

CONST-VIBRATIONS Listserv Newsletter #13

Use of Response Spectra and Principles of Structural Dynamics to Verify the Shape of the Z Curve and to Supply Guidance for Future Research

This newsletter provides a detailed explanation of why the Z curve in Figure 1 should have an ascending bound for excitation frequencies below 3 to 4 Hz as shown by the gold line. The error of the descending bound for excitation frequencies below 3 Hz is evident in the graph itself as all the observations of threshold (cosmetic) cracking (black dots) occur at peak particle velocities (PPV) above 12.7 mm/s (0.5 ips) at frequencies below 5 Hz.

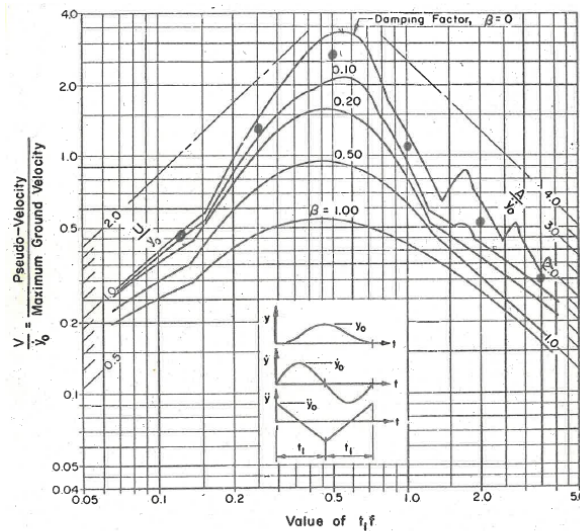
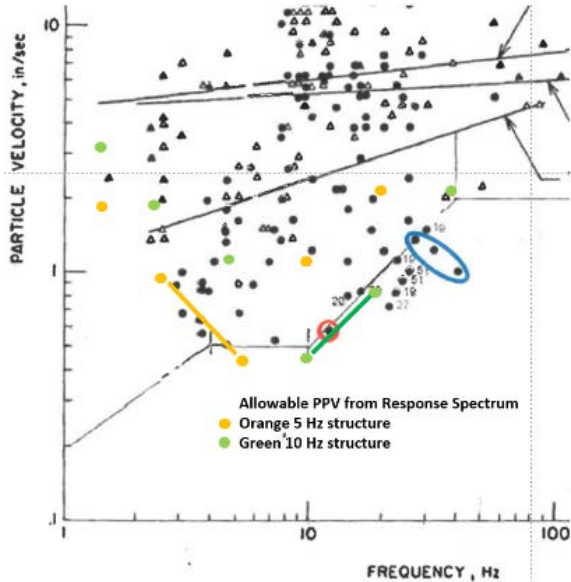


FIG. 2.44b DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A FULL-CYCLE PARABOLIC VELOCITY PULSE

**Figure 1 (left) Figure 54 from RI 8507 from Newsletter #3 annotated with Z curve (thin black line), USBM observations of cracking (structure numbers), basket shaking cases (blue circle), example of Dvorak data (red circle). Allowable PPV bounds calculated from the spectrum of responses of structures to a full cycle (reversing) pulse from Figure 2 (right) appear in gold and green for 5 and 10 Hz structures respectively**

According to the principles of structural dynamics the allowable PPV to prevent cosmetic cracking for 5 to 10 Hz structures should rise with declining excitation frequencies below 5 Hz (gold) just as it does for increasing frequencies above 10 Hz (green). The principle is demonstrated by employing the normalized response spectrum for a single reversing pulse shown in Figure 2 (to right of Fig. 1 above) to directly reproduce the shape of the Z curve (Veltso and Newmark, 1964). The following exercise is undertaken to demonstrate the reasonableness of the flat bottomed “V” shape of the bounds, not the exact numbers. However, it is remarkable how closely the values resembled those of the Z curve itself.

