_{cmto} = [t_o/(a + bt_o)]f_{cm28}$			(A-17)
	$f_{cmto} = 29.3$ MPa	(A-17)	$f_{cmto} = 4253$ psi	(A-17)
Mean elastic modulus at age t_o	$E_{cmto} = 0.043\gamma_c^{1.5}f_{cmto}$	(A-16)	$E_{cmto} = 33\gamma_c^{1.5}f_{cmto}$	(A-16)
	$E_{cmto} = 26{,}441$ MPa	(A-16)	$E_{cmto} = 3{,}811{,}908$ psi	(A-16)
Elastic compliance	$J(t_o,t_o) = 1/E_{cmto}$			(A-15)
	$J(t_o,t_o) = 37.82 \times 10^{-6}$ (1/MPa)	(A-15)	$J(t_o,t_o) = 0.262 \times 10^{-6}$ (1/psi)	(A-15)

b) *Creep coefficient* $\phi(t,t_o)$

	SI units	in.-lb units
Nominal ultimate creep coefficient	$\phi_u = 2.35$ (A-19)	
Age application of load factor	$\gamma_{c,to} = 1.25t_o^{-0.118}$ (A-22)	
	$\gamma_{c,to} = 0.916$ (A-22)	
Ambient relative humidity factor	$\gamma_{c,RH} = 1.27 - 0.67h$ if $h \geq 0.4$ (A-24)	
	$\gamma_{c,RH} = 0.801$ (A-24)	
Volume-to-surface ratio factor	$\gamma_{c,vs} = 2/3[1 + 1.13e^{(-0.0213(V/S))}]$ (A-25)	$\gamma_{c,vs} = 2/3[1 + 1.13e^{(-0.54(V/S))}]$ (A-25)
	$\gamma_{c,vs} = 0.756$ (A-25)	$\gamma_{c,vs} = 0.754$ (A-25)
Slump of fresh concrete factor	$\gamma_{c,s} = 0.82 + 0.00264s$ (A-28)	$\gamma_{c,s} = 0.82 + 0.067s$ (A-28)
	$\gamma_{c,s} = 1.018$ (A-28)	$\gamma_{c,s} = 1.018$ (A-28)
Fine aggregate factor	$\gamma_{c,\psi} = 0.88 + 0.0024\psi$ (A-29)	
	$\gamma_{c,\psi} = 0.976$ (A-29)	
Air content factor	$\gamma_{c,\alpha} = 0.46 + 0.09\alpha \geq 1$ (A-30)	
	$\gamma_{c,\alpha} = 1.000$ (A-30)	
Cumulative correction factor	$\gamma_c = \gamma_{c,to}\gamma_{c,RH}\gamma_{c,vs}\gamma_{c,s}\gamma_{c,\psi}\gamma_{sh,\alpha}$ (A-21)	
	$\gamma_c = 0.551$ (A-21)	$\gamma_c = 0.549$ (A-21)
Ultimate shrinkage strain	$\phi_u = 2.35\gamma_c$ (A-20)	
	$\phi_u = 1.29$ (A-20)	$\phi_u = 1.29$ (A-20)
Creep coefficient time function	$f(t - t_o) = [(t - t_o)^{\Psi}/(d + (t - t_o)^{\Psi})]$	
Creep coefficients	$\phi(t,t_o) = [(t - t_o)^{\Psi}/(d + (t - t_o)^{\Psi})]\phi_u$ (A-18)	

	SI units			in.-lb units		
$\psi = 0.6$	t, days	$f(t - t_c)$	$\phi(t,t_o)$	t, days	$f(t - t_c)$	$\phi(t,t_o)$
$d = 10$ days	14	0.000	0.000	14	0.000	0.000
	28	0.328	0.424	28	0.328	0.423
	60	0.499	0.646	60	0.499	0.643
	90	0.573	0.742	90	0.573	0.740
	180	0.682	0.883	180	0.682	0.880
	365	0.771	0.998	365	0.771	0.995

c) *Compliance* $J(t,t_o) = 1/E_{cmto} + \phi(t,t_o)/E_{cmto}$

	SI units			in.-lb units		
t, days	$J(t_o,t_o)$, $\times 10^{-6}$	$\phi(t,t_o)/E_{cmto}$, $\times 10^{-6}$	$J(t,t_o)$ (1/MPa), $\times 10^{-6}$	$J(t_o,t_o)$, $\times 10^{-6}$	$\phi(t,t_o)/E_{cmto}$, $\times 10^{-6}$	$J(t,t_o)$ (1/psi), $\times 10^{-6}$
14	37.82	0	37.82	0.262	0	0.262
28	37.82	16.04	53.86	0.262	0.111	0.373
60	37.82	24.42	62.24	0.262	0.169	0.431
90	37.82	28.08	65.90	0.262	0.195	0.457
180	37.82	33.41	71.24	0.262	0.231	0.493
365	37.82	37.75	75.58	0.262	0.261	0.523

Note that when the effect of the volume-surface ratio is considered in the time function of the creep coefficient as $d = 26e^{0.0142(V/S)} = 108$ days ($f = 26e^{0.36(V/S)} = 110$ days) and $\psi = 1$, the creep coefficient and the compliance rate of development are initially smaller than when the effect of the volume-surface ratio is not considered; however, after 365 days under load, they are similar.

C.2—Bažant-Baweja B3 model solution

C.2.1 *Estimated concrete properties*

		SI units	in.-lb units	
Mean 28-day strength	f_{cm28} =	33.3 MPa	4830 psi	Table 5.3.2.2 ACI 318-05
Mean 28-day elastic modulus	E_{cm28} =	27,318 MPa	3,961,297 psi	(A-39)

C.2.2 *Estimated concrete mixture*

		SI units	in.-lb units	
Cement type		I		
Maximum aggregate size		20 mm	3/4 in.	
Cement content	c =	409 kg/m^3	690 lb/yd^3	
Water content	w =	205 kg/m^3	345 lb/yd^3	Table 6.3.3 ACI 211.1-91
Water-cement ratio	w/c =	0.50	(4-1)	
Aggregate-cement ratio	a/c =	4.23		
Fine aggregate percentage	ψ =	40%		
Air content	α =	2%		Table 6.3.3 ACI 211.1-91
Slump	s =	75 mm	2.95 in.	
Unit weight of concrete	γ_c =	2345 kg/m^3	3953 lb/yd^3	146 lb/ft^3*

*Table A1.5.3.7.1 and 6.3.7.1 of ACI 211.1-91.

