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Creep and Shrinkage of Concrete

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97 PRELIMINARY GUIDELINES AND RECOMMENDATION FOR CHARACTERIZING CREEP AND SHRINKAGE IN STRUCTURAL DESIGN CODES

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Abstract
Preliminary Guidelines and Recommendations, which have been formulated by a drafting group in Subcommittee 1 of RILEM Committee TC107, but have not yet been approved by the Committee, are reported. The Guidelines reflect the recent advance in experimental and theoretical research, and are formulated with computer analysis in mind and with a view toward increased long-term safety and durability of concrete structures. The second part of the present report presents one recent creep and shrinkage prediction model, serving as an example of a model that satisfies the Guidelines. This model is intended for a RILEM Recommendation.
Keywords: Concrete creep, shrinkage, prediction, design codes, material properties, constitutive models, testing, structural analysis, statistics.

1 Guidelines for Formulation of Functions Characterizing Creep and Shrinkage

The experimental results accumulated during the last several decades, as well as theoretical and numerical studies of the physical phenomena involved in creep and shrinkage of concrete, such as moisture and temperature diffusion, have led to the following guidelines for creep and shrinkage prediction models to be used in design codes (or design recommendations). These guidelines, which evolved from the RILEM TC67 (1988) conclusions and were produced by RILEM Committee TC107 in collaboration with RILEM Committee TC114 (chaired by I. Carol), include a number of requirements and consistency conditions which should be satisfied by any creep or shrinkage prediction functions in order to avoid conflict with experimental evidence, agree with solidly established theoretical concepts, and achieve mathematical consistency.

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1. The creep of concrete in the service stress range can be characterized in terms of the compliance function. Its use is made possible by the fact that the creep of concrete (in contrast to creep of metals, ice or clay) can be approximately considered as *linear* with regard to stress, following the *principle of superposition*. The principle of superposition agrees with test results very well if there is no drying. At drying, and especially if cracking takes place, there are appreciable departures from the principle of superposition, but they have to be neglected in simple design code formulations because of their complexity.

2. The design codes should specify the *compliance function* $J(t,t')$ rather than the creep coefficient ϕ (ϕ = ratio of creep to the initial 'elastic' strains, t = current age of concrete, t' = age at loading). For structural creep analysis, of course, it is often more convenient to use the creep coefficient, but its value can always be easily calculated from the compliance function specified in the code ($\phi = EJ - 1$, E = conventional 'elastic' modulus, characterizing the truly instantaneous deformation plus short-time creep). One reason for preferring J is that the E-values specified in the codes are not defined on the basis of the initial strains measured in typical creep tests. A more profound reason is that concrete creep in the range of short load durations from 0.1 s to .1 day is already quite significant, which means that $1/E$ is inevitably an arbitrarily chosen point t_0 on the smoothly rising creep curve for unit stress. Depending on t_0, the corresponding E-values vary widely (and different creep data correspond to very different choices of t_0). But what matters for the results of structural creep analysis is the values of J, not ϕ and E. If the creep coefficient ϕ is given to the structural analyst, there is always the danger that he might combine it with some non-corresponding value of E, which then implies an incorrect J. When J is specified, this kind of mistake is prevented.

3. Considering concrete as a homogeneous material, creep and shrinkage should be considered, strictly speaking, as phenomena associated with a point of that continuum. The evolution of such *intrinsic creep* and *intrinsic shrinkage* is affected by factors such as the specific moisture content in the pores or temperature, which can vary from point to point in a cross section of the structure. Therefore, the compliance function and shrinkage in general will be nonuniform throughout the cross section and the intrinsic creep will not be linearly distributed. If, however, the cross section remains plane (which usually is true for long prismatic members such as beams or columns), some internal stresses will be generated to make the strain at each point of the cross section conform to a linear strain distribution.

   For most practical purposes, however, the *mean (or average)* creep and shrinkage can be defined for the cross section as a whole. They have the meaning of the average cross-section compliances or average shrinkage

strains, regardless of the associated internal stress and the inherent cracking at each particular point. Although, strictly speaking, such average creep or shrinkage depend on the type of loading (i.e., the ratio of the bending moment to the normal force), approximate average properties can be established for all types of loading. This is in general the meaning of the formulas characterizing creep and shrinkage in contemporary design codes. This type of formulas are useful, but it must be emphasized that they cannot be considered as material properties unless members in a sealed condition and at constant uniform temperature are considered. Rather, they are the properties of the cross section. Consequently, the formulas depend on the properties of the cross section, such as its size and shape, and are influenced by the nonhomogeneity of creep and shrinkage within the cross section. This inevitably causes that good prediction formulas are much more complicated than the constitutive law for a material point.

4. *Drying* plays a fundamental role in creep and shrinkage. It is the direct cause of shrinkage—if no drying occurs, no shrinkage occurs (except for autogeneous shrinkage, which is of chemical origin and is usually negligible). Drying also affects creep, increasing the creep strain significantly with respect to a similar situation without drying. In a cross section, the drying process is governed by the nonlinear theory of moisture diffusion through the pores in concrete. Its effects increase as the environmental relative humidity decreases. Of course, no drying occurs when the specimen is sealed. In the case of immersion in water (relative humidity 100%) there is swelling or negative shrinkage, which is usually rather small (the inhibition of water also causes some small nonuniformity in pore humidities throughout the cross section, residual stress and possibly microcracking).

5. *Basic creep* is defined as the creep at no drying and at constant temperature. Under such conditions, the behavior at all points in the cross section is the same, and therefore the basic creep can be considered as an intrinsic material property. *Drying creep* is defined as the increase of the creep strain over the basic creep, when drying takes place. This component of creep vanishes for sealed conditions, which is approximately at 100% environmental relative humidity (the small humidity drop called self-desiccation, caused by hydration, is neglected). Therefore, under sealed conditions, the creep for any cross section should be the same and equal to the basic creep regardless of cross-section size and shape.

6. *Diffusion theory*, linear as well as nonlinear, indicates that the drying times $\hat{t}$ required to achieve a similar degree of overall drying in a cross-section (i.e., the same relative water loss) increase with the square of its size. The shrinkage formulas must give shrinkage as a function of the relative time $\theta = \hat{t}/\tau_{sh}$ rather than the actual drying time $\hat{t}$ where τ_{sh} represents a coefficient with the meaning of the time necessary to achieve a certain percentage of the

final shrinkage, e.g., the *shrinkage half-time* or the time necessary to achieve half of the final shrinkage; τ_{sh} should be proportional to the square of the cross-section size. Furthermore, τ_{sh} should depend on the shape of the cross section as indicated by the solution of diffusion theory. The incorporation of τ_{sh} implies that, for a fixed duration of drying, the mean shrinkage in the cross section should decrease with increasing size, with practically no shrinkage at all beyond a certain size.

7. *Diffusion theory* further indicates that, asymptotically for short drying times $\hat{t}$, the shrinkage strain ϵ_{sh} should grow in proportion to $\sqrt{\hat{t}}$. This property agrees well with tests[2] Also, according to the diffusion theory, the final shrinkage value ϵ_{sh_∞} should be asymptotically approached as an exponential, that is, the difference of the shrinkage strain from the final value should asymptotically decrease as e^{-ct} where c = constant and t = time.

8. Similarly, the mean *drying creep* of a cross section, for a fixed load duration, should also decrease with increasing size and almost vanish at a certain size beyond which only basic creep remains. It is reasonable that the decrease of shrinkage and drying creep with the cross section size be governed by the same law, since both are caused by the same physical process, that is, the drying in the cross section. Therefore, like shrinkage, the drying creep term should have the following properties:

   (a) It should be specified as a function of θ rather than $\hat{t}$.

   (b) Asymptotically for short times, it should be proportional to $\sqrt{\hat{t}}$.

