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DIFFUSIVE INSTABILITIES IN DILATING AND COMPACTING GEOMATERIALS

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Abstract. This chapter reviews and extends analyses of diffusive instabilities in inelastically deforming geomaterials. The onset of these instabilities is connected with the conditions for shear localization in the limiting cases of drained (constant pore pressure) and undrained (constant fluid mass) deformation and depends on whether inelastic volume change is dilation or compaction. Rice [1975] showed that homogeneous shear deformation of a layer was stiffer for undrained than for drained conditions but was unstable in the sense that the magnitude of infinitesimal spatial nonuniformities begins to grow exponentially in time when the condition for localization is met in terms of the underlying drained response. As the condition for localization in terms of the undrained response is passed, infinitesimal spatial perturbations experience infinitely rapid decay and then infinitely rapidly growth. For materials that dilate with inelastic shearing the condition for localization is met for the drained response before it is met for the undrained response. For materials that compact and for which the shear yield stress increases with normal stress, the undrained response is softer than the drained and conditions for localization are met for undrained response before drained. If the shear yield stress decreases with normal stress, as for materials modeled by a "cap" on the yield surface, results for compacting materials are identical to those for the dilating materials. Generalization of the layer results to arbitrary deformation states reveals the same relation for the onset of diffusive instability: spatial nonuniformities begin to grow exponentially when the condition for localization is met in terms of the underlying drained response. In contrast to the result for the layer, the growth rate of perturbations does not necessarily become unbounded when the condition for localization is met in terms of the undrained response. The difference is due to a lack of symmetry in the constitutive tensors that is typical of geomaterials. Explicit expressions are given for the undrained response in terms of the drained for an elastic-plastic relation with yield stress and flow potential depending on

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first and second stress invariants. For this relation and the limit of incompressible solid constituents, the lack of symmetry just-mentioned disappears. If the fluid constituent is also incompressible, the analysis confirms a result of Runesson et al. [1996] that the undrained response is independent of mean stress and the predicted direction of shear bands is 45° to the principal axes of stress.

1 Introduction

In contrast to metals, inelastic deformation of geomaterials typically involves volume change. In low porosity rocks, dilatancy (volume increase) can occur during inelastic shearing under compressive mean stress because of local tensile microcracking at the tips of sliding fissures or at local property mismatches, and from uplift in sliding over asperity contacts on fissure surfaces. In soils, dilatancy results from rearrangement of close-packed particles due to shearing. Compaction in high porosity rocks can result from the collapse of pore structures due to shearing or high mean stresses and, at very high mean stresses, from grain crushing. Compaction of low density soils occurs when shearing causes a closer packed arrangement of particles.

When the geomaterial is fluid-saturated, inelastic volume changes tend to cause a change in pore fluid pressure. If the deformation is slow enough and drainage from the boundaries is possible, alterations in pore pressure will be equilibrated by fluid mass flow. In this drained limit, the pore pressure is constant. For volume changes that occur without allowing drainage from material elements, the pore pressure changes. This undrained limit can occur if volume changes occur too rapidly (though still slow enough so that inertia is not significant) to allow time for fluid mass flow or during homogeneous deformation if fluid flow from the boundaries of the body is prevented.

Because the inelastic response of geomaterials is affected by the mean effective stress, that is, the total compressive mean stress minus the pore pressure, alterations in pore pressure will either inhibit or promote further inelastic straining. Consequently, the inelastic response differs in the drained and undrained limits and, for intermediate cases, is coupled to the diffusion of pore fluid. Conditions for failure and, in particular, for localization of deformation depend on this coupling.

In a seminal analysis, Rice [1975] examined the coupling of pore fluid diffusion and inelastic response for combined shear and compression of a layer that exhibited dilatant volume changes. He showed that dilatancy during homogeneous shearing of the layer without allowing fluid drainage from the boundaries prevented caused a reduction of pore fluid pressure and, thus, an increase in effective compressive stress. This increase inhibited further inelastic deformation. But Rice [1975] proceeded to show that homogeneous dilatantly strengthened response becomes unstable when the condition for localization of deformation [Rudnicki and Rice, 1975] is met in terms of the underlying drained (constant pore pressure) response. For the dilatant behavior and layer model considered in Rice [1975], this condition occurs when the drained shear stress versus shear strain curve reaches a peak even though the undrained response curve is still rising. When this condition is met, spatial non-uniformities grow exponentially in time with the smallest wavelengths growing the fastest. Because spatially nonuniform deformation causes fluid flow in response to pore pressure gradients, homogeneous undrained response cannot be realized beyond this point.

Dilatant strengthening has been widely observed in granular materials dating back to the experiments of Reynolds [1885]. More recently, it has been observed in laboratory tests on both rocks [Brace and Martin, 1968; Martin, 1980] and soils[Mokni and Desrues, 1999]. Vardoulakis [1985, 1986, 1996a, b] has adapted Rice's analysis for the biaxial deformation of both dilating and compacting water saturated sand and used it to interpret laboratory observations on the development of localization of deformation. Recent theoretical analyses [Runesson et al., 1996; 1998] have examined conditions for localization in the limit of undrained deformation.

In this article, I review Rice's [1975] analysis and discuss more generally its implications, in particular, for compacting materials. Compaction can result not only from inelastic shearing but also from purely hydrostatic stress. The inelastic response of materials that compact under hydrostatic stress is often modeled by a "cap" on the yield surface enclosing the hydrostatic axis [Dimaggio and Sandler, 1971; Wong et al., 1997]. The presence of this cap implies that inelastic shearing is enhanced, rather than inhibited, by an increase in mean compressive stress. Recently, Issen and Rudnicki [2000] have shown that for compacting materials the conditions for localization admit solutions not only for shear bands, but also for compaction bands. Compaction bands, narrow planar zones of compacted material that form perpendicular to the largest compressive principal stress, have been observed both in the field [Mollema and Antonellini, 1996] and in laboratory experiments [Olsson, 1999].

For the deformation state considered in Rice [1975] the conditions for localization are met at the peak of the shear stress versus shear strain curve. For more general deformation states, Rudnicki and Rice [1975] have shown that the condition for localization may be met before or after peak stress. Here, I show the conclusions of Rice [1975] concerning the stability of undrained deformation can be generalized to arbitrary deformation states. A previous analysis of this type has been outlined in Rudnicki [1983] but simplifies the description of fluid flow. Rudnicki [1983] assumes the pore pressure is uniform in a planar band and in the surrounding material, and, in addition, that fluid mass exchange between the layer and the surrounding material is proportional to the difference in pore fluid pressures. These simplifications are avoided here by resolving the perturbations from the uniform fields in terms of Fourier components relative to the putative band. The implications of the results are examined for a general elastic plastic model and used to illuminate the role of pore fluid compressibility on conditions for localization in undrained deformation [Runesson et al., 1996].

The next section briefly summarizes Rice's [1975] analysis. Succeeding sections develop the extensions to compacting materials and arbitrary deformation states.