By setting an allowable strain, Figure 2 (right of Z curve above) can be employed to determine allowable PPV for a range of excitation frequencies above and below the natural frequency of the structure(s), which are one and two story in this case. Since shear strain is relative displacement divided by height ( $= U/H$ ), allowable  $U$  for a two story, 5 Hz structure is twice that for a single story, 10 Hz structure because it is twice as high. Upon setting an allowable  $U$  of 0.016 and 0.008 in. for 5 and 10 Hz structures, displacement,  $y_0$ , can be found as  $0.016/(U/y_0 \text{ graph})$  &  $0.008/(U/y_0 \text{ graph})$  respectively for 5

and 10 Hz structures); allowable PPVs can be calculated as  $= 2\pi f_g y_0$ . The allowable U of 0.008 ips for a 10 Hz structure from RI 8507 was discussed in Newsletter #3.

The process of conversion of the black dots in Figure 1-right to the gold and green dots in Figure 1-left is shown in the table below for 5% damped 5 and 10 Hz structures ( $f_s$ ) responding to the full cycle velocity pulse shown in the inset in Figure 2. Dominant excitation frequencies ( $f_g$ ) are above and below the 5 and 10 Hz natural frequencies. For instance for a 5 Hz structure and  $f_g = 21$  Hz,  $U/y_0$  from Figure 1-right is 0.09, and thus allowable PPV  $= 2\pi f_g y_0 = 2\pi * 21 * [0.016/0.09] = 2.3$  ips (58 mm/s)

$t_1 f_s' s (=f_s/2f_g)$	0.125	0.25	0.5	1	2	3.5
$U/y_0$ from Figure 2	0.09	1.2	1.2	0.27	0.07	0.023
$f_g$ for 5 Hz Structure	21	10	5	2.5	1.3	0.7
Allowable PPV $= 2\pi f_g [0.016/(U/y_0)]$	2.3	0.84	0.42	0.93	1.80	3.12
$f_g$ for 10 Hz Structure	42	20	10	5	2.6	1.4
Allowable PPV $= 2\pi f_g [0.008/(U/y_0)]$	2.3	0.84	0.42	0.93	1.80	3.12

Lower bounds of allowable PPV are determined by responses of differing structures depending whether the excitation frequencies are above or below the 5 and 10 Hz range of structural natural frequency. When excitation frequencies are above 10 Hz, say  $\sim 20$  Hz, the green dots in Figure 1, allowable PPV's for the 10 Hz structure, are lower than the gold dots for the 5 Hz structure. When excitation frequencies are below 5 Hz, say  $\sim 2.5$  Hz, the gold dots for the 5 Hz structure are below the green dots for the 10 Hz structure. The green and gold solid lines in Figure 1 connect the dots to make these lower bounds more evident.

The green line in Figure 1 lies along a line of constant ground displacement, which is consistent with the lower bound of response spectra of ground motions of cases of cosmetic cracking reported in RI 8507 and discussed in Newsletter #3. The gold line lies along a line of constant acceleration, which, if it were 0.03 g would encapsulate all of the low frequency cases from RI 8507 as introduced in Newsletter #4.

The upward sloping line on the Z curve ( $f_g > 10$  Hz) in Figure 1 provides the relationship between U and  $y_0$  necessary to calculate the allowable relative displacement, U, from the normalized response in Figure 2. As shown in Figure 2 any structure with  $f_s \ll f_g$  (natural frequency,  $f_s$ , less than excitation frequency,  $f_g$ , and where  $t_1 f$  is less than 0.15) U is equal to  $y_0$ . From the field observations made in RI 8507, U critical is 0.008 to 0.010 in. This relationship then provides the means to calculate the green and gold PPVs in Figure 1 as  $PPV = 2\pi f_g y_0$ , where  $y_0 = 0.008/(U/y_0 \text{ graph})$  for a 1 story structure.

## REFERENCES

Veltos, A.S. and Newmark, N.M. (1964) Design Procedures for Shock Isolation Systems of Underground Protective Structures: Vol III Response Spectra of Single-Degree-of-Freedom Elastic and Inelastic Systems, Technical Documentary Report No. RTD TDR 63-3096, Vol III, AD 444 989. This document can be found on line at <https://apps.dtic.mil/docs/citations/AD0444989> as of the date of this newsletter.