C.2.3 *Shrinkage strains* $\varepsilon_{sh}(t,t_c)$

	SI units	in.-lb units
Ambient relative humidity factor	$k_h = -0.2$ if $h = 1$ (Table A.6)	
	$k_h = 12.74 - 12.94h$ if $0.98 < h < 1$ (Table A.6)	
	$k_h = 1 - h^3$ if $h \le 0.98$ (Table A.6)	
	$k_h = 0.657$ (Table A.6)	
Cement type factor	$\alpha_1 = 1.000$ (Table A.7)	
Curing condition factor	$\alpha_2 = 1.000$ (Table A.8)	
Nominal ultimate shrinkage	$\varepsilon_{s\infty} = -\alpha_1\alpha_2[0.019w^{2.1}f_{cm28}{}^{-0.28} + 270] \times 10^{-6}$ (A-33)	$\varepsilon_{s\infty} = -\alpha_1\alpha_2[0.02565w^{2.1}f_{cm28}{}^{-0.28} + 270] \times 10^{-6}$ (A-33)
	$\varepsilon_{s\infty} = -780 \times 10^{-6}$ (A-33)	$\varepsilon_{s\infty} = -781 \times 10^{-6}$ (A-33)
Member shape factor	$k_s = 1.000$ (Table A.9)	
Shrinkage half-time	$\tau_{sh} = 0.085t_c{}^{-0.08}f_{cm28}{}^{-0.25}[2k_s(V/S)]^2$ (A-36)	$\tau_{sh} = 190.8t_c{}^{-0.08}f_{cm28}{}^{-0.25}[2k_s(V/S)]^2$ (A-36)
	$\tau_{sh} = 1211.323$ (A-36)	$\tau_{sh} = 1253.630$ (A-36)
Time dependence factor	$E_{cm607}/E_{cm(t_c+\tau_{sh})} = 1.16742/[(t_c + \tau_{sh})/(4 + 0.85(t_c + \tau_{sh}))]$ (A-32) & (A-34)	
	$E_{cm607}/E_{cm(t_c+\tau_{sh})} = 0.996$ (A-32) & (A-34)	$E_{cm607}/E_{cm(t_c+\tau_{sh})} = 0.996$ (A-32) & (A-34)
Ultimate shrinkage strain	$\varepsilon_{sh\infty} = -\varepsilon_{s\infty}E_{cm607}/E_{cm(t_c+\tau_{sh})}$ (A-32)	
	$\varepsilon_{sh\infty} = -777 \times 10^{-6}$ (A-32)	$\varepsilon_{sh\infty} = -778 \times 10^{-6}$ (A-32)
Shrinkage time function	$S(t - t_c) = \tanh[(t - t_c)/\tau_{sh}]^{0.5}$ (A-35)	
Shrinkage strains	$\varepsilon_{sh}(t,t_c) = -\varepsilon_{sh\infty}k_h\tanh[(t - t_c)/\tau_{sh}]^{0.5}$ (A-31)	

	t, days	$S(t-t_c)$	$\varepsilon_{sh}(t,t_c)$, × 10^{-6}	t, days	$S(t-t_c)$	$\varepsilon_{sh}(t,t_c)$, × 10^{-6}
	7	0.000	0	7	0.000	0
	14	0.076	–39	14	0.075	–38
	28	0.131	–67	28	0.129	–66
	60	0.206	–105	60	0.203	–104
	90	0.256	–131	90	0.252	–129
	180	0.361	–184	180	0.355	–182
	365	0.496	–253	365	0.489	–250

C.2.4 *Compliance* $J(t,t_o) = q_1 + C_o(t,t_o) + C_d(t,t_o,t_c)$

a) *Instantaneous compliance* $q_1 = 0.6/E_{cm28}$

	SI units	in.-lb units
Instantaneous compliance	$q_1 = 1/E_o = 0.6/E_{cm28}$ (A-38)	
	$q_1 = 21.96 \times 10^{-6}$(1/MPa)	$q_1 = 0.152 \times 10^{-6}$(1/psi)

b) *Compliance function for basic creep* $C_o(t,t_o) = q_2Q(t,t_o) + q_3 ln[1 + (t-t_o)^n] + q_4 ln(t/t_o)$

Aging viscoelastic term $q_2Q(t,t_o)$

SI units		in.-lb units	
$q_2 = 185.4 \times 10^{-6} c^{0.5} f_{cm28}{}^{-0.9}$	(A-41)	$q_2 = 86.814 \times 10^{-6} c^{0.5} f_{cm28}{}^{-0.9}$	(A-41)
$q_2 = 159.9 \times 10^{-6}$ (1/MPa)	(A-41)	$q_2 = 1.103 \times 10^{-6}$ (1/psi)	(A-41)
$Q_f(t_o) = [0.086(t_o)^{2/9} + 1.21(t_o)^{4/9}]^{-1}$			(A-43)
$Q_f(t_o) = 0.246$			(A-43)
$m = 0.5$			
$n = 0.1$			
$r(t_o) = 1.7(t_o)^{0.12} + 8$			(A-45)
$r(t_o) = 10.333$			(A-45)
Aging viscoelastic term		Aging viscoelastic term	
$Z(t,t_o) = (t_o)^{-m}\ln[1 + (t-t_o)^n]$			(A-44)
$Q(t,t_o) = Q_f(t_o)[1 + \{Q_f(t_o)/Z(t,t_o)\}^{r(t_o)}]^{-1/r(t_o)}$			(A-42)

t, days	$Z(t,t_o)$	$Q(t,t_o)$	$q_2Q(t,t_o)$ (1/MPa), × 10^{-6}	t, days	$Z(t,t_o)$	$Q(t,t_o)$	$q_2Q(t,t_o)$ (1/psi), × 10^{-6}
14	0.000	0.000	0	14	0.000	0.000	0
28	0.223	0.216	34.59	28	0.223	0.216	0.239
60	0.241	0.228	36.41	60	0.241	0.228	0.251
90	0.249	0.232	37.02	90	0.249	0.232	0.255
180	0.262	0.236	37.78	180	0.262	0.236	0.261
365	0.275	0.240	38.30	365	0.275	0.240	0.264

Nonaging viscoelastic term $q_3\ln[1 + (t-t_o)^n]$

SI units		in.-lb units	
$q_3 = 0.29(w/c)^4 q_2$			(A-46)
$q_3 = 2.924 \times 10^{-6}$ (1/MPa)	(A-46)	$q_3 = 0.020 \times 10^{-6}$ (1/psi)	(A-46)
$n = 0.1$			

Nonaging viscoelastic term			Nonaging viscoelastic term		
t, days	$\ln[1 + (t - t_o)^n]$	$q_3\ln[1 + (t - t_o)^n]$ (1/MPa), $\times 10^{-6}$	t, days	$\ln[1 + (t - t_o)^n]$	$q_3\ln[1 + (t - t_o)^n]$ (1/psi), $\times 10^{-6}$
14	0.000	0	14	0.000	0
28	0.834	2.44	28	0.834	0.017
60	0.903	2.64	60	0.903	0.018
90	0.933	2.73	90	0.933	0.019
180	0.981	2.87	180	0.981	0.020
365	1.029	3.01	365	1.029	0.021