2 Rice's Analysis

2.1 Formulation

The geometry of the problem considered by Rice [1975] is shown in Figure 1: plane strain deformation of a layer extending indefinitely in the x-direction. Displacements in the x and y directions are $u(y,t)$ and $v(y,t)$, respectively, where t is time. Only the normal strain $\epsilon(y,t) = \partial v/\partial y$ (positive in extension) and the shear strain $\gamma(y,t) = \partial u/\partial y$ are nonzero. Stresses work-conjugate to ϵ and γ are normal stress σ (positive in compression) and shear stress τ. Equilibrium (in the absence of body forces) requires that σ and τ be uniform. Hence, they are functions only of time. Other reaction stresses exist to maintain the constraints of zero strain in the x and out-of-plane directions

Constitutive relations relate increments of ϵ and γ to increments of σ and τ. For constant pore pressure, Rice [1975] gives these as follows:

$$d\gamma = \frac{d\tau}{G} + \frac{1}{H}(d\tau - \mu d\sigma) \tag{1a}$$

$$d\varepsilon = -\frac{d\sigma}{M} + \frac{\beta}{H}(d\tau - \mu d\sigma) \tag{1b}$$

The first term in each expression is the elastic portion of the increment; G and M are elastic moduli. The second terms in (1) are the inelastic portions. For constant σ, the hardening modulus H is related to $H_{\tan}$, the slope of a curve of τ versus γ,

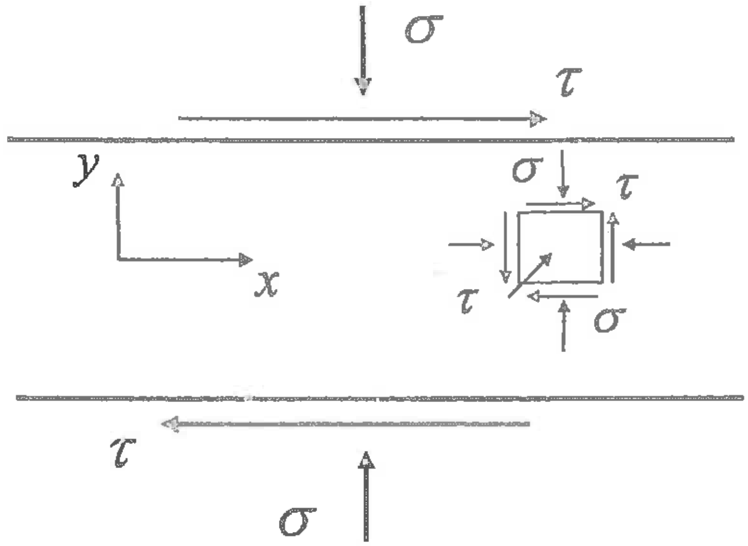

Figure 1: Geometry of the layer problem analyzed by Rice [1975].

by

$$H_{\text{tan}} = H/(1 + H/G) \tag{2}$$

Thus, $H \approx H_{\text{tan}}$, for $H \ll G$. The yield surface is the boundary of the stress states (τ, σ) that cause only elastic response for a given state of inelastic deformation; μ, the local slope of this surface, is referred to as a friction coefficient (Figure 2). Thus, when $H > 0$, deformation increments tending to make $d\tau \leq \mu d\sigma$ are purely elastic and the second terms in (1) are dropped. (When $H < 0$, elastic unloading corresponds to $d\tau \geq \mu d\sigma$.) Thus, increases in compressive normal stress inhibit further inelastic deformation. Inelastic increments of volume strain $d^p\epsilon$ are related to inelastic increments of shear strain $d^p\gamma$ by

$$d^p\epsilon = \beta d^p\gamma \tag{3}$$

where β is a dilatancy factor.

When the pore pressure is not constant, the constitutive relations (1) are modified by replacing the increment of normal stress by an increment of the effective normal stress, a linear combination of $d\sigma$ and dp. Experiments on failure of rocks (e.g., Paterson [1978]) suggest that $\sigma - p$ is the appropriate form for the effective

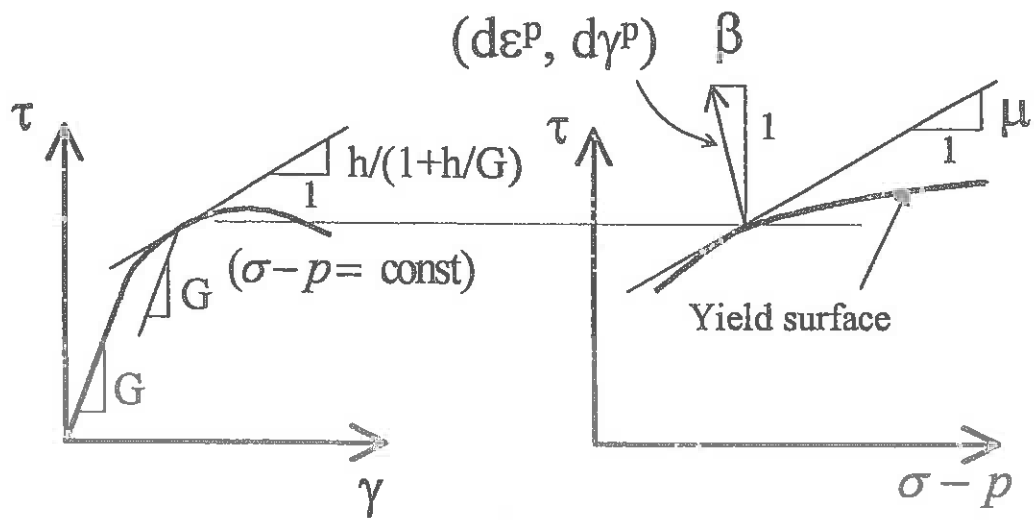

Figure 2: Geometric interpretation of the constitutive parameters used by Rice [1975].

stress for inelastic response. In addition, Rice [1977] has argued on theoretical grounds that this is the appropriate form for inelasticity due to microcracking from the tips of sharp fissures and to frictional sliding on surfaces with small real contact areas. Generally, a different form is needed for elastic deformation [Nur and Byerlee, 1971; Rice and Cleary, 1976]. But, when the solid and fluid constituents are much less compressible than the porous matrix, as for most soils, $\sigma - p$ is also the appropriate form for elastic straining. In this case, equations (1) become

$$d\gamma = \frac{d\tau}{G} + \frac{1}{H}[d\tau - \mu d(\sigma - p)] \qquad (4a)$$

$$d\varepsilon = -\frac{d(\sigma - p)}{M} + \frac{\beta}{H}[d\tau - \mu d(\sigma - p)] \qquad (4b)$$

