STABILITY OF UNDRAINED DEFORMATION OF FLUID-SATURATED GEO-MATERIALS†

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ABSTRACT - The presence of pore fluids in geomaterials can alter deformation processes and facilitate or delay material failure. Dilation/contraction of geomaterials in the course of inelastic undrained deformation causes a reduction/increase in pore pressure altering the effective compressive stress. This results in an increase/decrease in shear stress that can be sustained by the material over the corresponding case in drained deformation. We use a perturbation approach to investigate whether undrained conditions inhibit or promote localization or diffused instability as compared to the drained case. The effect of the material strain-rate sensitivity on the undrained stability is evaluated.

INTRODUCTION: Inelastic volume changes in fluid-saturated geomaterials tend to cause a change in pore fluid pressure. Under drained conditions, pore pressure remains constant as its alterations are equilibrated by the pore fluid flow. Under undrained conditions, changes in pore pressure persist. Pore pressure drop in undrained shearing of dilatant geomaterials causes reduction of effective compressive stress and, consequently, the increase of the shear stress sustained by the material (dilatant strengthening) over the corresponding drained shear strength [Rice, 1975]. Undrained shear stress "weakening" over the corresponding drained strength is observed in contracting geomaterials (Fig. 1). Rice [1975] has used perturbation analysis to show that undrained dilatant strengthening is limited by instability at the state (C), Fig. 1, corresponding to the peak stress in the underlying drained response, state (T). For contractive materials, Vardoulakis [1985, 1996] has predicted strong instability of undrained deformation at small strains corresponding to the shear stress peak (T_-), Fig. 1. However, neither of these predictions agree with the results from displacement-controlled biaxial compression experiments on saturated sands under undrained conditions (e.g., Han and Vardoulakis [1991]). These experiments indicate that loss of stability (manifested by subsequent localization of deformation into shear bands) occurs at plastic strains significantly higher than the ones corresponding to the state (C) for dense (dilatant) sands, and to the state (T_-) for loose (contractive) sands. In this paper, we reconsider the earlier theoretical analyses (and assumptions) to resolve the above disagreement between the theory and experiment.

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PROCEDURES, RESULTS AND DISCUSSION: We consider a problem initially formulated by Rice [1975] of a plane strain shearing of a layer of width 2ℓ extending indefinitely in the x-direction. Only the normal strain $\varepsilon = \partial v / \partial y$ (positive in extension) and the shear strain $\gamma = \partial u / \partial y$ are non-zero. Conjugate stresses are normal stress σ (positive in compression) and shear stress τ. Constant shear rate $\dot{\gamma}$ and normal stress σ are prescribed at the boundary. From equilibrium, stress is uniform throughout the layer (i.e. function of time only). Other reaction stresses exist to maintain the constraints of plane strain and zero strain in x direction. Under monotonic elasto-plastic loading, constitutive relations are taken as:

$$\textit{elasticity: } \gamma^e = \tau / G, \;\; \varepsilon^e = -\sigma' / M; \qquad \textit{plasticity: } \tau = \mu_* \sigma', \;\; d\varepsilon^p = \beta_* d\gamma^p \qquad \textbf{(1)}$$

where $\sigma' = \sigma - p$ is normal effective stress, p is pore pressure, G and M are elastic moduli, and, μ_* and β_* are mobilized internal friction and dilatancy coefficients in the yield condition and flow rule, respectively. Material shear strain rate sensitivity is modeled via

$$\mu_* = \mu(\gamma^p)(1 + S_\mu \ln(\dot{\gamma}/\dot{\gamma}_o)), \qquad \beta_* = \beta(\gamma^p)(1 + S_\beta \ln(\dot{\gamma}/\dot{\gamma}_o)) \qquad \textbf{(2)}$$

where $\mu(\gamma^p)$ and $\beta(\gamma^p)$ are mobilized friction and dilatancy at a constant rate $\dot{\gamma}_o$. Flow of incompressible pore fluid is governed by the continuity and Darcy's laws (e.g., Rice [1975]). In undrained homogeneous deformation, the volumetric strain is zero and the response is characterized by the undrained plastic modulus [Rice, 1975]: $(d\tau / d\gamma^p)_u = h_u = h + \beta\mu M$, where $h = (d\mu(\gamma^p) / d\gamma^p)\sigma'$ is the plastic modulus of frictional material [Vardoulakis, 1985] (which, at constant pore pressure, corresponds to the drained plastic modulus). Undrained "dilatant strengthening" ($\beta > 0$) and "contractive weakening" ($\beta < 0$) then follow, Fig. 1. A small spatial perturbation is introduced into the otherwise homogeneous undrained solution to study its stability. In earlier analyses of Rice [1975] and Vardoulakis [1985], perturbations are assumed to evolve much faster than the background homogeneous solution (or, equivalently, with much smaller wavelength [Rice, 1975]). The homogenous solution is then assumed constant over the range of deformations for which the evolution of perturbation is considered. In order to address the disparity between the theoretical and experimental results on undrained instabilities, we relax the small wavelength assumption. Thus, we acknowledge that the perturbation wavelength cannot be arbitrarily small or large, as it is bounded by the material grain size from below and by the specimen dimensions from above. This signifies that perturbation growth rate is always bounded (unlike in the earlier analyses), and perturbation evolution is coupled to the evolution of the homogeneous solution. The plastic shear strain perturbation solution is given by $\tilde{\gamma}^p = \Gamma(t;\lambda)\cos(y/\lambda)$, where cosine specifies the spatial variation with wavelength