Aging flow term $q_4\ln(t/t_o)$

SI units			in.-lb units		
$q_4 = 20.3 \times 10^{-6}(a/c)^{-0.7}$ (A-47)			$q_4 = 0.14 \times 10^{-6}(a/c)^{-0.7}$ (A-47)		
$q_4 = 7.396 \times 10^{-6}$ (1/MPa) (A-47)			$q_4 = 5.106 \times 10^{-8}$ (1/psi) (A-47)		
Aging flow term			Aging flow term		
t, days	$\ln(t,t_o)$	$q_4\ln(t/t_o)$ (1/MPa), $\times 10^{-6}$	t, days	$\ln(t,t_o)$	$q_4\ln(t/t_o)$ (1/psi), $\times 10^{-6}$
14	0.000	0	14	0.000	0
28	0.693	5.13	28	0.693	0.035
60	1.455	10.76	60	1.455	0.074
90	1.861	13.76	90	1.861	0.095
180	2.554	18.89	180	2.554	0.130
365	3.261	24.12	365	3.261	0.167

Compliance function for basic creep $C_o(t,t_o) = q_2Q(t,t_o) + q_3\ln[1 + (t - t_o)^n] + q_4\ln(t/t_o)$

SI units					in.-lb units				
$C_o(t,t_o) = q_2Q(t,t_o) + q_3\ln[1 + (t - t_o)^n] + q_4\ln(t/t_o)$ (A-40)									
t, days	$q_2Q(t,t_o)$	$q_3\ln[1 + (t - t_o)^n]$	$q_4\ln(t/t_o)$, $\times 10^{-6}$	$C_o(t,t_o)$ (1/MPa), $\times 10^{-6}$	t, days	$q_2Q(t,t_o)$	$q_3\ln[1 + (t - t_o)^n]$	$q_4\ln(t/t_o)$, $\times 10^{-6}$	$C_o(t,t_o)$ (1/psi), $\times 10^{-6}$
14	0	0	0	0	14	0	0	0	0
28	34.59	2.44	5.13	42.15	28	0.239	0.017	0.035	0.291
60	36.41	2.64	10.76	49.81	60	0.251	0.018	0.074	0.344
90	37.02	2.73	13.76	53.51	90	0.255	0.019	0.095	0.369
180	37.78	2.87	18.89	59.54	180	0.261	0.020	0.130	0.411
365	38.30	3.01	24.12	65.42	365	0.264	0.021	0.167	0.451

c) *Compliance function for drying creep* $C_d(t,t_o,t_c) = q_5[\exp\{-8H(t)\} - \exp\{-8H(t_o)\}]^{0.5}$

SI units	in.-lb units
$q_5 = 0.757f_{cm28}^{-1}\lvert\varepsilon_{sh\infty} \times 10^6\rvert^{-0.6}$ (A-49)	
$q_5 = 419.3 \times 10^{-6}$ (1/MPa) (A-49)	$q_5 = 2.889 \times 10^{-6}$ (1/psi) (A-49)
$S(t_o - t_c) = \tanh[(t_o - t_c)/\tau_{sh}]^{0.5}$ (A-53)	
$S(t_o - t_c) = 7.587 \times 10^{-2}$ (A-53)	$S(t_o - t_c) = 7.459 \times 10^{-2}$ (A-53)
$H(t_o) = 1 - (1 - h)S(t_o - t_c)$ (A-51)	
$H(t_o) = 0.977$ (A-51)	$H(t_o) = 0.978$ (A-51)
$S(t - t_c) = \tanh[(t - t_c)/\tau_{sh}]^{0.5}$ (A-52)	

$H(t) = 1 - (1 - h)S(t - t_c)$ (A-50)									
$f(H) = [\exp\{-8H(t)\} - \exp\{-8H(t_o)\}]^{0.5}$									
$C_d(t,t_o,t_c) = q_5[\exp\{-8H(t)\} - \exp\{-8H(t_o)\}]^{0.5}$ (A-48)									
t, days	$S(t - t_c)$	$H(t)$	$f(H)$, $\times 10^{-2}$	$C_d(t,t_o,t_c)$ (1/MPa), $\times 10^{-6}$	t, days	$S(t - t_c)$	$H(t)$	$f(H)$, $\times 10^{-2}$	$C_d(t,t_o,t_c)$ (1/psi), $\times 10^{-6}$
14	0.076	0.977	0	0	14	0.075	0.978	0	0
28	0.131	0.961	0.754	3.16	28	0.129	0.961	0.746	0.022
60	0.206	0.938	1.216	5.10	60	0.203	0.939	1.202	0.035
90	0.256	0.923	1.475	6.19	90	0.252	0.925	1.458	0.042
180	0.361	0.892	1.988	8.34	180	0.355	0.893	1.964	0.057
365	0.496	0.851	2.646	11.10	365	0.489	0.853	2.613	0.076

d) *Compliance* $J(t,t_o) = q_1 + C_o(t,t_o) + C_d(t,t_o,t_c)$

SI units					in.-lb units				
$J(t,t_o) = q_1 + C_o(t,t_o) + C_d(t,t_o,t_c)$ (A-37)									
t, days	q_1, $\times 10^{-6}$	$C_o(t,t_o)$, $\times 10^{-6}$	$C_d(t,t_o,t_c)$, $\times 10^{-6}$	$J(t,t_o)$ (1/MPa), $\times 10^{-6}$	t, days	q_1, $\times 10^{-6}$	$C_o(t,t_o)$, $\times 10^{-6}$	$C_d(t,t_o,t_c)$, $\times 10^{-6}$	$J(t,t_o)$ (1/psi), $\times 10^{-6}$
14	21.96	0	0	21.96	14	0.152	0	0	0.152
28	21.96	42.15	3.16	67.27	28	0.152	0.291	0.022	0.464
60	21.96	49.81	5.10	76.87	60	0.152	0.344	0.035	0.530
90	21.96	53.51	6.19	81.66	90	0.152	0.369	0.042	0.563
180	21.96	59.54	8.34	89.84	180	0.152	0.411	0.057	0.619
365	21.96	65.42	11.10	98.48	365	0.152	0.451	0.076	0.678

C.3—CEB MC90-99 model solution

C.3.1 *Estimated concrete properties*

		SI units	in.-lb units	
Mean 28-day strength	$f_{cm28} =$	33.0 MPa	4786 psi	(A-73)
Strength constant	$f_{cmo} =$	10 MPa	1450 psi	(A-72)
Mean 28-day elastic modulus	$E_{cm28} =$	32,009 MPa	4,642,862 psi	(A-72)

C.3.2 *Estimated concrete mixture*

		SI units	in.-lb units	
Cement type		N		
Maximum aggregate size		20 mm	3/4 in.	
Cement content	$c =$	406 kg/m^3	685 lb/yd^3	
Water content	$w =$	205 kg/m^3	345 lb/yd^3	Table 6.3.3 ACI 211.1-91
Water-cement ratio	$w/c =$	0.504	(4-1)	
Aggregate-cement ratio	$a/c =$	4.27		
Fine aggregate percentage	$\psi =$	40%		
Air content	$\alpha =$	2%		Table 6.3.3 ACI 211.1-91
Slump	$s =$	75 mm	2.95 in.	
Unit weight of concrete	$\gamma_c =$	2345 kg/m^3	3953 lb/yd^3	146* lb/ft^3

*Table A1.5.3.7.1 and 6.3.7.1 of ACI 211.1-91.