An additional constitutive relation is Darcy's law which, in the absence of body forces, has the following form:

$$q = -\rho\kappa\frac{\partial p}{\partial y} \qquad (5)$$

where q is fluid mass flow rate per unit area in the y direction and ρ is the fluid mass density. The coefficient κ is often expressed as the ratio of a permeability, with dimensions of length squared (frequently measured in darcies; 1 darcy = 10^{-8} cm^2) to the fluid viscosity. If the fluid phase is incompressible, the density is constant and the equation expressing conservation of fluid mass is

$$\frac{\partial (q/\rho)}{\partial y} + \frac{\partial \varepsilon}{\partial t} = 0 \tag{6}$$

Substituting (5) into (6) yields an equation relating gradients in pore pressure to changes in volumetric strain:

$$\frac{\partial}{\partial y}\left[\kappa \frac{\partial p}{\partial y}\right] = \frac{\partial \varepsilon}{\partial t} \tag{7}$$

2.2 Undrained Homogeneous Deformation

If drainage from the boundaries of the layer is prevented and the deformation and properties are uniform, (5) yields $q = 0$ throughout the layer and, from (7), increments in volume strain are zero. Setting $d\varepsilon = 0$ in (4) yields the following expression for the change in effective normal stress:

$$d(\sigma - p) = \frac{\beta M}{H + \mu \beta M} d\tau \tag{8}$$

Since M is an elastic modulus and positive, the pore pressure decreases for dilation ($\beta > 0$) at fixed normal stress. Substitution of (8) into (4a) reveals that the slope of the shear stress versus shear strain curve (no longer at constant effective normal stress) is still given by (2) but with the hardening modulus H replaced by the augmented value:

$$H_{undrained} = H + \mu \beta M \tag{9}$$

2.3 Instability

Rice [1975] proceeds to show, however, that the homogeneous, undrained solution is unstable with respect to small spatial perturbations in the strain or pore pressure. In particular, linearization of the governing equations about the undrained homogeneous solution yields the homogeneous diffusion equation for the perturbations in pore pressure $\tilde{p}$:

$$\frac{\partial^2 \tilde{p}}{\partial y^2} = \frac{1}{c} \frac{\partial \tilde{p}}{\partial t} \tag{10}$$

where the diffusivity c is given by

$$c = \frac{\kappa M H}{H + \mu \beta M} \tag{11}$$

and the constitutive parameters are to be evaluated at homogeneous undrained deformation. Perturbations with a Fourier wavelength λ grow exponentially at a rate

$$r = -4\pi^2 c/\lambda^2 \tag{12}$$

If both β and μ are positive, then, since $M > 0$, $H_{undrained} > H$ and the homogeneous, undrained response is dilatantly hardened. But, because c (11) passes through zero from positive to negative when $H = 0$, the magnitude of spatial perturbations, instead of decaying exponentially, grow exponentially (12). As noted by Rice [1975] this is analogous to running the heat equation backwards in time: non-uniformities become more localized rather than more diffuse with time. Thus, dilatantly hardened response becomes unstable when the *underlying drained response* passes through a peak. Since H is generally a decreasing function of inelastic deformation, $H = 0$, corresponding to a peak in the drained τ vs. γ curve (at constant σ) will occur before a peak in the undrained response curve, $H_{undrained} = 0$.

The Appendix of Rice [1975] develops the analysis for arbitrarily compressible solid and fluid constituents. The effect is to replace the elastic modulus M in (8), (9) and (11) by a modified value

$$M' = \left\{ \frac{1}{M} + \frac{\phi}{K_f} - \frac{1}{M_s} - \frac{\phi}{N_s} \right\}^{-1} \tag{13}$$

where ϕ is the apparent void volume fraction, K_f is the bulk modulus of the pore fluid and M_s and N_s are additional moduli associated with the solid constituents. When both the solid and fluid constituents are effectively incompressible, $M' = M$. If the solid constituents are incompressible, i.e., M_s, $N_s \gg M$,

$$M' = \frac{M}{1 + M\phi/K_f} \tag{14}$$

If the fluid is very compressible $K_f/\phi \ll M$ and $M' \approx K_f/\phi$. Thus, the dilatant hardening effect vanishes in the limit that the pore fluid bulk modulus goes to zero.

2.4 Discussion

Rice [1975] remarks that if initial material non-uniformities or variation of constitutive parameters on the time scale of perturbation evolution are included, these introduce additional inhomogeneous terms into (10). In the linearized analysis, these terms are linear in the perturbation magnitudes and proportional to the stress or strain rate of the uniform solution. For infinitesimal wavelengths, $\lambda \to 0$, the growth rate in (12) is unbounded and these additional terms do not affect the conditions for instability. In actuality, the perturbation wavelength is limited from below by the material grain size l_g. Consequently, the growth rate r is bounded from above by $r_{\max} = -4\pi^2 c/l_g^2$. Thus, Rice's [1975] analysis corresponds to the case when the perturbation growth rate is much larger than the layer deformation rate, $r_{\max} \gg \dot{\gamma}$. That is, perturbation growth occurs "instantaneously" on the time scale of the layer deformation and the onset of perturbation growth ($c = 0$) will coincide with instability.

For very low permeability rocks (or fast loading rates), the diffusion length scale $l_d = (-c/\,\dot{\gamma})^{1/2}$ may be comparable to the grain size, yielding $r_{\max} \sim \dot{\gamma}$. In this case, the actual instability, defined as a certain increase of the perturbation magnitude over its initial value, will be delayed from the onset of perturbation growth. To estimate the delay of undrained instability, Garagash and Rudnicki [2000] have extended Rice's analysis to include the coupling between the perturbation and the evolution of the constitutive parameters with uniform background deformation.

Rudnicki [1984b] examined the effect of an initial non-uniformity by considering the shear of a weakened layer embedded in an infinite body. Both the layer and the surrounding material deform non-elastically but the peak stress in the layer is slightly less than that in the surrounding material. The pore pressure is assumed to be uniform in both the layer and the surrounding material and the fluid mass exchange is assumed to be proportional to the difference. The development of instability in time depends on the ratio of the rate of imposed farfield strain-rate $\dot{\gamma}_\infty$ to the rate of exchange of fluid mass between flow from the layer, d. In the limit $\dot{\gamma}_\infty\,/d \to 0$, the pore pressure in the layer is the same as in the surrounding material and instability occurs at the peak of the drained stress-strain curve. For finite $\dot{\gamma}_\infty\,/d$, instability is delayed until the weakened layer reaches the peak of its dilatantly hardened stress-strain curve. For small $\dot{\gamma}_\infty\,/d$, as appropriate for most applications, an asymptotic analysis predicts that the time delay is given by $(\alpha d)^{2/3}(\lambda/\,\dot{\gamma}_\infty)^{1/3}\Delta^{-1/6}$, where λ is the half-width of the peak of the stress-strain curve, Δ is the difference in the peak stresses of the weakened layer and the surrounding material divided by λ times the elastic shear modulus and α is a nondimensional measure of the strength of dilatant hardening. These delay times are less than a few hours for tectonic strain rates and less than a few tens of seconds for typical laboratory strain-rates.