   (c) Also, it should asymptotically approach the final value as e^{-kt} where k = constant.

9. Because the *basic creep* and the *drying creep* have different properties, depend on different variables and originate from different physical mechanisms, they should be given by separate terms in the creep model. Therefore, the formula for the creep curves must contain a separate term for the drying creep, which is additive to the basic creep term. Although other more complicated possibilities might be conceivable, a summation of the basic and drying terms in the creep formula appears to be acceptable. Unlike the basic creep term, the drying creep term should approach a final asymptotic value.

[2]This is true, however, only for properly conducted tests, in which the first length reading should be taken right after the stripping of the mold, approximately within 15 minutes. Unfortunately, in many measurements the first reading has been taken too late, even as late as several hours after the stripping. In that case, a significant initial shrinkage deformation (possibly 3% to 10% of the final shrinkage) has been missed. Such distorted data may then give the false impression that the initial shrinkage curve does not follow the $\sqrt{\hat{t}}$-law.

10. The model of *basic creep* should be able to fit the existing experimental data throughout the entire ranges of the ages at loading and the load durations. It should exhibit the following characteristic features:

    (a) According to test results, the creep curves do not possess any final asymptotic value.

    (b) A power function of the load duration, with the exponent around 1/8, fits very well the data available for short durations, while a linear function of the logarithm of the load duration, with a slope independent of the age at loading, fits very well the data available for long durations. The creep model should satisfy these properties asymptotically.

    (c) The transition from the short-time and long-time asymptotic creep curves, which is quite gradual, should be centered at a certain creep value rather than at a certain creep duration, which means that for an older concrete the transition occurs later.

11. The *aging* property of creep, that is the decrease of creep strain at a fixed load duration with the age at loading, is well described by a power function of age with a small negative exponent, approximately -1/3. This function satisfies the experimental observation that a significant aging effect continues through a very broad range of ages at loading, certainly from the moment of set to over 10 years. This property contrasts with the age effect on strength or elastic modulus, which becomes unimportant after several months of age. For this reason, the functions suitable for describing those properties (e.g. $t/(t + \text{const.})$ do not work well for the age effect on creep.

12. Although not strictly necessary on the theoretical basis (i.e., thermodynamic restrictions), it is appropriate and convenient that the creep model exhibit *no divergence* of the creep curves for different ages at loading. There is no clear experimental evidence of such a phenomenon, and models showing such divergence can lead to various unpleasant difficulties when used for complex load histories in combination with the principle of superposition (for instance, giving a reversal of the creep recovery that follows load removal).

13. The creep formulation should also have a form suited for numerical computation. In the interest of efficient numerical solution of large structural systems, it is necessary to expand the creep formula (i.e. the compliance function) into a Dirichlet series, which makes it possible to characterize creep in a *rate-type* form corresponding to a Kelvin or Maxwell chain model. Every compliance function can be expanded into Dirichlet series (or converted to a Maxwell chain model), but for some there are more difficulties than for others because the expansion process:

    - requires a computer solution rather than a simple evaluation from a formula,

- leads to an ill-posed problem with a nonunique solution, and
- yields negative values of the chain moduli within short periods of time.

Creep formulations that avoid these unpleasant characteristics should be preferred for the codes.

The source of the aforementioned difficulties is the age-dependence of creep properties, which in general requires considering the chain moduli to be age-dependent. These difficulties disappear with a creep formulation in which the chain moduli are age-independent, same as in classical viscoelasticity. Such a desirable property of the creep model can be achieved if the age dependence is introduced separately from the chain model, by means of some transformations of time. More than one such time transformation is known to be necessary, and it should preferably be based on some reasonable physical model for the ***solidification process*** that causes the age dependence of creep.

14. An important requirement for any creep and shrinkage model is ***continuity*** in the general sense. Small variations of the dimensions, environmental conditions, loading times, etc., should lead to small variations in the creep and shrinkage predictions, without no finite jumps.

15. Because the available creep data do not, and cannot, cover all possible practical situations, it is important that even a simple prediction model be based to the greatest extent possible on a sound theory. In that case one has the best chance the creep model would perform correctly in such experimentally unexplored situations. This means the model should be based on, and agree with, what is known about the basic ***physical processes*** involved—particularly diffusion phenomena, solidification process, theory of activation energy, etc.

16. Even though the creep and shrinkage model for a design code must be sufficiently simple, it should be compared to ***all the test data*** that are relevant and exist in the literature. The model parameters should be calibrated on the basis of these data by optimum fitting according to the method of least squares. This task is made feasible by the computer and is greatly facilitated by the existence of a comprehensive ***computerized data bank*** for concrete creep and shrinkage (such a bank was compiled in 1978 by Bažant and Panula, was extended by Müller and Panula as part of a collaboration between ACI and CEB creep committees, and is now being further updated, extended and refined by a subcommittee of RILEM Committee TC107 chaired by H. Müller). No model, even the simplest one, should be incorporated in a code without evaluating and calibrating it by means of such a comprehensive data bank.

17. *Selective* use of test data sets from the literature does not prove validity of any model (unless a sufficiently large number of data sets were selected at random, by casting dice). Many examples of the deception by such a practice have been documented. For instance, as shown by Bažant and Panula (1978), by selecting 25 among the existing 36 creep data sets from the literature, the coefficient of variation of errors of the BP creep model dropped from 18.5% to 9.7%, and by selecting only 8 data sets, it further dropped to 4.2%; likewise, by selecting 8 among the existing 12 shrinkage data sets, the coefficient of variation dropped from 31.5% to 7.9% (no reason for suspecting the ignored data sets from being faulty could be seen from the viewpoint of experimental method).

18. Design codes should specify, as a matter of principle, the ***coefficient of variation*** ω of prediction errors of the model compared to all the data that exist in the aforementioned data bank. The values of ω should also be given separately for basic creep, creep at drying, and shrinkage. The value of ω automatically results from the computer comparison of the model with the data bank.

19. Design codes should require that creep-sensitive structures be analyzed and designed for a specified *confidence limit*, such as 95%. This means designing the structure in such a way that the probability of not exceeding a certain specified value of response (for example, maximum 50-year deflection, maximum bending moment or maximum stress) would be 95%. The response values that are not exceeded with a 95% probability can be easily calculated with a computer by the sampling method, in which the statistical information is obtained by repeatedly running a deterministic creep analysis program for a small number of randomly generated sets (samples) of the uncertain parameters in the creep prediction model.

20. The notoriously high uncertainty of long-time creep and shrinkage predictions can be significantly reduced by recalibrating the most important coefficients of the model according to short-time tests of creep and shrinkage of the given concrete, and then extrapolating to long times. This practice should be recommended in the code.

21. The unknown material parameters in the model should be as few as possible and should preferably be involved in such a manner that the identification of material parameters from the given test data (i.e., data fitting) be amenable to *linear* regression. This not only greatly simplifies the identification of material parameters and extrapolation of short-time tests, but it also makes it possible to obtain easily the coefficients of variation characterizing the uncertainty of the material parameters and the predictions. However, this desirable property should not be imposed at the expense of accuracy of the model. It seems that for creep this property can be achieved without compromising accuracy, but for shrinkage this does not seem to be the case.

One important influencing factor, namely temperature, has been ignored in the foregoing guidelines. The same has been true of all the existing design codes or recommendations. Although the effect of constant and uniform temperature is known quite well (it can be described by two activation energies, one controlling the rate of creep and the other the rate of hydration), there is a problem in regard to the statistical variability of environmental temperature, and its daily and seasonal fluctuations. In the future codes and recommendations, however, the effect of temperature ought to be taken into account.