C.3.3 *CEB MC90 shrinkage strains* $\varepsilon_{sh}(t,t_c)$

	SI units	in.-lb units
Cement type factor	$\beta_{sc} = 5$	(Table A.10)

<table>
<tr><td rowspan="2">Concrete strength factor</td><td colspan="6">$\varepsilon_s(f_{cm28}) = [160 + 10\beta_{sc}(9 - f_{cm28}/f_{cmo})] \times 10^{-6}$ (A-56)</td></tr>
<tr><td colspan="6">$\varepsilon_s(f_{cm28}) = 445 \times 10^{-6}$ (A-56)</td></tr>
<tr><td rowspan="4">Ambient relative humidity factor</td><td colspan="6">$\beta_{RH}(h) = -1.55[1 - (h/h_o)^3]$ for $0.4 \le h < 0.99$ (A-57)</td></tr>
<tr><td colspan="6">$\beta_{RH}(h) = 0.25$ for $h \ge 0.99$ (A-57)</td></tr>
<tr><td colspan="6">$h_o = 1$</td></tr>
<tr><td colspan="6">$\beta_{RH}(h) = -1.018$ (A-57)</td></tr>
<tr><td rowspan="2">Notional shrinkage coefficient</td><td colspan="6">$\varepsilon_{cso} = \varepsilon_s(f_{cm28})\beta_{RH}(h)$ (A-55)</td></tr>
<tr><td colspan="3">$\varepsilon_{cso} = -453 \times 10^{-6}$ (A-55)</td><td colspan="3">$\varepsilon_{cso} = -453 \times 10^{-6}$ (A-55)</td></tr>
<tr><td rowspan="3">Shrinkage time function</td><td colspan="6">$\beta_s(t - t_c) = [\{(t - t_c)/t_1\}/\{350([(V/S)/(V/S)_o]^2 + (t - t_c)t_i\}]^{0.5}$ (A-58)</td></tr>
<tr><td colspan="6">$t_1 = 1$ day</td></tr>
<tr><td colspan="3">$(V/S)_o = 50$ mm</td><td colspan="3">$(V/S)_o = 2$ in.</td></tr>
<tr><td>Shrinkage strains</td><td colspan="6">$\varepsilon_{sh}(t,t_c) = \varepsilon_{cso}\beta_s(t - t_c)$ (A-54)</td></tr>
<tr><td rowspan="8"></td><td>t, days</td><td>$\beta_s(t - t_c)$</td><td>$\varepsilon_{sh}(t,t_c)$, $\times 10^{-6}$</td><td>t, days</td><td>$\beta_s(t - t_c)$</td><td>$\varepsilon_{sh}(t,t_c)$, $\times 10^{-6}$</td></tr>
<tr><td>7</td><td>0.000</td><td>0</td><td>7</td><td>0.000</td><td>0</td></tr>
<tr><td>14</td><td>0.071</td><td>–32</td><td>14</td><td>0.071</td><td>–32</td></tr>
<tr><td>28</td><td>0.122</td><td>–55</td><td>28</td><td>0.122</td><td>–55</td></tr>
<tr><td>60</td><td>0.191</td><td>–87</td><td>60</td><td>0.191</td><td>–87</td></tr>
<tr><td>90</td><td>0.237</td><td>–107</td><td>90</td><td>0.237</td><td>–107</td></tr>
<tr><td>180</td><td>0.332</td><td>–150</td><td>180</td><td>0.332</td><td>–150</td></tr>
<tr><td>365</td><td>0.451</td><td>–205</td><td>365</td><td>0.451</td><td>–205</td></tr>
</table>

C.3.4 *CEB MC90-99 shrinkage strains* $\varepsilon_{sh}(t,t_c)$

a) *Autogenous shrinkage* $\varepsilon_{cas}(t)$

<table>
<tr><td></td><td colspan="3">SI units</td><td colspan="3">in.-lb units</td></tr>
<tr><td>Cement type factor</td><td colspan="6">$\alpha_{as} = 700$ (Table A.11)</td></tr>
<tr><td rowspan="2">Notional autogenous shrinkage</td><td colspan="6">$\varepsilon_{caso}(f_{cm28}) = -\alpha_{as}[(f_{cm28}/f_{cmo})/\{6 + (f_{cm28}/f_{cmo})\}]^{2.5} \times 10^{-6}$ (A-63)</td></tr>
<tr><td colspan="3">$\varepsilon_{caso}(f_{cm28}) = -52.5 \times 10^{-6}$ (A-63)</td><td colspan="3">$\varepsilon_{caso}(f_{cm28}) = -52.5 \times 10^{-6}$ (A-63)</td></tr>
<tr><td rowspan="2">Autogenous shrinkage time function</td><td colspan="6">$\beta_{as}(t) = 1 - \exp[-0.2(t/t_i)^{0.5}]$ (A-64)</td></tr>
<tr><td colspan="6">$t_1 = 1$ day</td></tr>
<tr><td>Autogenous shrinkage strains</td><td colspan="6">$\varepsilon_{cas}(t) = \varepsilon_{caso}(f_{cm28})\beta_{as}(t)$ (A-62)</td></tr>
<tr><td rowspan="9"></td><td>t, days</td><td>$\beta_{as}(t)$</td><td>$\varepsilon_{cas}(t)$, $\times 10^{-6}$</td><td>t, days</td><td>$\beta_{as}(t)$</td><td>$\varepsilon_{cas}(t)$, $\times 10^{-6}$</td></tr>
<tr><td>0</td><td>0.000</td><td>0</td><td>0</td><td>0.000</td><td>0</td></tr>
<tr><td>7</td><td>0.411</td><td>–22</td><td>7</td><td>0.411</td><td>–22</td></tr>
<tr><td>14</td><td>0.527</td><td>–28</td><td>14</td><td>0.527</td><td>–28</td></tr>
<tr><td>28</td><td>0.653</td><td>–34</td><td>28</td><td>0.653</td><td>–34</td></tr>
<tr><td>60</td><td>0.788</td><td>–41</td><td>60</td><td>0.788</td><td>–41</td></tr>
<tr><td>90</td><td>0.850</td><td>–45</td><td>90</td><td>0.850</td><td>–45</td></tr>
<tr><td>180</td><td>0.932</td><td>–49</td><td>180</td><td>0.932</td><td>–49</td></tr>
<tr><td>365</td><td>0.978</td><td>–51</td><td>365</td><td>0.978</td><td>–51</td></tr>
</table>

b) *Drying shrinkage* $\varepsilon_{cds}(t,t_c)$

<table>
<tr><td></td><td>SI units</td><td>in.-lb units</td></tr>
<tr><td rowspan="2">Cement type factors</td><td colspan="2">$\alpha_{ds1} = 4$ (Table A.11)</td></tr>
<tr><td colspan="2">$\alpha_{ds2} = 0.12$ (Table A.11)</td></tr>
</table>