If dilatant hardening causes a sufficient reduction of the pore fluid pressure, exsolution of dissolved gases or cavitation of the fluid may occur and the pore fluid bulk modulus will be dramatically reduced. Equations (13) and (9) indicate that the dilatant hardening effect diminishes with reduction of the pore fluid bulk modulus and vanishes in the limit $K_f \to 0$. Rudnicki and Chen [1988] have proposed that cavitation is a limit to strong dilatant hardening associated with slip on a weakening frictional surface and suggest that this effect is consistent with observations by Martin [1980] of pore fluid stabilization of rock failure. In addition, in undrained biaxial experiments on a quartz sand, Mokni and Desrues [1999] have observed that the formation of shear bands in dilatant specimens does not occur until cavitation of the pore fluid. The numerical simulations of Schrefler et al. [1996] also indicate the importance of cavitation in the formation of shear bands.

3 Application to Compacting Materials

Although Rice [1975] considers only dilating materials ($\beta > 0$), he notes that for loosely packed granular materials $\beta < 0$. This will also be the case for high porosity rocks that compact when sheared. For $\beta < 0$, (8) indicates that the effective compressive normal stress decreases and (9) that $H_{undrained} < H$. This phenomenon could be described as *compaction softening*. Consequently, the denominator of (11) will pass through zero before the numerator. The exponent r in (12) reveals that spatial perturbations are damped infinitely fast and then grow infinitely fast as $H_{undrained}$, the denominator of (11), passes through zero. This singular jump in diffusivity suggests that variation of constitutive parameters with the uniform background deformation should be included in the perturbation solution. By including this variation, Garagash and Rudnicki [2000] show that undrained deformation of compacting material is stable (the perturbation magnitude vanishes) as $H_{undrained}$ passes through zero if the ratio of the diffusion length $l_d = (-c/\dot{\gamma})^{1/2}$ to the maximum perturbation wavelength (defined by the layer width $2h$) is larger than a critical value defined by the dependence of the inelastic moduli H and $H_{undrained}$ on the deformation state:

$$4\pi^2(l_d/h)^2 > -\frac{1}{H}\frac{dH_{undrained}}{d\gamma^p} \tag{15}$$

This inequality indicates that the maximum perturbation wavelengths ($h/2\pi$) are most unstable for compaction softening in contrast to dilatant hardening for which the shortest wavelengths grow most rapidly (12). This suggests that failure will occur by a diffuse mode rather than a localized mode, as for dilatant hardening.

The stability criterion (15) implies an interesting scale effect: compaction softening is stable near the peak shear stress ($H_{undrained} = 0$) if the specimen size h is small enough. This result is consistent with the small scale laboratory results of Han and Vardoulakis [1991] and Finno et al. [1995]. If the stability criterion is not satisfied, the perturbation magnitude becomes algebraically singular at $H_{undrained} = 0$. Vardoulakis [1996b] has applied Rice's analysis to study the stability of biaxial deformation of water-saturated sand and noted that the instability at $H_{undrained} = 0$ can also be mitigated by the introduction of rate-dependence in the material constitutive behavior.

The discussion of compaction in the preceding paragraphs assumes $\mu > 0$ and the physical interpretation of μ as a friction coefficient would seem to preclude the possibility that $\mu < 0$. But, μ enters (1) as the local slope of the yield surface. As sketched in Figure 2, the yield surface is open on the normal stress axis. Consequently, purely normal stress σ does not cause inelastic deformation, as appropriate for low porosity rocks. For loose soils or highly porous rock, inelastic compaction (volume decrease) is due not only to inelastic shearing but may be caused by purely hydrostatic (or in the one dimensional layer model here, by purely normal) stress. In this case, the yield surface is closed on the normal stress axis, as depicted in Figure 3. So-called "cap" models were introduced by Dimaggio and Sandler [1971]. As sketched in Figure 3, $\mu < 0$ is appropriate for this class of models and the magnitude of μ becomes large as the σ axis is approached. Because the stability condition contains only the product $\beta\mu$, the case of a material that compacts under a stress state on the cap of the yield surface reduces to that analyzed by Rice [1975].

The form of equation (3) assumes that all inelastic volume strain is associated with inelastic shear strain. Consequently, no inelastic volume strain would occur for loading by purely normal stress. For high porosity rocks and loose soils, this will not be a complete description and a term contributing inelastic volume strain for purely (effective) normal stress can be added to (3)

$$d\varepsilon^p = \beta d\gamma^p - \frac{1}{k} d(\sigma - p) \tag{16}$$

where k is an inelastic modulus that is the slope of a drained ($dp = 0$) curve of σ versus ε^p at constant shear stress ($d\tau = 0$). The effect of including this term is to modify the elastic contribution to the volumetric strain in (4b) and in the stability analysis to replace the elastic modulus M (or M') by the effective value

$$M'' = \frac{k}{1 + k/M} \tag{17}$$

For typical cases, $k \geq 0$ but $k \ll M''$. Decreasing k will reduce the difference between the drained and undrained hardening moduli. Thus, for small k, differences in drained versus undrained deformation may be difficult to observe unless

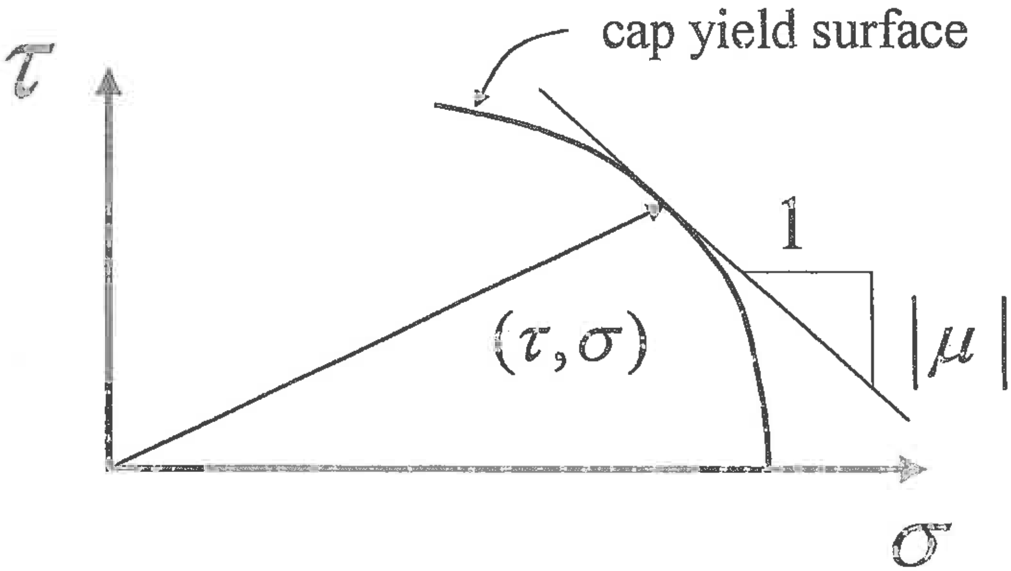

Figure 3: Schematic illustration of a cap on the yield surface. Note that $\mu = d\tau/d\sigma < 0$.

the stress state is near enough to the σ axis so that values of μ are large and negative.