It is highly desirable that the models incorporated in design codes or design recommendations would satisfy the foregoing basic guidelines. Unfortunately, a close examination of these models shows that this is not the case for two of the most widely applied codes—ACI and CEB. The ACI model (ACI Committee 209 recommendation) has the advantage of simplicity and easy application. However, it does not satisfy almost any of the aforementioned conditions. In the case of the CEB model, an important effort has been made in recent years to improve the situation. While the old CEB model in CEB-78 Model Code also did not satisfy many of the requirements listed, the new version CEB-90 does not contradict the main requirements of diffusion theory. However, conflict still remains with a few points, e.g. 10b, and the model does not posses the convenient features described by points 9, 12 and 13..

2 Simple Prediction Model Satisfying the Guidelines[3]

None of the classical models for creep and shrinkage prediction satisfies the foregoing guidelines completely. It is likely that there exist various possible forms of prediction models that satisfy these guidelines while at the same time giving good agreement with the bulk of the existing experimental evidence.

One such model, called the BP-KX model, has recently been developed by Bažant, Panula, Kim and Xi (1991-92) [1]-[5]. This model, however, is relatively complex because it attempts to cover a very broad range of conditions and take into account all the known influencing parameters. A model of that complexity is not suitable for a design code or a standard design recommendation.

The complexity of this model is not due to inherent or artificially contrived theoretical complexity. Rather, it is caused merely by the fact that the range covered by the model is very broad and all the known influencing factors are taken into account, with a broad range of variation. Nevertheless, in most practical situations, these factors have similar values and can be considered to vary only over a rather limited range. For such situations, the prediction model can be considerably simplified. Design codes or recommendations, of course, should be

[3]Prepared by Zdeněk P. Bažant, Walter P. Murphy Professor of Civil Engineering, Northwestern University, Evanston, Illinois 60201, U.S.A.; Yunping Xi, Postdoctoral Research Associate; and Sandeep Baweja, Graduate Research Assistant. This model, proposed to Subcommittee 1 of RILEM Committee TC107, resulted from a simplification of the BP-KX model developed by Z.P. Bažant, J.-K. Kim, L. Panula (deceased 1989) and Y. Xi.

relatively simple, covering only the range of typical situations.

A model that is suitable as a standard recommendation and is sufficiently simple has recently been obtained by means of simplification of the full BP-KX model by Bažant, Panula, Kim and Xi [6]. It will now be presented, as an example of a model satisfying the presently formulated guidelines. It should be kept in mind, however, that the full BP-KX model, which was calibrated by virtually all the existing, sufficiently documented test data from the literature (consisting of 347 data series), will be needed for special situations and for structures of high sensitivity to the effects of creep and shrinkage, such as bridges or roof shells of a record span, buildings of a record height or nuclear power plant structures.

2.1 Formulas for Shrinkage

Mean shrinkage strain in the cross section:

$$\epsilon_{sh}(t,t_0) = \epsilon_{sh\infty} k_h S(\hat{t}/\tau_{sh}) \quad ; \qquad \hat{t} = t - t_0 \tag{1}$$

Time curve:

$$S(\hat{t}/\tau_{sh}) = \tanh\sqrt{\hat{t}/\tau_{sh}} \tag{2}$$

Humidity dependence:

$$k_h = \begin{cases} 1-h^3 & \text{for} \quad h \le 0.98 \\ -0.2 & \text{for} \quad h = 1 \\ \text{linear interpolation} & \text{for} \quad 0.98 \le h \le 1 \end{cases} \tag{3}$$

Size dependence:

$$\tau_{sh} = 0.033\ D^2, \qquad D = \frac{2v}{s} \tag{4}$$

Here t = time, representing the age of concrete, t_0 = age when drying begins, $\hat{t}$ = duration of drying (all the times must be given in days), $\epsilon_{sh\infty}$ = ultimate shrinkage strain, h = relative humidity of the environment ($0 \le h \le 1$), τ_{sh} = shrinkage halftime, D = effective cross-section thickness in millimeters, v/s = volume-to-surface ratio in millimeters.

The final shrinkage strain may be approximately predicted from the mix composition and strength of concrete as follows:

$$\epsilon_{sh\infty} = \alpha_1\alpha_2\left\{1.12(w/c)^{1.5}c^{1.1}f_c'^{-0.2}[1-(a/c)/(\rho_c/c)]+0.16\right\} \qquad (\text{in } 10^{-3}) \tag{5}$$

where

$$\alpha_1 = \begin{cases} 1.0 & \text{for type I cement;} \\ 0.85 & \text{for type II cement;} \\ 1.1 & \text{for type III cement.} \end{cases} \tag{6}$$

and

$$\alpha_2 = \begin{cases} 0.75 & \text{for steam}-\text{cured specimens;} \\ 1.0 & \text{for specimens cured in water or 100\% R.H.;} \\ 1.4 & \text{for specimens sealed during curing.} \end{cases} \tag{7}$$

Here c = cement content in lb/ft^3 (lb/ft^3 = 16.03 kg/m^3), w/c = water/cement ratio by weight, f'_c = 28-day standard cylinder strength in psi, a/c = aggregate-cement ratio by weight, $\rho_c/c = 1 + (w/c) + (a/c)$ relative concrete density, and $a/c = (g/c) + (s/c)$ where g/c is the gravel-to-cement ratio and s/c is the sand-to-cement ratio (all by weight).

2.2 Formulas for Creep

The average compliance function for the cross-section of a long member, representing the strain at age t caused by a unit uniaxial stress applied at age t', is the sum of the instantaneous deformation, the basic creep and the additional creep due to drying, expressed as follows:

$$J(t,t') = q_1 + C_0(t,t') + C_d(t,t',t_0) \tag{8}$$

For function $C_0(t,t')$ there are two alternatives. The first and more rational alternative, which is given later in Section 2.7 (and was presented in Part 6 of [4]), is more accurate but less short (it is also more suitable for computer solutions and for conversion to a rate-type formulation and does not violate any of the preceding Guidelines).

The second alternative, which is given by Eq. (9) which follows, is not as accurate and theoretically justified as the first alternative, and exhibits deviations, although not large ones, from the condition of nondivergence (Guideline 12). But it is slightly shorter, which has been deemed very important for design applications by some members of the ACI and RILEM committees who discussed the present formulation. It is given by the log-double power law:

$$C_0(t,t') = q_0 \ln\left[1 + \psi_1(t'^{-m} + \alpha)(t-t')^n\right] \tag{9}$$

The function characterizing the drying creep term is described by means of function S that has been used for shrinkage, and is defined as:

$$C_d(t,t',t_0) = q_5 k_h \epsilon_{sh\infty}\left[\tanh\left(\frac{t-t_0}{2\tau_{sh}}\right)^{1/2} - \tanh\left(\frac{t'-t_0}{2\tau_{sh}}\right)^{1/2}\right]^{1/2} \tag{10}$$

Eqs. (8)-(10) contain three basic empirical parameters, q_1, q_0 and q_5, of the dimensions 1/psi (1 psi = 6895 Pa). The term containing q_0 characterizes the basic creep compliance, and the term containing q_5 the drying creep compliance, i.e. the apparent additional creep due to drying. Furthermore, q_1 is a function of E_0, E_0 is the asymptotic elastic modulus, which characterizes the strain for extremely short load durations and is obtained by extrapolating the short-time creep measurements to zero time (infinitely fast loading). Function $C_0(t,t')$ is the basic creep compliance and $C_d(t,t',t_0)$ is the additional creep compliance due to drying taking place simultaneously with creep; t = age at loading, t_0 = age at the start

of drying, and t = time = current age of concrete (all in days). S and k_h are functions already defined for shrinkage (see part 6 of [4]), and $\epsilon_{sh\infty}$ = final shrinkage strain for zero humidity and reference conditions.