Notional drying shrinkage coefficient	$\varepsilon_{cdso}(f_{cm28}) = [(220 + 110\alpha_{ds1})\exp(-\alpha_{ds2}f_{cm28}/f_{cmo})] \times 10^{-6}$ (A-66)	
	$\varepsilon_{cdso}(f_{cm28}) = 444 \times 10^{-6}$ (A-66)	$\varepsilon_{cdso}(f_{cm28}) = 444 \times 10^{-6}$ (A-66)
Ambient relative humidity factor	$h_o = 1$	
	$\beta_{\sigma 1} = [3.5f_{cmo}/f_{cm28}]^{0.1} \leq 1.0$ (A-69)	
	$\beta_{\sigma 1} = 1.000$ (A-69)	$\beta_{\sigma 1} = 1.000$ (A-69)
	$\beta_{RH}(h) = -1.55[1 - (h/h_o)^3]$ for $0.4 \leq h < 0.99\beta_{s1}$ (A-67)	
	$\beta_{RH}(h) = 0.25$ for $h \geq 0.99\beta_{s1}$ (A-67)	
	$\beta_{RH}(h) = -1.018$ (A-67)	$\beta_{RH}(h) = -1.018$ (A-67)
Drying shrinkage time function	$\beta_{ds}(t - t_c) = [\{(t - t_c)/t_1\}/\{350([(V/S)/(V/S)_o]^2 + (t - t_c)/t_i\}]^{0.5}$ (A-68)	
	$t_1 = 1$ day	
	$(V/S)_o = 50$ mm	$(V/S)_o = 2$ in.
Drying shrinkage strains	$\varepsilon_{cds}(t,t_c) = \varepsilon_{cdso}(f_{cm28})\beta_{RH}(h)\beta_{ds}(t - t_c)$ (A-65)	

t, days	$\beta_{ds}(t-t_c)$	$\varepsilon_{cds}(t,t_c)$, $\times 10^{-6}$	t, days	$\beta_{ds}(t-t_c)$	$\varepsilon_{cds}(t,t_c)$, $\times 10^{-6}$
7	0.000	0	7	0.000	0
14	0.071	–32	14	0.071	–32
28	0.122	–55	28	0.122	–55
60	0.191	–86	60	0.191	–87
90	0.237	–107	90	0.237	–107
180	0.332	–150	180	0.332	–150
365	0.451	–204	365	0.451	–205

c) *Total shrinkage strains* $\varepsilon_{sh}(t,t_c)$

SI units				in.-lb units			
$\varepsilon_{sh}(t,t_c) = \varepsilon_{cas}(t) + \varepsilon_{cds}(t,t_c)$ (A-61)							
t, days	$\varepsilon_{cas}(t)$, $\times 10^{-6}$	$\varepsilon_{cds}(t,t_c)$, $\times 10^{-6}$	$\varepsilon_{sh}(t,t_c)$, $\times 10^{-6}$	t, days	$\varepsilon_{cas}(t)$, $\times 10^{-6}$	$\varepsilon_{cds}(t,t_c)$, $\times 10^{-6}$	$\varepsilon_{sh}(t,t_c)$, $\times 10^{-6}$
0	0	—	0	0	0	—	0
7	–22	0	–22	7	–22	0	–22
14	–28	–32	–60	14	–28	–32	–60
28	–34	–55	–89	28	–34	–55	–89
60	–41	–86	–127	60	–41	–87	–128
90	–45	–107	–152	90	–45	–107	–152
180	–49	–150	–199	180	–49	–150	–199
365	–51	–204	–255	365	–51	–205	–256

C.3.5 *Compliance* $J(t,t_o)$

a) *Elastic compliance* $J(t_o,t_o)$

	SI units	in.-lb units
Cement type	N	
	$s = 0.25$ (Table A.12)	
Mean strength at age t_o	$\beta_e = \exp[s/2\{1 - (28/t_o)^{0.5}\}]$ (A-97)	
	$\beta_e = 0.950$ (A-97)	
	$f_{cmto} = \beta_e^2 f_{cm28}$ (A-96)	
	$f_{cmto} = 29.8$ MPa (A-96)	$f_{cmto} = 4315.1$ psi (A-96)
Mean elastic modulus at age t_o	$E_{cmto} = E_{cm28}\exp[s/2\{1 - (28/t_o)^{0.5}\}]$ (A-71)	
	$E_{cmto} = 30{,}394$ MPa (A-71)	$E_{cmto} = 4{,}408{,}587$ psi (A-71)

Elastic compliance	$J(t_o,t_o) = 1/E_{cmto}$ (A-70)	
	$J(t_o,t_o) = 32.90 \times 10^{-6}$ (1/MPa) (A-70)	$J(t_o,t_o) = 0.227 \times 10^{-6}$ (1/psi) (A-70)
Effect of temperature on modulus of elasticity	$E_{cm28}(T) = E_{cm28}(1.06 - 0.003T/T_o)$ (A-85)	$E_{cm28}(T) = E_{cm28}(1.06 - 0.003 \cdot [18.778T - 600.883]/T_o)$ (A-85)
	$E_{cm28}(T) = 32{,}009$ MPa (A-85)	$E_{cm28}(T) = 4{,}642{,}853$ psi (A-85)
	$E_{cmto}(T) = E_{cmto}(1.06 - 0.003T/T_o)$ (A-85)	$E_{cmto}(T) = E_{cmto}(1.06 - 0.003 \cdot [18.778T - 600.883]/T_o)$ (A-85)
	$E_{cmto}(T) = 30{,}394$ MPa (A-85)	$E_{cmto}(T) = 4{,}408{,}579$ psi (A-85)
Elastic compliance temperature adjusted	$J(t_o,t_o) = 1/E_{cmto}$ (A-70)	
	$J(t_o,t_o) = 32.90 \times 10^{-6}$ (1/MPa) (A-70)	$J(t_o,t_o) = 0.227 \times 10^{-6}$ (1/psi) (A-70)