Many geomaterials exhibit both compaction and dilation depending on the initial confining stress, initial porosity and load path. Examples include simulated fault gouge [Marone et al., 1990], loose sand [Finno et al., 1995] and limestone [Baud et al., 2000]. When these materials are fluid-saturated and deformed without allowing fluid flow from the boundaries, the evolution of localized zones depends on the local rate of fluid flow, the imposed rate of straining, and the transition from contraction to dilation. A simple analysis [Rudnicki, 1996] shows that small variations in the evolution of porosity with shear strain can dramatically alter the undrained response. Rudnicki et al. [1996] have suggested that compaction softening followed by dilatant hardening may be an explanation for the evanescent shear band structures observed in some experiments of Finno et al. [1995]. If compaction softening causes the onset of localized deformation in a narrow zone but gives way to dilatant hardening before full development of the band, formation of a shear band in another orientation may occur when the dilatant hardening response becomes unstable.

4 Generalization to Arbitrary Deformation States

In this section, I extend Rice's [1975] analysis to arbitrary deformation states. The analysis follows the lines of Rice's [1976] general treatment of conditions for localization of plastic deformation in rate-independent solids. That analysis is phrased in terms of measures of stress and strain appropriate for arbitrary deformation magnitudes and discusses localization both as a bifurcation from homogeneous deformation and as the growth of an initial non-uniformity. Here, for simplicity, I restrict attention to small strains and consider only small perturbations from completely uniform deformation. Runesson et al. [1998] have presented an analysis of localization for undrained conditions for finite strains. Generalizations of the present analysis to finite strains can be adapted from the treatments of Rudnicki [1983], Runesson et al. [1998] or Coussy [1995].

4.1 Localization in Rate-independent Solids

Consider a homogeneous solid deforming in homogeneous fashion described by a strain rate $\mathbf{d}^0$ and stress-rate $\dot{\sigma}^0$. Perturbations from these uniform fields are denoted by $\Delta\mathbf{d}$ and $\Delta\dot{\sigma}$. To investigate the possibility of localization in a planar band the perturbed fields are taken to vary with distance from a plane with normal $\mathbf{n}$. The requirements of equilibrium and continuous velocities place the following restrictions on the form of the perturbed fields [Rice, 1976]):

$$\Delta\mathbf{d} = \frac{1}{2}(\mathbf{ng}+\mathbf{gn}) \tag{18}$$

$$\mathbf{n}\cdot\Delta\dot{\sigma} = 0 \tag{19}$$

where $\mathbf{g}$ is a function of $\mathbf{n}\cdot\mathbf{x}$.

For nonlinear elastic or rate-independent elastic-plastic solids with a smooth yield surface and plastic potential, the strain-rate can be given by an incrementally linear relation:

$$\mathbf{d} = \mathbf{K} : \dot{\sigma} \tag{20}$$

where $(\mathbf{K}:\dot{\sigma})_{ij} = K_{ijkl}\dot{\sigma}_{kl}$. If $\mathbf{K}$ can be inverted, (20) can be written in the alternative form:

$$\dot{\sigma} = \mathbf{M} : \mathbf{d} \tag{21}$$

where $\mathbf{K}$ and $\mathbf{M}$ are mutual inverses with components satisfying

$$M_{ijkl}K_{klmn} = \frac{1}{2}(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm}) \tag{22}$$

and δ_{ij} is the Kronecker delta. The components of the incremental compliances K_{ijkl} and the moduli M_{ijkl} are symmetric with respect to interchange of the first two indices and the last two indices, i.e., $M_{ijkl} = M_{jikl}$ and $M_{ijkl} = M_{ijlk}$, but are not, in general, symmetric with respect to interchange of the first pair and the last pair, i.e., $M_{ijkl} \neq M_{klij}$. The last is typical of geomaterials for which inelastic strain increments are not perpendicular to the yield surface or, in other words, the plastic flow rule is not associated with the yield function. Substitution of (18) into (21) and the result into (19) yields

$$(\mathbf{n} \cdot \mathbf{M} \cdot \mathbf{n}) \cdot \mathbf{g} = \mathbf{n} \cdot (\mathbf{M}^0 - \mathbf{M}) : \mathbf{d}^0 \tag{23}$$

where $\mathbf{M}^0$ denotes the incremental moduli for the uniform fields. In the simplest case, the constitutive response of the perturbed field is identical with that of the uniform field and the right hand side of (23) vanishes. The first possibility for a non-trivial solution for $\mathbf{g}$ occurs when the determinant of the coefficient matrix vanishes:

$$\det(m_{jk}) = 0 \tag{24}$$

where $m_{jk} = (\mathbf{n} \cdot \mathbf{M} \cdot \mathbf{n})_{jk}$ If the constitutive responses of the perturbed and homogeneous fields differ, the condition (24) is still limiting in the sense that $\mathbf{g}$ determined from (23) will be unbounded when (24) is met.

4.2 Inclusion of Pore Fluid

When the solid is saturated with a fluid that can be described in terms of a single scalar pore pressure (see Cleary [1978] for a discussion of other possibilities), a term involving the rate of pore pressure must be appended to (20):

$$\mathbf{d} = \mathbf{K} : \dot{\sigma} + \mathbf{A}\,\dot{p} \tag{25}$$

Alternatively (25) can be written as

$$\dot{\sigma} = \mathbf{M} : \mathbf{d} - \alpha\,\dot{p} \tag{26}$$

where $\alpha = \mathbf{M} : \mathbf{A}$. The introduction of the pore pressure as an additional field variable requires an additional constitutive equation. This is conveniently taken to be a relation the rate of change of fluid mass content per unit volume of porous solid $m = \rho v$ where ρ is the density of homogeneous pore fluid and v is apparent void volume fraction. For arbitrarily compressible constituents, $\dot{m}$ can be expressed in terms of the rate of strain and pore pressure change:

$$\frac{\dot{m}}{\rho} = \mathbf{R} : \mathbf{d} + \eta\,\dot{p} \tag{27}$$

or in terms of the stress-rate and pore pressure change by substituting (25) into (27):

$$\frac{\dot{m}}{\rho} = \mathbf{R} : \mathbf{K} : \dot{\boldsymbol{\sigma}} + (\mathbf{R} : \mathbf{A} + \eta)\, \dot{p} \tag{28}$$

In (25), (26) and (27), $\mathbf{A}$, $\boldsymbol{\alpha}$ and $\mathbf{R}$ are symmetric because of the symmetry of $\mathbf{d}$ and $\dot{\boldsymbol{\sigma}}$.