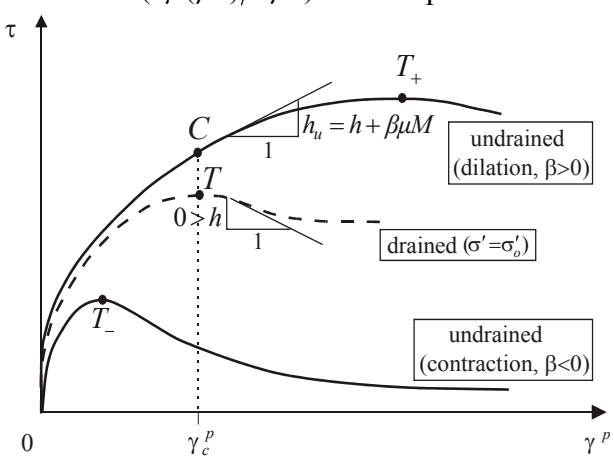

Fig. 1: Shear stress vs. inelastic shear strain.

$\lambda = \ell/(n\pi)$, n=1,2,… Amplitude $\Gamma(t;\lambda)$ can be expressed in terms of the plastic strain in the homogeneous solution, $\gamma^p(t)$,

$$\Gamma(\gamma^p;\lambda) = C(\mu/h_{u\lambda})\exp[-\Lambda^{-2}\int^{\gamma^p}(h/h_{u\lambda})d\gamma^p], \qquad h_{u\lambda} = h_u + \Lambda^{-2}S_\mu\tau \tag{3}$$

where C is a constant, and $\Lambda=\lambda/\ell_\kappa$ is the wavelength normalized with the characteristic pore fluid diffusion length $\ell_\kappa = \sqrt{\kappa M\dot{\gamma}^{-1}}$. For typical laboratory tests on sands (d~1 mm, ℓ~10 cm, ℓ_κ~10 m at $\dot{\gamma} \sim 10^{-4}$ s^{-1}), wavelength bounds are: $\Lambda_{\min} = d/\ell_\kappa \sim 10^{-4}$ and $\Lambda_{\max} = \ell/(\pi\ell_\kappa) \sim 10^{-2}$. Thus, even seemingly negligible rate-sensitivity S_μ, alters perturbation solution dramatically as it is amplified by the large factor Λ^{-2} in the expression for modified modulus $h_{u\lambda}$ in Eqn. (3). Exponential perturbation growth takes place when moduli h and $h_{u\lambda}$ are of different sign, Eqn. (3). For *dilatant* materials, onset of exponential perturbation growth takes place at the state (C), Fig. 1, where modulus h changes the sign to negative [Rice, 1975]. The extent of stable deformation past this state (before the perturbation has grown by at least one order of magnitude) is estimated from Eqn. (3) with Λ<<1: $\Delta\gamma^p_{stable} \sim \sqrt{-(S_\mu\tau(dh/d\gamma^p)^{-1})_{(C)}}$. The experimentally observed strain range past the (C)-state in undrained biaxial compression of dense sand specimens [Han and Vardoulakis, 1991] before the inception of shear banding was about 2%, which is of the same order of magnitude as the range (0.5%) estimated from the above formulae. For *contractive* materials, instability takes place at the state $h_{u\lambda} = 0$, Eqn. (3). (When strain rate-sensitivity is neglected [Vardoulakis, 1985], $h_{u\lambda} = h_u$, and instability is predicted at the state (T_-) of the peak shear stress, Fig. 1). Instability condition $h_{u\lambda} = 0$ for rate-hardening material (S_μ>0) will be first satisfied (if ever) for the perturbation with the maximum wavelength defined by the specimen dimensions. Consequently, contractive materials are susceptible to diffuse (i.e. on the specimen lengthscale) instabilities. Estimates from the data from the undrained biaxial compression of loose (contractive) sand specimens [Han and Vardoulakis, 1991] suggest that $h_{u\lambda}$ is always positive and, therefore, no instability is predicted. Naturally, no instabilities have been observed in these tests.

CONCLUSIONS: "Coupled" perturbation theory under the provisions of bounded perturbation wavelength is used for stability analysis of undrained 1D shear. Analysis suggests that material strain-rate sensitivity amplified by the pore fluid diffusion is the primary mechanism governing stability. Predictions are consistent with the experimentally observed undrained stability trends.

REFERENCES:

Han, C. and Vardoulakis, I.G. 1991, "Plane-strain compression experiments on water-saturated fine-grained sand", Geotechnique, **41**, 49.

Rice, J.R. 1975, "On stability of dilatant hardening for saturated rock masses", J. Geophysical Research, **80**, 1531.

Vardoulakis, I. 1985, "Stability and bifurcation of undrained, plane rectilinear deformations on water-saturated granular soils", Int. J. Num. Anal. Meth. Geomechanics, **9**, 399.

Vardoulakis, I. 1996, "Deformation of water-saturated sand: II.", Geotechnique **46**, 457.