b) *Creep coefficient* $\phi_{28}(t,t_o)$

	SI units	in.-lb units
Compressive strength factors	$\alpha_1 = [3.5f_{cmo}/f_{cm28}]^{0.7}$ (A-79)	
	$\alpha_2 = [3.5f_{cmo}/f_{cm28}]^{0.2}$ (A-79)	
	$\alpha_1 = 1.042$ (A-79)	$\alpha_1 = 1.042$ (A-79)
	$\alpha_2 = 1.012$ (A-79)	$\alpha_2 = 1.012$ (A-79)
Ambient relative humidity and volume-surface ratio factor	$\phi_{RH}(h) = [1 + \{(1 - h/h_o)\alpha_1/(0.1(V/S)/(V/S)_o\}]\alpha_2$ (A-76)	
	$h_o = 1$	
	$(V/S)_o = 50$ mm	$(V/S)_o = 2$ in.
	$\phi_{RH}(h) = 1.553$ (A-76)	$\phi_{RH}(h) = 1.553$ (A-76)
Concrete strength factor	$\beta(f_{cm28}) = 5.3/(f_{cm28}/f_{cmo})^{0.5}$ (A-77)	
	$\beta(f_{cm28}) = 2.918$ (A-77)	$\beta(f_{cm28}) = 2.917$ (A-77)
Temperature-adjusted age of loading	$t_{o,T} = \Sigma\Delta t_i \exp[13.65 - 4000/\{273 + (T(\Delta t_i/T_o))\}]$ (A-87)	$t_{o,T} = \Sigma\Delta t_i \exp[13.65 - 4000/\{273 + (18.778T(\Delta t_i) - 600.883/T_o)\}]$ (A-87)
	$T_o = 1$ °C	$T_o = 33.8$ °F
	$t_{o,T} = 14.0$ days (A-87)	$t_{o,T} = 14.0$ days (A-87)
	$t_o = t_{o,T}[9/\{2 - (t_{o,T}/t_{1,T})^{1.2}\} + 1]^{\alpha} \geq 0.5$ days (A-81)	
	$\alpha = 0$	
	$t_{1,T} = 1$ day	
	$t_o = 14.0$ days (A-81)	
Adjusted age of loading factor	$\beta(t_o) = 1/[0.1 + (t_o/t_1)^{0.2}]$ (A-78)	
	$\beta(t_o) = 0.557$ (A-78)	$\beta(t_o) = 0.557$ (A-78)
Notional creep coefficient	$\phi_o = \phi_{RH}(h)\beta(f_{cm28})\beta(t_o)$ (A-75)	
	$\phi_o = 2.524$ (A-75)	$\phi_o = 2.524$ (A-75)
Creep coefficient time function	$\alpha_3 = [3.5f_{cmo}/f_{cm28}]^{0.5}$ (A-84)	
	$\alpha_3 = 1.030$ (A-84)	$\alpha_3 = 1.030$ (A-84)
	$\beta_H = 150[1 + (1.2h/h_o)^{18}](V/S)/(V/S)_o + 250\alpha_3 \leq 1500\alpha_3$ (A-83)	
	$\beta_H = 570.470$ (A-83)	$\beta_H = 570.445$ (A-83)
	$\beta_c(t - t_o) = [(t - t_o)/t_1/\{\beta_H + (t - t_o)/t_1\}]^{0.3}$ (A-82)	
Creep coefficients	$\phi_{28}(t,t_o) = \phi_o\beta_c(t - t_o)$ (A-74)	

	t, days	$\beta_c(t-t_o)$	$\phi_{28}(t,t_o)$	t, days	$\beta_c(t-t_o)$	$\phi_{28}(t,t_o)$
	14	0.000	0.000	14	0.000	0.000
	28	0.326	0.824	28	0.326	0.824
	60	0.459	1.159	60	0.459	1.159
	90	0.526	1.328	90	0.526	1.328
	180	0.640	1.614	180	0.640	1.614
	365	0.749	1.890	365	0.749	1.889

	SI units		in.-lb units	
Effect of temperature conditions	$\phi_T = \exp[0.015(T/T_o - 20)]$	(A-91)	$\phi_T = \exp[0.015\{(18.778T - 600.883)/T_o - 20\}]$	(A-91)
	$\phi_T = 1.000$	(A-91)	$\phi_T = 1.000$	(A-91)
	$\phi_{RH,T} = \phi_T + [f_{RH}(h) - 1]\phi_T^{1.2}$			(A-90)
	$\phi_{RH,T} = 1.553$	(A-90)	$\phi_{RH,T} = 1.553$	(A-90)
	$\phi_o = \phi_{RH,T}\beta(f_{cm28})\beta(t_o)$			(A-75)
	$\phi_o = 2.524$	(A-75)	$\phi_o = 2.524$	(A-75)
Effect of high stresses	$\phi_{o,k} = \phi_o \exp[1.5(k_\sigma - 0.4)]$			(A-93)
	$\phi_{o,k} = 2.524$	(A-93)	$\phi_{o,k} = 2.524$	(A-93)
Notional creep coefficient temperature and stress adjusted	$\phi_o = \phi_{ck}$			
	$\phi_o = 2.524$		$\phi_o = 2.524$	
Effect of temperature conditions on creep coefficient time function	$\beta_T = \exp[1500/(273 + T/T_o) - 5.12]$	(A-89)	$\beta_T = \exp[1500/\{273 + (18.778T - 600.883)/T_o\}) - 5.12]$	(A-89)
	$\beta_T = 0.999$	(A-89)	$\beta_T = 0.999$	(A-89)
	$\beta_{H,T} = \beta_H\beta_T$			(A-88)
	$\beta_{H,T} = 570.159$	(A-88)	$\beta_{H,T} = 570.128$	(A-88)
	$\Delta\phi_{T,trans} = 0.0004(T/T_o - 20)^2$	(A-92)	$\Delta\phi_{T,trans} = 0.0004[(18.778T - 600.883)/T_o - 20]^2$	(A-92)
	$\Delta\phi_{T,trans} = 0.000$	(A-92)	$\Delta\phi_{T,trans} = 0.000$	(A-92)
	$\beta_c(t-t_o) = [(t-t_o)/t_1/\{\beta_H + (t-t_o)/t_1\}]^{0.3}$			(A-82)
Creep coefficients temperature and stress adjusted	$\phi_{28}(t,t_o,T) = \phi_o\beta_c(t-t_o) + \Delta\phi_{T,trans}$			(A-86)

	t, days	$\beta_c(t-t_o)$	$\phi_{28}(t,t_o,T)$	t, days	$\beta_c(t-t_o)$	$\phi_{28}(t,t_o,T)$
	14	0.000	0.000	14	0.000	0.000
	28	0.326	0.824	28	0.326	0.824
	60	0.459	1.159	60	0.459	1.159
	90	0.526	1.328	90	0.526	1.328
	180	0.640	1.615	180	0.640	1.614
	365	0.749	1.890	365	0.749	1.890

c) *Compliance* $J(t,t_o) = 1/E_{cmto} + \phi_{28}(t,t_o)/E_{cm28}$

SI units				in.-lb units			
$J(t,t_o) = 1/E_{cmto} + \phi_{28}(t,t_o)/E_{cm28}$							(A-70)
t, days	$J(t_o,t_o)$, $\times 10^{-6}$	$\phi_{28}(t,t_o)/E_{cm28}$, $\times 10^{-6}$	$J(t,t_o)$ (1/MPa), $\times 10^{-6}$	t, days	$J(t_o,t_o)$, $\times 10^{-6}$	$\phi_{28}(t,t_o)/E_{cm28}$, $\times 10^{-6}$	$J(t,t_o)$ (1/psi), $\times 10^{-6}$
14	32.90	0	32.90	14	0.227	0	0.227
28	32.90	25.74	58.65	28	0.227	0.178	0.404
60	32.90	36.20	69.10	60	0.227	0.250	0.476
90	32.90	41.49	74.39	90	0.227	0.286	0.513
180	32.90	50.44	83.34	180	0.227	0.348	0.575
365	32.90	59.04	91.94	365	0.227	0.407	0.634