The basic material parameters q_1, q_0 and q_5 appear in Eqs. (8)-(10) linearly, and so, when test data are available, they can be easily determined by linear regression. Wherever possible, parameters q_1, q_0 and q_5, or at least some of them, should be calibrated by fitting of the available test data for the particular concrete to be used, or at least a similar concrete used in a given geographic zone. Even short-time measurements allow a great improvement of prediction. An example of this will be given in section 2.5.

In the absence of test data for concrete to be used for the planned structure, one may predict the values of q_1, q_0 and q_5 as follows:

$$q_1 = \frac{0.68 \times 10^6}{E_{28}}, \qquad E_{28} = 57000(f'_c)^{1/2} \tag{11}$$

$$q_0 = 0.88 w^{1.58} (\log_{10} f'_c)^{-4.18} \tag{12}$$

$$\psi_1 = 9.32, \quad \alpha = 0.016, \quad m = 0.75, \quad n = 0.32 \tag{13}$$

For drying creep

$$q_5 = 40(f'_c)^{-1/2} \tag{14}$$

Here f'_c is the average standard 28-day cylinder strength in psi (1 psi = 6895 Pa); E_{28} (also in psi) is the conventional elastic modulus at 28 days (which is taken here according to the well-known ACI formula); w=specific water content of concrete mix (in lb. ft.$^{-3}$(1 lb. ft.$^{-3}$ = 16.02 kg m^{-3})). Note that $w = (w/c)c$ where w/c = water-cement ratio by weight, and c = specific cement content.

2.3. Comparison to BP-KX Formulas

Figure 1 (after Part 6 of [4]) shows a comparison of the Simplified BP-KX Model to the full BP-KX Model for typical values of the water-cement ratio and standard cylinder strength, with the following parameters being fixed: $c = 300$ kg/m^3, $s/c = 2$, $g/c = 3$. The parameters in Eqs. (4)-(6) have been obtained so as to give, in these figures from Part 6 of [4], the optimal least-square fits of the BP-KX formulas, while the formula for q_5 is taken from the BP-KX model. Figure 2 gives examples of comparison of the BP-KX to some typical test data.

2.4 Use of the Model in Computer Programs

Although in computer programs one should always prefer the full prediction model, there might be instances where the present short form is preferred. Efficient computer programs for concrete creep are written on the basis of a rate-type form of the creep law associated with either the Kelvin chain model or the Maxwell

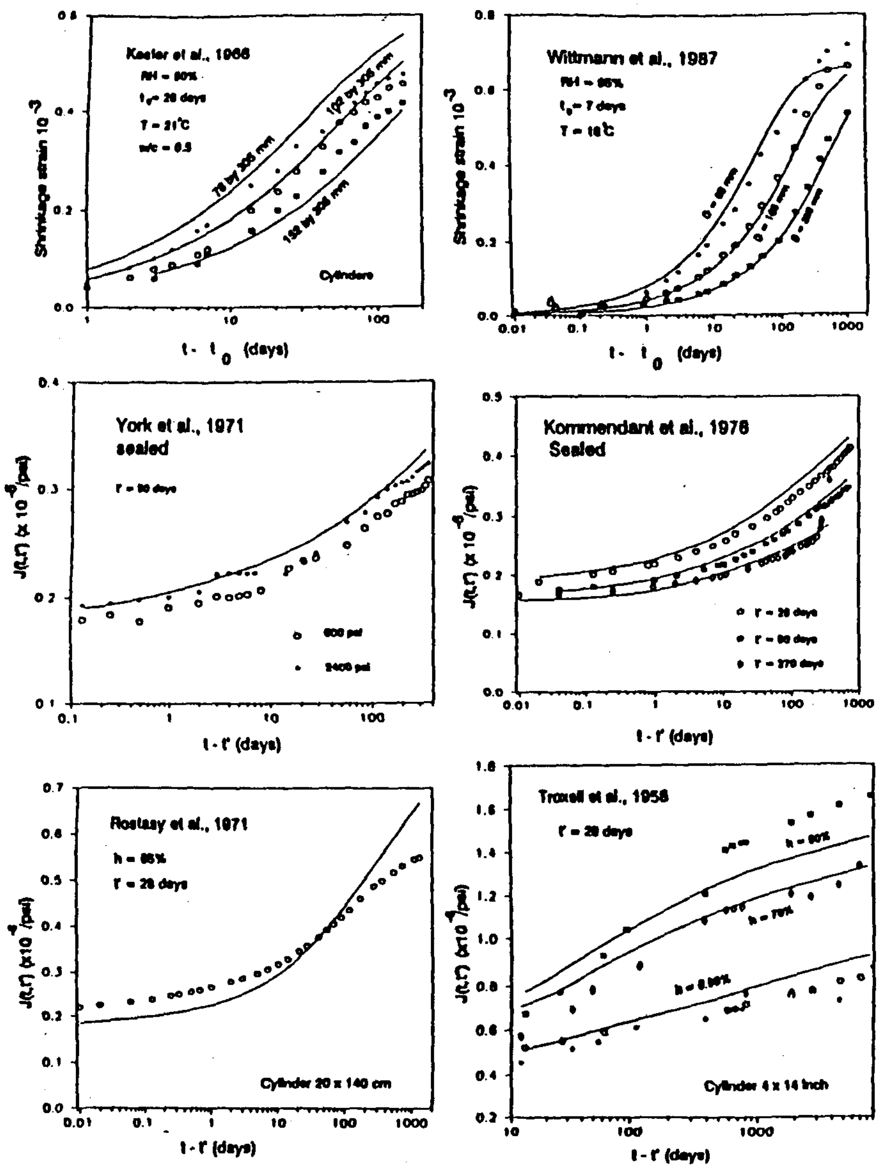

Fig. 1 Examples of Comparison of Simplified BP-KX Model with Test Data by Kesler et al. (top left), Wittmann et al. (top right), York et al. (middle left), Kommnedant et al. (middle right), Rostasy et al. (bottom left) and Troxell et al. (bottom right); after Part 6 of Bažant, Kim and Xi, 1992.