For drained response, the pore pressure is constant and (25) and (26) reduce to (20) and (21). The condition for localization is given by (24).

For undrained response, the fluid mass in material elements is constant, the right hand side of (27) is zero, and the pore pressure is given by

$$\dot{p} = -\eta^{-1}\mathbf{R} : \mathbf{d} \tag{29}$$

or, equivalently, in terms of the stress from (28)

$$\dot{p} = \frac{-\mathbf{R} : \mathbf{K} : \dot{\boldsymbol{\sigma}}}{\eta + \mathbf{R} : \mathbf{A}} \tag{30}$$

Substitution of (29) into (26) gives

$$\dot{\boldsymbol{\sigma}} = \mathbf{M}^{u} : \mathbf{d} \tag{31}$$

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where the undrained incremental moduli are given by

$$\mathbf{M}^{u} = \mathbf{M} + \eta^{-1}\mathbf{R}\boldsymbol{\alpha} \tag{32}$$

The condition for localization (24) evaluated in terms of the undrained moduli (32) is

$$\det(m^{u}_{ij}) = 0 \tag{33}$$

where $\mathbf{m}^{u} = \mathbf{n} \cdot \mathbf{M}^{u} \cdot \mathbf{n}$. The matrix $\mathbf{m}^{u}$ is related to $\mathbf{m}$ (24) by

$$\mathbf{m}^{u} = \mathbf{m} + \eta^{-1}\mathbf{r}\mathbf{a} \tag{34}$$

where $\mathbf{r} = \mathbf{n} \cdot \mathbf{R}$ and $\mathbf{a} = \mathbf{n} \cdot \boldsymbol{\alpha}$. The earlier discussion of the layer problem examined by Rice [1975] suggests that (33) may be met before or after (24) depending on the nature of the constitutive relation and, in particular, the inelastic volumetric deformation. If (24) is met before (33), Rice's [1975] analysis also suggests that undrained homogeneous deformation is unstable, in the sense that small spatial perturbations will grow exponentially in time. As a result, the condition (33) may be irrelevant since instability may occur well before it is met.

4.3 Diffusive Instability

In order to analyze the stability of undrained homogeneous deformation, an equation expressing fluid mass conservation (in the absence of sources) must be added to the field equations and an additional constitutive equation relating the fluid mass flux to gradients in pore pressure must be specified. Fluid mass conservation (in the absence of sources) is given by

$$\dot{m} + \nabla \cdot \mathrm{q} = 0 \tag{35}$$

where q is the mass flow rate per unit area of porous solid. Fluid mass flow is assumed to be related to the pressure gradient by Darcy's law:

$$\mathrm{q} = -\rho_0 \boldsymbol{\kappa} \cdot \nabla p \tag{36}$$

where $\boldsymbol{\kappa}$ is the ratio of a symmetric permeability tensor to the (scalar) fluid viscosity and ρ_0 is the (constant) fluid mass density in the reference state. Substitution of (36) into (35) yields

$$\dot{m} = \rho_0 \nabla \cdot \boldsymbol{\kappa} \cdot \nabla p \tag{37}$$

For simplicity, we assume, as in Rice [1975], that perturbations from undrained homogeneous deformation are small enough so that incremental constitutive parameters are the same for the uniform and perturbed fields. If this is not the case, additional inhomogeneous terms would enter the right hand sides of the equations. As in (23), these terms involve the differences of the constitutive parameters multiplied by the strain-rate and pore pressure rate in the uniform field. Forming the difference of (26) and substituting into (19) yields

$$\mathrm{m} \cdot \mathrm{g} - \mathrm{a}\Delta\dot{p} = 0 \tag{38}$$

where $\Delta\dot{p}$ is the difference between the pore pressure-rates in the uniform and perturbed fields and, again, $\mathrm{a} = \mathbf{n} \cdot \boldsymbol{\alpha}$. Because the unperturbed fields are homogeneous and undrained, both sides of (37) are identically zero. The kinematic restriction on the perturbed strain-rate field (18) suggests that Δp depends on distance $\mathbf{n} \cdot \mathbf{x}$ perpendicular to the orientation of a planar band and, hence, can be decomposed into Fourier components of the form $\exp(\iota\lambda\mathbf{n} \cdot \mathbf{x})$. This type of deformation mode has also been discussed for sands by Vardoulakis [1996b]. For this decomposition, using (27) in (37). Since both sides of (37) are zero for the unperturbed (homogeneous, undrained) field, the difference fields also satisfy (37). Using the Fourier decomposition on the right hand side and assuming the permeability is uniform yields

$$\Delta\dot{m} = \rho_0 \kappa \Delta p \tag{39}$$

where $\kappa = \lambda^2 \mathbf{n}\cdot\boldsymbol{\kappa}\cdot\mathbf{n}$. Forming the difference of (27) and substituting into (39) yields

$$\mathbf{r}\cdot\mathbf{g} + \eta\Delta\dot{p} + \kappa\Delta p = 0 \tag{40}$$

The vector $\mathbf{g}$ can be eliminated from (38) by writing the inverse of $\mathbf{m}$ as $\mathbf{k}/m$ where $\mathbf{k}$ is the matrix of co-factors and $m = \det(m_{ij})$. Substituting the result into (40) yields

$$[\mathbf{r}\cdot\mathbf{k}\cdot\mathbf{a} + \eta m]\,\Delta\,\dot{p} + m\kappa\Delta p = 0 \tag{41}$$

This is a linear ordinary differential equation for the time evolution of the perturbed pressure field. The solutions change from exponentially decaying to exponentially growing as m passes through zero from positive to negative values (at least if the term in square brackets is positive). Since the vanishing of m is the condition for localization in terms of the drained incremental moduli (24), this condition controls the growth rate of spatial perturbations as in the layer analysis of Rice [1975].

The similarity with Rice's [1975] analysis suggests that the square bracket in (41) will vanish when the condition for localization is met in terms of the undrained moduli (33). Using $\mathbf{m}^{-1} = \mathbf{k}/m$ in (34), pre- and post-multiplying by $\mathbf{a}$ and dividing through by $a^2 = \mathbf{a}\cdot\mathbf{a}$ yields

$$\mathbf{a}\cdot\mathbf{k}\cdot\mathbf{r} + m\eta = \eta\frac{\mathbf{a}}{a}\cdot\mathbf{k}\cdot\mathbf{m}^u\cdot\frac{\mathbf{a}}{a} \tag{42}$$

Using this expression to eliminate $m\eta$ in (41) gives

$$\left[\eta\frac{\mathbf{a}}{a}\cdot\mathbf{k}\cdot\mathbf{m}^u\cdot\frac{\mathbf{a}}{a} + \mathbf{r}\cdot\mathbf{k}\cdot\mathbf{a} - \mathbf{a}\cdot\mathbf{k}\cdot\mathbf{r}\right]\Delta\,\dot{p} + m\kappa\Delta p = 0 \tag{43}$$

If $\mathbf{R} = \boldsymbol{\alpha}$ (so that $\mathbf{a} = \mathbf{r}$) or if $\mathbf{k}$ is symmetric, then the second term in brackets vanishes and the coefficient of $\Delta\dot{p}$ is expressed in terms of the matrix entering the localization condition in terms of the undrained moduli. Even in this case, however, it is not clear that the sign of this term changes when (33) is met. For geomaterials, $\mathbf{M}$ is generally not symmetric with respect to interchange of the first and last pair of indices. Consequently, neither $\mathbf{m}$ nor $\mathbf{k}$ will be symmetric. In addition, for the particular form of a constitutive relation discussed in the next section, this lack of symmetry of $\mathbf{M}$ causes $\mathbf{R}$ and $\boldsymbol{\alpha}$ to differ.