Compliance temperature and stress adjusted

SI units				in.-lb units			
$J(t,t_o) = 1/E_{cmto(T)} + \phi_{28}(t,t_o)/E_{cm28(T)}$							(A-70)
t, days	$J(t_o,t_o)$, $\times 10^{-6}$	$\phi_{28}(t,t_o)/E_{cm28}$, $\times 10^{-6}$	$J(t,t_o)$ (1/MPa), $\times 10^{-6}$	t, days	$J(t_o,t_o)$, $\times 10^{-6}$	$\phi_{28}(t,t_o)/E_{cm28}$, $\times 10^{-6}$	$J(t,t_o)$ (1/psi), $\times 10^{-6}$
14	32.90	0	32.90	14	0.227	0	0.227
28	32.90	25.75	58.65	28	0.227	0.178	0.404
60	32.90	36.21	69.11	60	0.227	0.250	0.476
90	32.90	41.50	74.40	90	0.227	0.286	0.513
180	32.90	50.44	83.34	180	0.227	0.348	0.575
365	32.90	59.04	91.94	365	0.227	0.407	0.634

C.4—GL2000 model solution

C.4.1 *Estimated concrete properties*

		SI units	in.-lb units	
Mean 28-day strength	$f_{cm28} =$	32.5 MPa	4689 psi	(A-94)
Mean 28-day elastic modulus	$E_{cm28} =$	28,014 MPa	4,060,590 psi	(A-95)

C.4.2 *Estimated concrete mixture*

		SI units	in.-lb units	
Cement type		I		
Maximum aggregate size		20 mm	3/4 in.	
Cement content	$c =$	402 kg/m^3	676 lb/yd^3	
Water content	$w =$	205 kg/m^3	345 lb/yd^3	Table 6.3.3 ACI 211.1-91
Water-cement ratio	$w/c =$	0.510	(4-1)	
Aggregate-cement ratio	$a/c =$	4.33		
Fine aggregate percentage	$\psi =$	40%		
Air content	$\alpha =$	2%		Table 6.3.3 ACI 211.1-91
Slump	$s =$	75 mm	2.95 in.	
Unit weight of concrete	$\gamma_c =$	2345 kg/m^3	3953 lb/yd^3	146 lb/ft^3*

*Table A1.5.3.7.1 and 6.3.7.1 of ACI 211.1-91.

C.4.3 *Shrinkage strains* $\varepsilon_{sh}(t,t_c)$

	SI units	in.-lb units
Cement type factor	$k = 1.000$	(Table A.14)
Ultimate shrinkage strain	$\varepsilon_{shu} = 900k[30/f_{cm28}]^{0.5} \times 10^{-6}$ (A-99)	$\varepsilon_{shu} = 900k[4350/f_{cm28}]^{0.5} \times 10^{-6}$ (A-99)
	$\varepsilon_{shu} = 865 \times 10^{-6}$ (A-99)	$\varepsilon_{shu} = 867 \times 10^{-6}$ (A-99)
Ambient relative humidity factor	$\beta(h) = (1 - 1.18h^4)$	(A-100)
	$\beta(h) = 0.717$	(A-100)
Shrinkage time function	$\beta(t-t_c) = [(t-t_c)/\{t-t_c+0.12(V/S)^2\}]^{0.5}$ (A-101)	$\beta(t-t_c) = [(t-t_c)/\{t-t_c+77(V/S)^2\}]^{0.5}$ (A-101)
Shrinkage strains	$\varepsilon_{sh}(t,t_c) = \varepsilon_{shu}\beta(h)\beta(t-t_c)$	(A-98)

t, days	$\beta(t-t_c)$	$\varepsilon_{sh}(t,t_c)$, $\times 10^{-6}$	t, days	$\beta(t-t_c)$	$\varepsilon_{sh}(t,t_c)$, $\times 10^{-6}$
7	0.000	0	7	0.000	0
14	0.076	47	14	0.075	47
28	0.131	81	28	0.129	80
60	0.206	128	60	0.203	126
90	0.254	158	90	0.251	156
180	0.355	220	180	0.351	218
365	0.479	297	365	0.475	295

C.4.4 *Compliance* $J(t,t_o)$

a) *Elastic compliance* $J(t_o,t_o)$

	SI units		in.-lb units	
Cement type	I			
	$s = 0.335$			(Table A.14)
Mean strength at age t_o	$\beta_e = \exp[s/2\{1 - (28/t_o)^{0.5}\}]$			(A-97)
	$\beta_e = 0.933$			(A-97)
	$f_{cmto} = \beta_e^2 f_{cm28}$			(A-96)
	$f_{cmto} = 28.3$ MPa	(A-96)	$f_{cmto} = 4081.1$ psi	(A-96)
Mean elastic modulus at age t_o	E_{cmto} (MPa) $= 3500 + 4300(f_{cmto})^{0.5}$	(A-95)	E_{cmto} (psi) $=$ 500,000 + 52,000$(f_{cmto})^{0.5}$	(A-95)
	$E_{cmto} = 26{,}371$ MPa	(A-95)	$E_{cmto} = 3{,}821{,}929$ psi	(A-95)
Elastic compliance	$J(t_o,t_o) = 1/E_{cmto}$			(A-102)
	$J(t_o,t_o) = 37.92 \times 10^{-6}$ (1/MPa)	(A-102)	$J(t_o,t_o) = 0.262 \times 10^{-6}$ (1/psi)	(A-102)