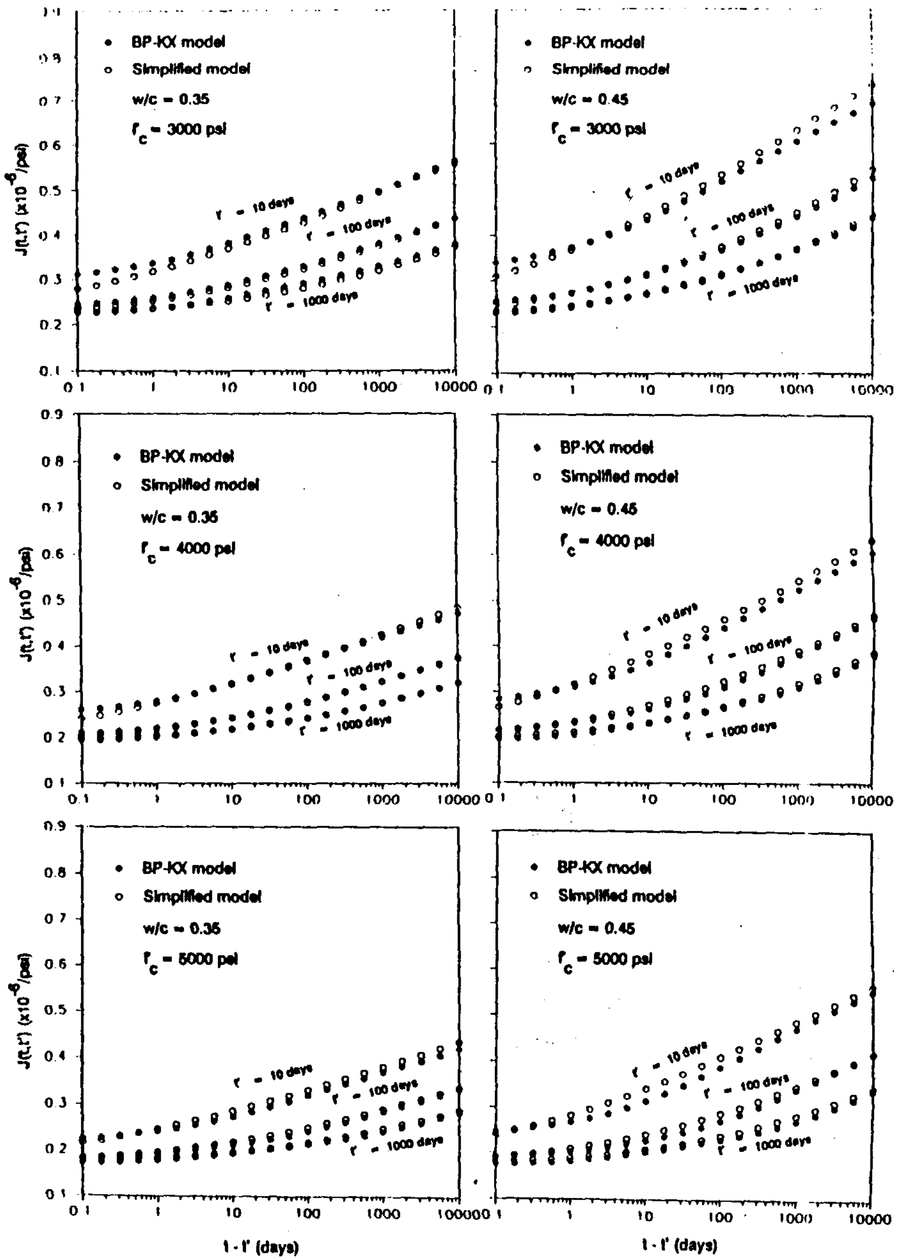

Fig. 2 Comparisons of the Simplified and Full BP-KX Models for $w/c = 0.35$ (left) and $w/c = 0.45$ (right).

chain model [4]. In that case, it is convenient to approximate the log-double power law involved in the present model by the equivalent formulas for the full BP-KX model. This conversion may be approximately carried out by replacing $C_0(t,t')$ given by Eq. (2) with the sum of the terms involving q_2, q_3 and q_4 in BP-KX model (Part 6 of [4]) in which $q_2 = q_0(q_2/q_0)$, $q_3 = q_0(q_3/q_0)$, $q_4 = q_0\ (q_4/q_0)$ where the ratios are taken according to the formulas for q_2, q_3 and q_4 given in the BP-KX model (Part 6). Then the formulas given in [3] may be used to obtain the equivalent rate type model based on Kelvin chain with aging.

2.5 Improvement of Prediction by Linear Regression of Short-Time Data

The greatest source of uncertainty of creep and shrinkage prediction model is the dependence of material parameters on the composition of concrete. This uncertainty can often be greatly reduced by carrying out short-time measurements of the creep and shrinkage of the given concrete and adjusting the values of the parameters q_1 and q_0 in Equations (8)-(9) accordingly, instead of determining them from Equations (4)-(5). Compared to other models, including the original BP Model, the solidification theory which is the basis of the present model has the advantage that the adjusted values of the parameters q_1 and q_0 can be easily obtained by linear regression of the short-time test data. Measurements of short-time data are especially important for special concretes, such as high-strength concrete, because for them the available formulae for taking the composition of concrete into account may involve large errors.

To illustrate the procedure, consider the drying creep data of Rostasy et al. [9], for which the prediction of the present short form is not too good, as is clear from Fig. 3 (for information on these data see [4]). We now pretend we know only the data for the first 2 days of creep duration, which are shown by the solid circles. We want to use these data points to determine parameters q_1 and q_0. For this purpose, we rewrite Equation 1 as

$$J(t,t') = q_1 + q_0\, F(t,t',t_0) \tag{15}$$

where

$$F(t,t',t_0) = \ln[1+\psi_1(t'^{-m}+\alpha)(t-t')^n] + \frac{q_5}{q_0}\, k_h\, \epsilon_{sh\infty}\left[\tanh\left(\frac{t-t_0}{2\tau_{sh}}\right)^{1/2} - \tanh\left(\frac{t'-t_0}{2\tau_{sh}}\right)^{1/2}\right]^{1/2} \tag{16}$$

where the ratio q_5/q_0 is evaluated according to Equations (12) and (14) from the composition of the given concrete. If the data agreed with our prediction model exactly, the plot of $J(t,t')$ versus $F(t,t',t_0)$ would have to be a single straight line for all t, t' and t_0; see Fig. 3. Therefore, the vertical deviations of the data points from this straight line represent errors, which are regarded as random and are to be minimized by regression. So we consider the plot of the known (measured)

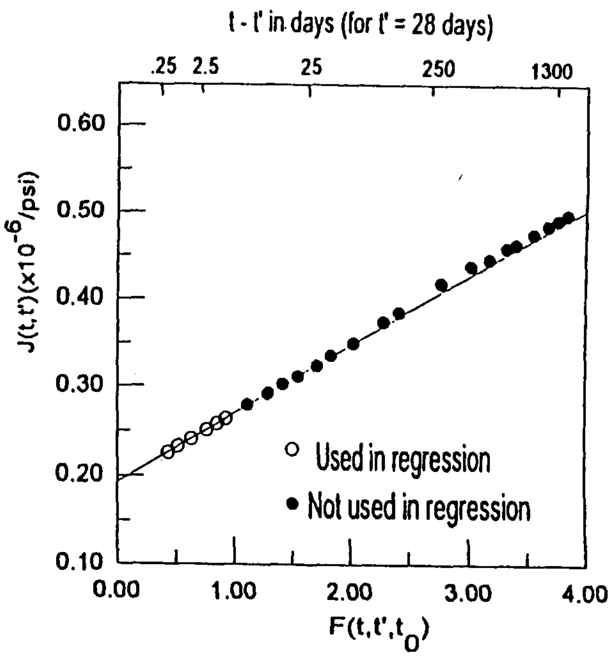

Fig. 3 Example of Updating the Model Parameters by Means of Regression of Short-Time Measurements of Creep.

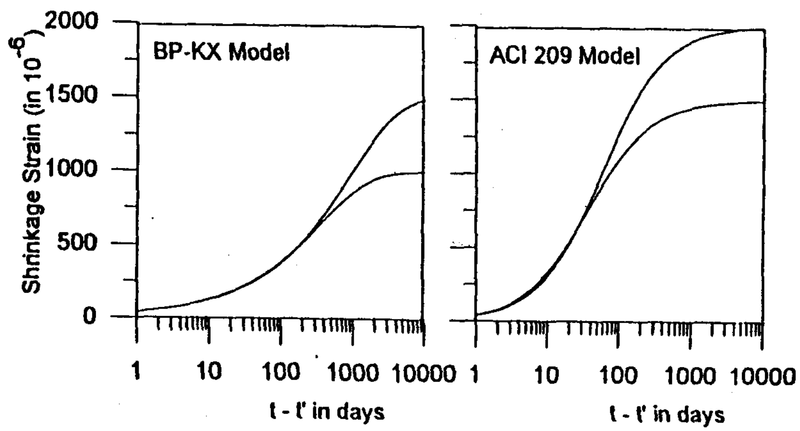

Fig. 4 Example of Shrinkage Curves Giving Nearly the Same Initial Shrinkage but Very Different Final Values (left : $\epsilon = 1500\tanh(\hat{t}/1500)^{1/2}$ and $1000\tanh(\hat{t}/600)^{1/2}$; and right : $\epsilon = 2000\hat{t}/(55+\hat{t})$ and $1500\hat{t}/(35+\hat{t})$).

values $Y = J(t,t')$ (up to $t - t' = 2$ days) versus the corresponding values of $X = F(t,t',t_0)$ calculated from Equation (16), and pass through these points the regression line $Y = AX + B$. Then the slope A and the Y-intercept B of this line give the values of q_1 and q_0 that are optimum in the sense of the least-square method; $A = q_0$ and $B = q_1$.