5 Application to an Elastic-Plastic Relation

This section illustrates the analysis of the preceding section by specializing to a particular form of elastic plastic constitutive relation. The strain-rate for plastic-loading is given by

$$\mathrm{d} = \mathbf{C} : \dot{\boldsymbol{\sigma}} + \frac{1}{h}\mathbf{P}\,\mathbf{Q} : \dot{\boldsymbol{\sigma}} \tag{44}$$

where the first term is the elastic contribution, $\mathbf{C}$ is the tensor of elastic compliances, $\mathbf{P}$ is a tensor specifying the direction of inelastic strain increments in stress space, $\mathbf{Q}$ is the tensor giving the normal to the yield surface in stress space, and h is an inelastic hardening (softening) modulus. Deformation increments that tend to make $\mathbf{Q} : \dot{\boldsymbol{\sigma}} \leq 0$ for $h > 0$ cause elastic unloading and for these the second term is dropped. (If $h < 0$, increments tending to make $\mathbf{Q} : \dot{\boldsymbol{\sigma}} > 0$ cause elastic unloading). Thus, the tensor $\mathbf{K}$ in (20) is given by

$$\mathbf{K} = \mathbf{C} + \frac{1}{h}\mathbf{P}\mathbf{Q} \tag{45}$$

The inverse of $\mathbf{K}$, $\mathbf{M}$ in (21) is given by

$$\mathbf{M} = \mathbf{L} - \frac{\mathbf{p}\,\mathbf{q}}{h + \mathbf{Q} : \mathbf{L} : \mathbf{P}} \tag{46}$$

where $\mathbf{L} = \mathbf{C}^{-1}$ is the tensor of elastic moduli, $\mathbf{p} = \mathbf{L} : \mathbf{P}$ and $\mathbf{q} = \mathbf{L} : \mathbf{Q}$.

The constitutive relation used by Rudnicki and Rice [1975] in their study of localization has the form of (44) but the yield function and plastic potential depend only on the first and second stress invariants. In this case, $\mathbf{P}$ and $\mathbf{Q}$ are given by

$$\mathbf{P} = \mathbf{N} + \frac{1}{3}\beta\,\mathbf{I} \tag{47a}$$

$$\mathbf{Q} = \mathbf{N} + \frac{1}{3}\mu\,\mathbf{I} \tag{47b}$$

where $\mathbf{N} = \mathbf{s}/2\overline{\tau}$, $\mathbf{s} = \boldsymbol{\sigma} - (1/3)\,\mathrm{tr}\,\boldsymbol{\sigma}\,\mathbf{I}$ is the deviatoric stress tensor, $\overline{\tau} = (\mathbf{s} : \mathbf{s}/2)^{1/2}$ is the Mises equivalent shear stress, $\mathbf{I}$ is the identity tensor and μ and β have interpretations similar to those in (1). If the elasticity is assumed to be isotropic with shear and bulk moduli, G and K, respectively, and $\mathbf{P}$ and $\mathbf{Q}$ are given by (50)

$$\mathbf{p} = 2G\mathbf{N} + \beta K\,\mathbf{I} \tag{48a}$$

$$\mathbf{q} = 2G\mathbf{N} + \mu K\,\mathbf{I} \tag{48b}$$

$$\mathbf{Q} : \mathbf{L} : \mathbf{P} = G + \mu\beta K \tag{48c}$$

Rudnicki [1984a, 1985] has extended the formulation to include pore fluid effects in the form of (44) used by Rudnicki and Rice [1975]. He assumes isotropic poroelasticity for the elastic strain increments and inelastic strain increments are given by the second term of (44) with the stress replaced by the effective stress $\sigma + p\mathrm{I}$. As already noted, much experimental evidence is consistent with this form and Rice [1977] has argued that it is appropriate for inelasticity arising from microcracking and frictional slip on surfaces with small real areas of contact. For isotropic poroelasticity, Nur and Byerlee [1971] have shown that the proper form of the effective stress is $\sigma + p\zeta\mathrm{I}$. The Biot porous media parameter ζ (often denoted α) is equal to $1 - K/K_s'$, where K is the elastic bulk modulus of the porous solid and K_s' is an additional modulus related to the bulk modulus of the solid constituents [Rice and Cleary, 1976]. Thus, if the solid constituents are incompressible, $K_s' \gg K$ and $\zeta = 1$ This is a suitable approximation for soils, in which the compressibility of the grains is much less than that of the porous matrix. Under the same conditions for which $\sigma + p\mathrm{I}$ is the form of the effective stress for inelastic deformation, Rice [1977] has also argued that the inelastic increment of the apparent void volume fraction ($\dot{v}$, where $v = m/\rho$) is equal to the inelastic volume strain increment. Under these circumstances, the tensors A in (25) and α in (26) are given by

$$\mathrm{A} = \zeta\mathrm{C} : \mathrm{I} + h^{-1}\mathrm{P}\,\mathrm{tr}(\mathrm{Q}) \tag{49a}$$

$$\alpha = \zeta\mathrm{I} + \frac{\mathrm{p}\,\mathrm{tr}(\mathrm{Q})(1-\zeta)}{h + \mathrm{Q} : \mathrm{L} : \mathrm{P}} \tag{49b}$$

The tensor R and the coefficient η entering (27) are given by

$$\mathrm{R} = \zeta\mathrm{I} + \frac{\mathrm{q}\,\mathrm{tr}(\mathrm{P})(1-\zeta)}{h + \mathrm{Q} : \mathrm{L} : \mathrm{P}} \tag{50a}$$

$$\eta = \frac{\zeta(1-\zeta B)}{KB} + \frac{\mathrm{tr}\,\mathrm{P}\,\mathrm{tr}\,\mathrm{Q}(1-\zeta)^2}{h + \mathrm{Q} : \mathrm{L} : \mathrm{P}} \tag{50b}$$

where B is Skempton's coefficient, the negative of the ratio of pore pressure to mean normal stress during undrained elastic deformation [Rice and Cleary, 1976]. The rate of change of pore pressure during undrained deformation (30) is given by

$$\dot{p} = -\frac{1}{(\zeta/BK) + h^{-1}\,\mathrm{tr}\,\mathrm{P}\,\mathrm{tr}\,\mathrm{Q}}\left\{\zeta\mathrm{C} : \mathrm{I} + h^{-1}\,\mathrm{tr}\,\mathrm{P}\;\mathrm{Q}\right\} : \dot{\sigma} \tag{51}$$

For isotropic elasticity, $\mathrm{C} : \mathrm{I} = \mathrm{I} : \mathrm{C} = (1/3K)\mathrm{I}$ and $\mathrm{Q} : \mathrm{L} : \mathrm{P} = G + \mathrm{tr}(\mathrm{Q})\,\mathrm{tr}(\mathrm{P})K$.