b) *Creep coefficient* $\phi_{28}(t,t_o)$

SI units				in.-lb units			
$J(t,t_o) = 1/E_{cmto} + \phi_{28}(t,t_o)/E_{cm28}$							(A-102)
Effect of drying before loading factor				Effect of drying before loading factor			
$\Phi(t_c) = 0.961$			(A-104) & (A-105)	$\Phi(t_c) = 0.962$			(A-104) & (A-105)
Basic creep coefficient							
1st term	$2[(t - t_o)^{0.3}/\{(t - t_o)^{0.3} + 14\}]$						
2nd term	$[7/t_o]^{0.5}[(t - t_o)/\{(t - t_o) + 7\}]^{0.5}$						
t, days	1st term	2nd term	Basic creep coefficient	t, days	1st term	2nd term	Basic creep coefficient
14	0.000	0.000	0.000	14	0.000	0.000	0.000
28	0.272	0.577	0.850	28	0.272	0.577	0.850
60	0.368	0.659	1.026	60	0.368	0.659	1.026
90	0.415	0.677	1.092	90	0.415	0.677	1.092
180	0.497	0.693	1.190	180	0.497	0.693	1.190
365	0.586	0.700	1.286	365	0.586	0.700	1.286
Drying creep coefficient							
Ambient relative humidity factor	$2.5(1 - 1.086h^2)$						
	1.170						
Time function	$f(t,t_o) = [(t - t_o)/\{(t - t_o) + 0.12(V/S)^2\}]^{0.5}$			Time function	$f(t,t_o) = [(t - t_o)/\{(t - t_o) + 77(V/S)^2\}]^{0.5}$		
t, days	$f(t,t_o)$	Drying creep coefficient 3rd term		t, days	$f(t,t_o)$	Drying creep coefficient 3rd term	
14	0.000	0.000		14	0.000	0.000	
28	0.107	0.126		28	0.106	0.124	
60	0.192	0.225		60	0.190	0.222	
90	0.244	0.285		90	0.241	0.282	
180	0.349	0.408		180	0.345	0.403	
365	0.476	0.556		365	0.471	0.551	
Creep coefficient							
$\phi_{28}(t,t_o) = \Phi(t_c) \times$ [basic + drying creep]							(A-103)
t, days	Basic + drying creep	$\phi_{28}(t,t_o)$		t, days	Basic + drying creep	$\phi_{28}(t,t_o)$	
14	0.000	0.000		14	0.000	0.000	
28	0.975	0.937		28	0.974	0.936	
60	1.251	1.203		60	1.248	1.201	

90	1.377	1.324	90	1.374	1.321
180	1.598	1.536	180	1.593	1.532
365	1.843	1.771	365	1.837	1.767

c) *Compliance* $J(t,t_o) = 1/E_{cmto} + \phi_{28}(t,t_o)/E_{cm28}$

SI units				in.-lb units			
$J(t,t_o) = 1/E_{cmto} + \phi_{28}(t,t_o)/E_{cm28}$ (A-102)							
t, days	$J(t_o,t_o)$, $\times 10^{-6}$	$\phi_{28}(t,t_o)/E_{cm28}$, $\times 10^{-6}$	$J(t,t_o)$ (1/MPa), $\times 10^{-6}$	t, days	$J(t_o,t_o)$, $\times 10^{-6}$	$\phi_{28}(t,t_o)/E_{cm28}$, $\times 10^{-6}$	$J(t,t_o)$ (1/psi), $\times 10^{-6}$
14	37.92	0	37.92	14	0.262	0	0.262
28	37.92	33.46	71.38	28	0.262	0.231	0.492
60	37.92	42.93	80.85	60	0.262	0.296	0.557
90	37.92	47.25	85.17	90	0.262	0.325	0.587
180	37.92	54.82	92.74	180	0.262	0.377	0.639
365	37.92	63.22	101.1	365	0.262	0.435	0.697

C.5—Graphical comparison of model predictions

C.5.1 *Shrinkage strains* $\varepsilon_{sh}(t,t_c)$

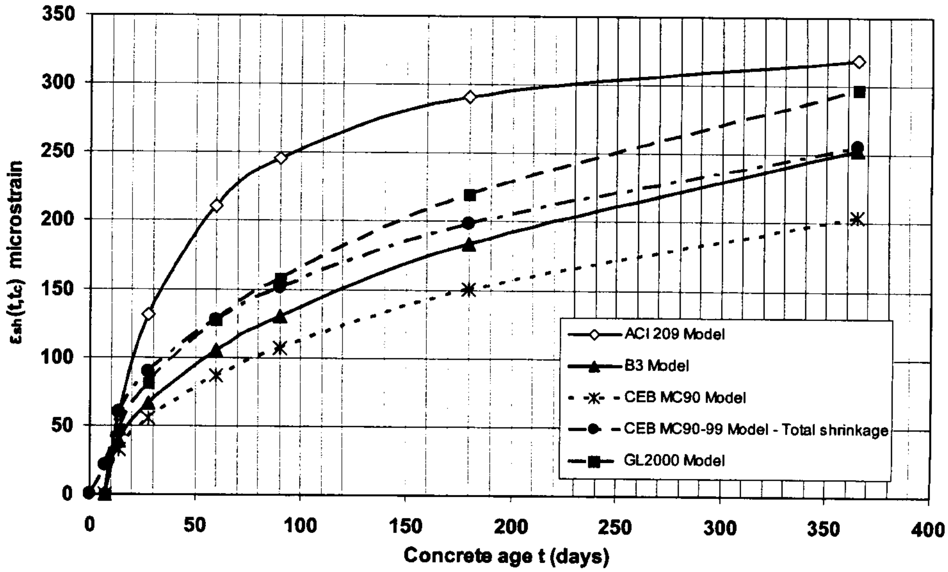

Fig. C.1—Shrinkage strain predictions.

C.5.2 *Compliance* $J(t,t_o)$

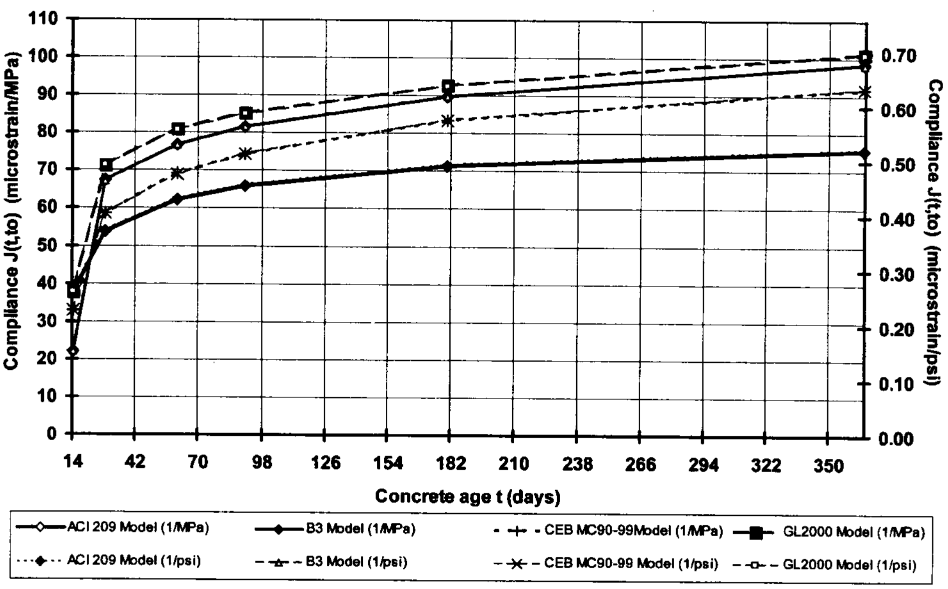

Fig. C.2—Compliance predictions.