The linear regression calculations are made according to the well-known normal equations $q_0 = [n\sum(F_iJ_i) - (\sum F_i)(\sum J_i)][n\sum(F_i^2) - (\sum F_i)^2]^{-1}$ and $q_1 = \bar{J} - q_0\bar{F}$ where subscripts $i = 1, 2, \ldots n$ label the known data points, n is their total number, $F = F(t,t',t_0), J = J(t,t'), \bar{J}$ = mean value of all the measured J and $\bar{F}$ = mean value of all the corresponding F. Obviously, the improvement of long-time predictions achieved by short-time measurements is in this example very significant. The well-known formulas of linear regression also yield the coefficient of variation of q_0 and q_1, which in turn provides the coefficient of variations of $J(t,t')$ for any given t and t'.

An experimentalist planning creep measurements should note that the use of short-time data for improving the predictions is more successful if the creep measurements begin at very short times after loading and, as already pointed out, if the shrinkage measurements also begin within a very short time after stripping of the mold (preferably within a minute after loading or stripping). The reason is that the creep curves have been found to be smooth through the entire range from 0.001 s to 30 years, and the initial shrinkage curves have been shown to follow a simple law, being initially proportional to $\sqrt{\hat{t}}$.

If, in our example of a short-time creep test, the first reading were taken only 1 day after loading, as is often done, we would have in Figure 3 only the last three solid points, and obviously from these three points it would be impossible to determine the slope of the regression line with any certainty. To get an equally significant prediction improvement, it would then be necessary to measure creep up to at least 20 days of creep.

As for the shrinkage test, the data are seriously distorted if the first reading is not taken immediately after stripping. Because such initial distortion increases the initial curvature of the shrinkage curve in the log $\hat{t}$ plot, one could get the wrong impression that Ross' hyperbola (ACI 1971 model) might fit the data better than the curve $\sqrt{\hat{t}}$.

In the case that the time range of the measured data is so short that it is impossible to determine both q_0 and q_1, it is proper to take $q_0 = 1$ and determine only one parameter, q_1, by matching the measured data. This can be achieved by choosing q_1 so that the centroid of the measured values would coincide with the centroid of the corresponding predicted values.

In the preceding example, we had only one measured creep curve. But the method can of course be applied even when there are several short-time creep curves for different t', for example $t' = 10$ days and 30 days. Then all the values measured during the initial periods of both tests, for example 30-day periods, are used in the calculations of $F(t,t',t_0)$.

The shrinkage predictions can also be improved on the basis of short-time measurements. This problem, however, is not as easy because the regression is not linear. Nevertheless, the following simple procedure is often possible.

In the short-form model for shrinkage, which is the same as the simplified model from Part 6 of [4], we ignore Eqs. (5)-(7) giving the effect of concrete composition and introduce into Eqs. (4), (1) and (2) parameters q_5 and q_6 as follows:

$$\tau_{sh} = 0.033 q_6 D^2 \qquad \epsilon_{sh}(t,t_0) = q_7\epsilon_c \tag{17}$$

So we have

$$q_7 = \frac{\epsilon_{sh}(t,t_0)}{\epsilon_c} \qquad \epsilon_c = k_h \tanh\sqrt{\frac{t-t_0}{\tau_{sh}}} \tag{18}$$

(note that if our model described the test data exactly, we would have $q_6 = q_7 = 1$). Then we select a sequence of trial values for q_6, preferably a geometric progression such as $q_6 = 0.10, 0.18, 0.32, 0.56, 1.0, 1.8, 3.2, 5.6, 10$ (for which $q_{6,i+1} = 10^{1/4} q_{6,i}$, $i = 1, 2, \ldots$). Since ideally $q_7\epsilon_c - \epsilon_{sh} = 0$, we want to minimize for each q_6 value the sum $S = \sum_i \Delta_i{}^2 = \sum_i (q_7\epsilon_{ci} - \epsilon_{sh_i})^2$ where i = 1, ...N labels the measured points and the corresponding predicted values. From the minimizing condition $\partial S/\partial q_7 = 0$ we get $q_7 = (\sum_i \epsilon_{sh_i}\epsilon_{ci})/(\sum_i \epsilon_{ci}{}^2)$. For this value we then calculate the value of S, and repeat it for all q_6 values. Then we select the case for which S is minimum.

There are, however, two caveats to mention:

- The foregoing procedure can work *only* if the first deformation measurement has been taken right after the stripping the mold (within about one minute). If it has been taken much later, a significant part of the initial shrinkage has been missed. This then causes that the apparent initial shrinkage measurements do not evolve in proportion to $\sqrt{\hat{t}}$, which means that there exists no simple law to extrapolate such distorted data.

- If the time range of the measured short-time shrinkage data is not sufficiently long, the problem of determining parameter q_7 which characterizes the final shrinkage value is ill-posed, that is, very different q_7 can give almost equally good fits of short-time data (Fig. 4). To overcome this problem, it may often be necessary that the data range would reach to drying times for which the slope of the shrinkage curve in the logarithmic time scale begins to decrease. Otherwise, one cannot calculate q_7; rather one must fix $q_7 = 1$ and optimize only the value of q_6.

The second caveat is illustrated in Fig. 4. If the data do not reach significantly beyond the point of crossing of the two curves, and if the scatter is not much smaller than the spread of the two curves before the crossing point, both final values seen in the figure agree with the data equally well. The problem occurs not only for the shrinkage formula of the BP-KX model (Fig. 4, left) but also for other expressions, which do not give a very good shape of shrinkage curves, for example Ross' hyperbola used in ACI 209 model (right). For cylinders of 6 in. (15 cm) diameter, reliable determination of the final value would thus require shrinkage tests of 3 years duration, which is of course unacceptable for a designer. One may however exploit the fact that the shrinkage halftime decreases with the square of specimen diameter and an increase of temperature accelerates shrinkage (but an increase above about 50° C would not be advisable for other reasons). In this manner, using cylinders of 2 in. diameter (if the aggregate size does not exceed about 0.75 in.), one could reduce the aforementioned time to about 50 days.

The problem can be alleviated by Bayesian statistical approach, exploiting prior knowledge of long-time data for similar concretes; but this can help only partly. Using this approach, one can reduce the duration of short-time shrinkage tests to about one-tenth of the aforementioned derivations, but with the penalty or a rather sophisticated probabilistic analysis exploiting other existing data [14].

2.6 Statistics of Errors of Simplified BP-KX Model and of Its Short Form

In [4] (parts 1 to 5), the coefficients of variation ω of the vertical deviations of the full BP-KX prediction model from the available test data have been presented. Their values should be used in design for estimating the errors of prediction. This is important because the design of structures should not be based on the predicted mean values of creep and shrinkage effects in structures, but on their confidence limits with a certain specified probability of not being exceeded, such as 95%.

The simplified BP-KX model from Part 6 of [4] and the present short form of this model have been also been compared to the data used in Parts 1 to 3 of [4], which cover essentially all the available sufficiently documented data that exist in the literature. The comparisons for shrinkage, basic creep and drying creep are given in Tables 1-2 where $\bar{\omega}$ and $\tilde{\omega}$ are the coefficients of variation of the vertical deviations from the various available data sets for the simplified BP-KX model from Part 6 and of the present short-form model, respectively. The last line of each table gives the overall coefficients of variation, defined as $\omega_{all} = [(\sum_k \omega_k^2)/N]^{1/2}$ where N is the number of data sets considered and $\omega_k (k = 1, ...N)$ are the coefficients of variation for the individual data sets listed (the references to these data sets can be found in Parts 1-3 of [4]).