The compliance tensor for undrained deformation can be formed from these results

$$\mathrm{K}^u = \mathrm{C} - B\zeta\mathrm{C} : \mathrm{I}\mathrm{I} + \frac{1}{h_{\mathrm{und}}}\left\{\mathrm{P}' + \frac{1}{3}(1-B)\,\mathrm{tr}\,\mathrm{P}\,\mathrm{I}\right\}\left\{\mathrm{Q}' + \frac{1}{3}(1-B)\,\mathrm{tr}\,\mathrm{Q}\,\mathrm{I}\right\} \tag{52}$$

where $h_{\text{und}} = h + BK \operatorname{tr}\mathbf{P} \operatorname{tr}\mathbf{Q}/\zeta$ and the primes on $\mathbf{P}$ and $\mathbf{Q}$ denote the deviatoric parts. For isotropic elasticity, the first two terms correspond to replacing K, the elastic bulk modulus for drained deformation by K_u the value for undrained deformation, where

$$K_u = K/(1 - \zeta B) \tag{53}$$

The contribution to the inelastic strain-rate, the last term in (52), is identical in form to that for drained deformation (44), with h replaced by h_{und} , $\operatorname{tr}\mathbf{P}$ replaced by $(1 - B)\operatorname{tr}\mathbf{P}$ and $\operatorname{tr}\mathbf{Q}$ by $(1 - B)\operatorname{tr}\mathbf{Q}$. If the expression for the compliance is specialized to pure shear, the result is exactly analogous to that of Rice [1975]: The undrained response is identical in form to that for drained but with h replaced by h_{und}. Since B, K, and ζ^{-1} are positive, h_{und} is greater or less than h depending on the sign of $\operatorname{tr}\mathbf{P} \operatorname{tr}\mathbf{Q}$, in a manner identical to the dependence on the sign of $\beta\mu$ in Rice [1975].

These expressions can also be used to examine the role of the compressibilities of the individual solid and fluid constituents. If the solid constituents are incompressible, $\zeta = 1$, and $\boldsymbol{\alpha} = \mathbf{R} = \mathbf{I}$. Consequently, the results simplify considerably. In particular $\mathbf{a} = \mathbf{r}$ in equation (34) and the term $\mathbf{r}\cdot\mathbf{k}\cdot\mathbf{a} - \mathbf{a}\cdot\mathbf{k}\cdot\mathbf{r}$ vanishes in (43).

The compressibility of the pore fluid enters only through the Skempton coefficient B. For isotropic poroelasticity, B can be expressed as

$$B = \frac{\zeta}{\zeta + v_0 K(1/K_f - 1/K_s'')} \tag{54}$$

where v_0 is the apparent void volume fraction in the reference state, K_f is the pore fluid bulk modulus, and K_s'' is another modulus related to the bulk modulus of the solid constituents [Rice and Cleary, 1976]. If the solid constituents are incompressible, K_s', $K_s'' \gg K$. If the fluid is also incompressible, $K_f \gg v_0 K$, then $B = 1$. In this limit, the bulk modulus is eliminated from (52) and so is $\operatorname{tr}\mathbf{P}$. Consequently, both the elastic and inelastic components are incompressible, consistent with the limit of incompressible constituents. Furthermore, $\operatorname{tr}\mathbf{Q}$ is eliminated from (52). Therefore, any dependence of the yield function on mean stress for drained response is exactly compensated by changes in pore pressure during undrained response. Thus, the undrained response is incompressible and does not depend on the mean stress. For such a solid, shear bands are predicted to occur at 45° from the principal axes, at least if $\mathbf{P}' = \mathbf{Q}'$. Runesson et al. [1996] obtain this result by direct calculation for undrained deformation of a porous elastic plastic solid with an incompressible pore fluid. The result pertains, however, only when the solid constituents are also incompressible. Although Runesson et al. [1996]

do not explicitly assume that the solid constituents are incompressible, they use $\boldsymbol{\sigma} + p\mathbf{I}$ as the effective stress for both inelastic and elastic deformation. As discussed earlier, this form is consistent with linear poroelasticity only when the solid constituents are incompressible.

6 Concluding Discussion

Application of Rice's [1975] results to materials that compact with inelastic shearing suggest that the accompanying pore pressure changes will be destabilizing. That is, the effective hardening modulus for undrained response will be less than that for drained response and conditions for localization will be met for undrained constitutive response before they are met in terms of the drained response. But this conclusion appears to apply only when the yield stress in pure shear increases with increasing mean stress. For highly porous materials that compact with mean stress, the yield surface typically has a cap and, as a result, the yield stress in pure shear decreases with increasing compressive mean stress. In this case, the conditions for linearized stability of undrained deformation revert to those obtained by Rice [1975]: undrained deformation is stiffer than the drained response, but homogeneous undrained response becomes unstable when conditions for localization are met in terms of the drained response.

The conclusions of Rice [1975] based on analysis of a layer subject to simple shear and compression are shown to apply to arbitrary deformations, although conditions for localization are not, in general, met at the peak of the stress versus strain curve as they are for the layer. Spatial perturbations from the undrained solution grow exponentially when the condition for localization is met in terms of the drained response. In the layer problem, the growth rate is unbounded immediately after the condition for localization is met in terms of the undrained response. This is not necessarily the case for arbitrary three dimensional deformations because of the lack of symmetry in the constitutive tensors that is typical of geomaterials.

For elastic plastic solids, the correspondence with the simple layer analysis is direct. The effective hardening modulus governing shear for undrained conditions is the sum of the value for drained response and a term that is the product of an elastic bulk modulus and the mean parts of tensors giving the plastic flow direction and the normal to the yield surface. The incremental compliance for undrained deformation of an elastic plastic solid gives some insight into the surprising result of Runesson et al. [1996]: if the pore fluid is completely incompressible, zones of localization are predicted to occur at 45° to the principal axes regardless of the form of the yield function and plastic potential (at least, if the deviatoric portions of the tensors giving the plastic flow direction and the normal to yield surface coincide). This results because, if the solid constituents are also incompressible, undrained response is completely incompressible and the induced changes in pore pressure exactly compensate for any dependence of the yield function on the mean stress.

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