Note that for the simplified BP-KX model the overall coefficients of variation are larger, albeit not much larger, than those listed in Parts 1-3 of [4] for the full BP-KX model, and those for the short form are still larger, albeit again not much larger. Further note that if, among the 20 data sets used for shrinkage, only 12 most favorable data sets were selected, the ω_{all} value would be reduced from 37.2% to 23.3%. Likewise, if among the 15 data sets used for basic creep only 7 most favorable data sets were selected, the $\tilde{\omega}_{all}$ value would be reduced from 20.4% (for the short form) to 13.4%. These observations, which are similar to those discussed in more detail in Part 7 of [4], document the dangerous deception hidden in selective use of test data. Presenting a comparison with 12 or 7 data sets looks like plenty, yet it can be gravely misleading, unless the data to be used

BASIC CREEP

All Data

Test data	$\bar{\omega}$	$\tilde{\omega}$
Keeton	24.0	15.0
Kommendant et al	5.1	7.0
L'Hermite et al.	48.3	52.8
Rostasy et al.	9.1	11.0
Troxell et al.	9.3	16.0
York et al.	11.7	12.2
McDonald	23.0	24.1
Maity and Meyers	16.5	25.7
Mossiossian and Gamble	18.0	17.0
Hansen and Harboe et al. (Ross Dam)	17.5	34.6
Browne et al. (Wylfa Vessel)	32.0	31.2
Hansen and Harboe et al. (Shasta Dam)	27.9	18.4
Brooks and Wainwright	8.7	20.0
Pirtz (Dworshak Dam)	26.7	37.1
Hansen and Harboe et al. (Canyon Ferry Dam)	31.6	24.1
Russel and Burg (Water Tower Place)	17.4	15.5
$\bar{\omega}_{all}$ or $\tilde{\omega}_{all}$	20.4	22.6

Restricted Parameter Range

Test data	$\bar{\omega}$	$\tilde{\omega}$
Keeton	24.0	15.0
Kommendant et al	5.1	7.0
L'Hermite et al.	35.2	39.9
Rostasy et al.	9.1	11.0
Troxell et al.	9.3	16.0
York et al.	11.3	12.2
McDonald	23.0	24.1
Hansen and Harboe et al. (Ross Dam)	16.5	32.6
Browne et al. (Wylfa Vessel)	29.1	36.1
Pirtz (Dworshak Dam)	26.7	24.1
Russel and Burg (Water Tower Place)	17.4	15.5
$\bar{\omega}_{all}$ or $\tilde{\omega}_{all}$	18.8	21.2

Table 1 Coefficients of Variation of Deviations of the Model from Various Basic Creep Data Test Data from the Literature (quoted in [4]); $\bar{\omega}$—for the Simplified BP-KX model from Part 6 of [4], $\tilde{\omega}$—for the Short Form Model from Part 7 of [4].

SHRINKAGE

Test data	ϖ
Hummel et al.	30.1
Rüsch et al.(1)	31.6
Wesche et al.	33.4
Rüsch et al.(2)	39.9
Wischers and Dahms	28.6
Hansen and Mattock	25.5
Keeton	56.5
Troxell et al.	73.5
Aschl and Stökl	53.9
Stökl	35.1
L'Hermite et al.	88.6
York et al.	33.0
Hilsdorf	27.7
L'Hermite and Mamillan	58.4
Wallo et al.	19.7
Lambotte and Mommens	37.3
Weigler and Karl	34.1
Wittman et al.	17.5
Ngab et al.	12.1
McDonald	14.3
Russel and Burg(Water Tower Place)	30.9
ϖ_{all}	37.2

DRYING CREEP

All Data

Test data	ϖ	$\bar{\omega}$
Hansen and Mattock	65.6	99.8
Keeton	18.4	10.3
Troxell et al.	8.4	10.9
L'Hermite et al	17.9	11.3
Rostasy et al.	16.0	21.1
York et al.	31.0	48.9
McDonald	37.0	50.5
Hummel	25.1	30.5
L'Hermite and Mamillan	37.7	27.3
Mossiossian and Gamble	8.0	9.1
Maity and Meyers	8.5	64.5
Russel and Burg (Water Tower Place)	18.7	16.2
ϖ_{all} or $\bar{\omega}_{all}$	24.4	33.4

Restricted Parameter Range

Test data	ϖ	$\bar{\omega}$
Keeton	18.4	10.3
Troxell et al.	8.4	10.9
L'Hermite et al.	17.9	11.0
Rostasy et al.	16.0	21.1
York et al.	31.0	48.9
McDonald	37.0	50.5
Hummel	28.9	30.5
L'Hermite and Mamillan	37.7	27.3
Russel and Burg (Water Tower Place)	14.9	15.1
ϖ_{all} or $\bar{\omega}_{all}$	22.8	25.0

Table 2 Same as Table 1 but for Drying Creep and Shrinkage.

were chosen by casting dice.

In the foregoing statistics (Tables 1-2), all the data points from the literature cited in [4] (Parts 1-3) have been considered. Naturally, it may be expected that a reduction in the coefficient of variation of errors could be achieved by limiting the ranges of parameters to the typical values encountered in practice. The following range limitation, which has been stated in Part 6 of [4] and corrected by the Errata at the end of this paper, has been considered:

For shrinkage:

$$3 \le t_0 \le 40 \text{ days} \tag{19}$$

For creep (basic and drying):

$$3 \le t_0 \le 40 \text{ days}, \quad 3 \le t' \le 365 \text{ days}, \quad 1 \le g/s \le 3.5 \tag{20,21,22}$$

and $t' \ge t_0$, where g/s is coarse aggregate to sand ratio.

The values of ϖ for simplified BP-KX Model from Part 6 of [4] and its short from in Part 7 of [4] have been recalculated for all the data sets excluding all the points outside the aforementioned range. The resulting overall coefficients of variation ω_{all} for such a limited range are listed in Tables 1-2. As can be seen, they are smaller than those for the full range of data, but not much smaller. This has been so for various range limitations studied. Apparently, restricting the range of parameters cannot reduce the errors of the prediction model significantly.

It is proper to describe the manner in which the ω values in Tables 1-2 have been calculated. In Part 6 of [1] presenting the original (1978-79) BP model, the sampling method was discussed in detail, and the same method has been used here.

Optimally, measurements should be taken at time intervals that are constant in the logarithmic scale of the creep or shrinkage duration [1,2,4], i.e. in $\log(t-t')$ or $\log(t-t_0)$ scales. The reason is that what matters is the percentage change of the duration rather than the actual change. For example, the differences between the creep values at 10 and 20 days, or those at 1000 and 2000 days, have about the same significance because their ratios are the same, namely 2. But the difference between the creep readings taken at 1000 and 1010 days is insignificant, most of it being due to random errors of the measuring device and test control. At the same time, the creep values at these two times are much more strongly correlated than those at 10 and 20 days, or at 1000 and 2000 days. Consequently, by taking readings at both 1000 and 1010 days one in effect doubles the weight of the reading, compared to those taken at 10 days and 20 days.

To avoid the subjective bias caused by such improper spacing of the data points in time, each measured time curve used in the 1978-79 BP model [2] had first been smoothed by hand and the data points to be used for statistics were then placed on the curves at equal intervals in log-time (this also approximately eliminated the random scatter of strain measurements, which should be excluded from structural analysis because the response of structures does not depend on the error of the measurements in the lab) (Part 6 of [2]).

In the present statistical calculations, a somewhat different but approximately equivalent approach has been adopted. The *log-time* scale has been divided into decades and the data points within each decade have been considered as one group. Then the data points in each group have been assigned a weight inversely proportional to the number of points in this group. By virtue of this approach, the statistics in Tables 1-2 are virtually free of subjective bias due to the experimentalist's choice of the reading times. They have thus been compensated for the subjective bias caused by crowding the readings in one part and taking sparse readings in another part of the scale.

2.7 Alternative More Accurate Expression for Compliance Function and Step-by-Step Numerical Algorithm

The aforementioned more accurate and more logical alternative to Eq. (9) for the basic creep term becomes much simpler when we specify the compliance function derivative $\dot{J}(t,t') = \partial J(t,t')/\partial t$ instead of the compliance function itself. For computer solutions this is actually more convenient because the step-by-step algorithm for creep structural analysis can be based directly on the expression for $\dot{J}(t,t')$ and the expansion into Dirichlet series and conversion of the stress-strain relation to a rate-type form can be best done directly on the basis of $\dot{J}(t,t')$. It is the expression for $\dot{J}(t,t')$ rather than $J(t,t')$ which results directly from the solidification theory. It has the following simple form

$$\dot{J}(t,t') = \frac{n(q_2 t^{-m} + q_3)}{(t-t') + (t-t')^{1-n}} + \frac{q_4}{t}, \qquad m = 0.5,\ n = 0.1 \tag{23}$$

in which t and t' must be given in days; q_2, q_3 and q_4 are three empirical parameters (constants) of the dimensions 1/psi (1 psi=6896 Pa). The terms containing q_2, q_3, and q_4 represent the rates of the aging viscoelastic compliance, the non-aging viscoelastic compliance and the flow compliance, respectively.

The integration of Eq.(23), which yields the compliance function $J(t,t')$, unfortunately cannot be carried out in a closed form because the function multiplying q_2 yields the binomial integral. Nevertheless, closed-form expressions which approximate the integral very accurately have been devised. With these approximate expressions, the basic creep compliance is given by

$$C_0(t,t') = q_2 Q(t,t') + q_3 \ln[1 + (t-t')^s] + q_4 \ln\left(\frac{t}{t'}\right) \tag{24}$$

in which

$$Q(t,t') = Q_f(t')\left[1 + \left(\frac{Q_f(t')}{Z(t,t')}\right)^r\right]^{-\frac{1}{r}}, \quad Z(t,t') = \frac{\ln[1+(t-t')^n]}{t'^m} \tag{25, 26}$$

$$Q_f(t') = [0.086(t')^{2/9} + 1.21(t')^{4/9}]^{-1} \qquad r = 1.7(t')^{0.12} + 8 \tag{27}$$

with t and t' given in days. In [1], the exponent $s = 1$ was used, although, according to the solidification theory [1] and Eq. (23), the correct value is $s = n = 0.1$. (The difference in optimum fits of data is not significant, but recalibration of the theory for $s = n$ will be presented in a forthcoming paper by Bažant and co-workers, along with simplifications of Eqs. 28–30 given below and some further improvements.).

The expression for $\dot{J}(t,t')$ in Eq. (23) satisfies the requirement for non-divergence of creep curves (Guideline 12) exactly. The approximate expressions (25)-(28) satisfy this requirement only approximately, but the violations of non-divergence are very small. For the log-double power law in (9) the violations are not so small, but still acceptable for practical purposes. For other models (ACI, CEB), they are significant.

The material parameters $q_1, q_2, ...q_5$ all appear in Eqs. (8) and (23) or (8) and (24) linearly, and so they can be easily determined from test data by linear regression. Wherever possible, parameters $q_1, ..., q_5$, or at least some of them, should be calibrated by fitting the test data available for the particular concrete to be used, or at least a similar concrete used in a given geographic zone. Even the use of short-time data, with calibration of only one or two parameters among $q_1, ...q_5$ is preferable to using no data at all. In the absence of test data, one may predict the values of $q_1, ...q_5$ from the concrete strength and composition using the following formulae:

$$q_2 = 0.011(w/c)^{0.8} c^{1.5}[1 - (a/c)(\rho_c/c)]^{-0.9}(0.001 f'_c)^{-0.5} - 0.39 \quad (\text{in } 10^{-6}) \tag{28}$$

$$q_3 = 0.025 q_2 \quad (\text{in } 10^{-6}) \tag{29}$$

$$q_4 = 0.072(w/c)^{2/3} c^{0.2}[1 - (a/c)/(\rho_c/c)]^{-0.39}(0.001 f'_c)^{0.46} \quad (\text{in } 10^{-6}) \tag{30}$$

For drying creep, $q_5 = 40/\sqrt{f'_c}$, and for the instantaneous (asymptotic) strain, $q_1 = 1/E_0 = 10^6/1.5E_{28}$ with $E_{28} = 57{,}000\ \sqrt{f'_c}$, which is the same as before; f'_c is the 28-day standard cylinder strength in psi (1 psi= 6895 Pa), and E_{28} (also in psi) is the conventional elastic modulus at 28 days (which is taken here according to the well-known ACI formula).

Figures 1-2 demonstrate some predictions with the present formulae and compare them with typical test data. These figures, however, do not represent a verification of the model. As stated in Guideline 16, the verification of the present model has been provided by the systematic analysis of all the existing sufficiently documented test data as presented in Parts 1 to 5 of [4].

The second-order accurate step-by-step computational algorithm for creep analysis of structures, *presented* in 1971 by Bažant (see Eqs. 2.29–2.33 on pages 116–117 of [8]), was expressed in terms of $J(t,t')$. However, only small increments of $J(t,t')$ over the time steps are used in this algorithm. Thus the algorithm can be easily expressed in terms of $\dot{J}(t,t')$ according to the following approximation, also of second-order accuracy:

$$J(t_r, t_{s-\frac{1}{2}}) - J(t_{r-1}, t_{s-\frac{1}{2}}) = \Delta t_r\, \dot{J}(t_{r-\frac{1}{2}}, t_{s-\frac{1}{2}}) \tag{31}$$

in which $\Delta t_r = t_r - t_{r-1}$; $t_r (r = 1, 2, ...)$ represent the discrete times, and $t_{r-\frac{1}{2}}$ is the midpoint of the interval (t_{r-1}, t_r) in the logarithmic time scale. This makes it possible to use the simple expression (Eq. 23) for $\dot{J}(t, t')$ and avoid the more complicated approximate expressions for $J(t, t')$.

2.8 Principal Advantages

1. The shrinkage and drying creep formulas agree with all the asymptotic requirements of nonlinear *diffusion* theory for the movement of moisture through concrete.
2. For creep, there are only five free material parameters, and they can all be identified from given test data by *linear* regression (i.e., no nonlinear optimization is needed).
3. The creep formula can be easily converted, by explicit formulae, to a rate-type (Kelvin chain) model with *nonaging* properties, the effect of aging being totally ascribed to transformations of time (this feature is very useful for finite element analysis).
4. The condition of *non-divergence* of the creep curves for different t_0 is automatically satisfied.
5. The additional creep due to drying is based on the shrinkage function, which is both the simplest and theoretically best justified (by the theory of stress-induced shrinkage).

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ERRATA FOR PARTS 2, 3 AND 6 [4]

1) In part 2 [4], p. 411, Eq. 12, replace exponent -0.46 by 0.46.
2) In part 2, p. 410, Eq. 9, $q_1 = 10^6/E_0 = 10^6/(1.5E_{28})$ in 10^{-6} psi.
3) In part 2, line 5 and line 8 below Eq. 1, replace t by t'.
4) In part 3, p. 219, the left-hand side of Eq. 5 should be $\epsilon_{sh\infty}$ rather than ϵ_{sh}.
5) In part 6, the parameter ranges in Eqs. 20 and 21 should be the same as in the present Eqs. 19–22.
6) In part 6, the right-hand side of Eq. 10 should read the same as in the present Eq. 10.
7) Finally, see the correction below Eq. 27, replacing $n = 1$ by $s = n = 0.1$.