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Localized Failure in Brittle Rock

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ABSTRACT. *Failure angles predicted by Rudnicki and Rice (75) are consistent with those observed in true triaxial tests on Westerly granite (Haimson and Chang 00). Predictions incorporate a variation of inelastic consitutive parameters (sum of a friction coefficient and dilatancy factor) with mean normal stress inferred from axisymmetric compression tests. The band angle tends to decrease as the deviatoric stress state varies from axisymmetric extension to axisymmetric compression, in contrast to the Coulomb condition which predicts no variation. The predictions underestimate the observed band angles, but they capture a roughly linear increase in band angle with increasing mean stress for fixed values of the least compressive stress and a decrease with increasing values of the least compressive stress.*

KEY WORDS: *localization, failure, fault angle, dilatancy, compaction, faulting*

1. Introduction

A prominent feature of localized brittle failure is the orientation of the failure plane or zone of localized deformation. Although failure is not always confined to a planar zone and sometimes occurs at multiple orientations, it is generally a relatively easy feature to identify in experiments. Understanding the relation of the orientation of the failure plane to the stress state and material properties is critical for a variety of applications. For example, in the field, the orientation of fractures is often used to constrain the principal stress directions that can only be inferred approximately from tectonic conditions. Furthermore fractures and failure zones significantly influence, or even control, the hydrologic properties of the rock mass. Consequently, understanding the factors that control their formation is critical to any applications that involve fluid injection or withdrawal.

Although the vast majority of testing on brittle rock has been done in axisymmetric configurations, applications and field conditions are seldom purely axisymmetric. Consequently, there remain questions about the role of the intermediate principal stress (σ_2) in failure. Haimson (06) and Mogi (07)) have discussed the role of σ_2 on the stress at failure. This paper focuses on the role of σ_2 on fault orientation because of its relatively easy observation in experiments. This focus on orientation also avoids issues about whether failure coincides with localization (faulting) and whether it occurs slightly before, at or after the peak of the stress strain curve (Bésuelle and Rudnicki 04). Classic results of Mogi (67) and data from recent true triaxial tests by Haimson and Chang (00) are compared with the Coulomb condition and predictions from a bifurcation approach to failure (Rudnicki and Rice 75).

2. Background

Figure 1 plots failure angles for Westerly granite in axisymmetric tests from Mogi (67) and from Haimson and Chang (00) against the mean normal stress

$$\sigma = -(\sigma_1 + \sigma_2 + \sigma_3) \tag{1}$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the principal stresses, taken to be positive in tension. The angle is defined as that between the normal to the failure plane and the direction of the most compressive principal stress (σ_3). Mogi (67) reports observations from both extension tests $(\sigma_2 = \sigma_3)$ and compression tests $(\sigma_2 = \sigma_1)$ conducted with axisymmetric apparatus. Data from Haimson and Chang (00) were obtained from a true triaxial machine. Compression data from both sources are consistent. As noted by Mogi (67) and others, failure angles are observed to be greater in axisymmetric extension than in axisymmetric compression. Also shown are least squares fits to the extension and compression data with slopes of –0.015 (extension) and –0.041

(compression) in degrees/MPa. These indicate a decrease in band angle with increasing mean stress that is more prominent for the compression data.

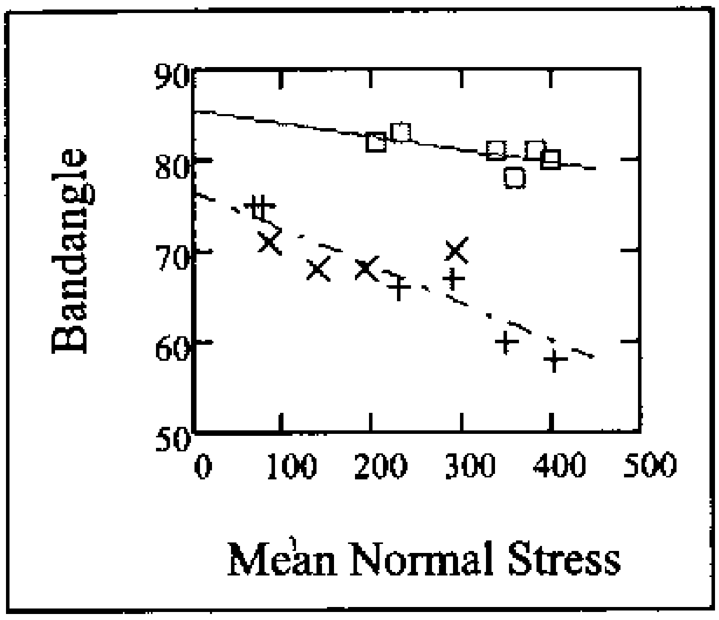

Figure 1. *Band angle vs. mean normal stress (MPa) from Mogi (67) x's compression, y's extension, and Haimson and Chang (00), +'s compression*

The classic Mohr-Coulomb failure criterion (Jaeger and Cook 69; Mogi 07) has the form

$$q = F_{MC}(p) \tag{2}$$

where $q = (\sigma_1 - \sigma_3)/2$ and $p = -(\sigma_1 + \sigma_3)/2$ are half the difference and sum of the greatest and least principal stresses. (The minus sign in the expression for p results from taking tensile stresses as positive). The criterion does not depend at all on the intermediate principal stress (σ_2) but Mogi (67) argued that the differences in band angle (and failure stress) between axisymmetric compression and extension shown in Figure 1 indicate such a dependence.

A complication with Mogi's (67) interpretation is that the intermediate principal stress σ_2 affects the plot in Figure 1 in two ways: it enters the mean normal stress, σ (1), and is different for compression and extension. But Figure 1 shows that even for similar values of σ, the band angle is significantly larger in extension than compression. The dependence of the abscissa on σ_2 could be eliminated by using p on the horizontal axis but the plot would be similar. In particular, different values of the band angle occur for extension and compression for roughly the same values of p.

Mogi's (67) observation that the orientation of the failure plane (and failure stress) depends on all three principal stresses suggests that, at least from the theoretical point of view, it is useful to express the dependence in terms of the stress invariants rather than the principal stresses themselves. For an isotropic material dependence on the principal stresses, without regard to their orientation, is equivalent to dependence on the three stress invariants. Although few rocks are, of course, precisely isotropic, this is a reasonably good assumption for some and often made in the interests of simplicity.

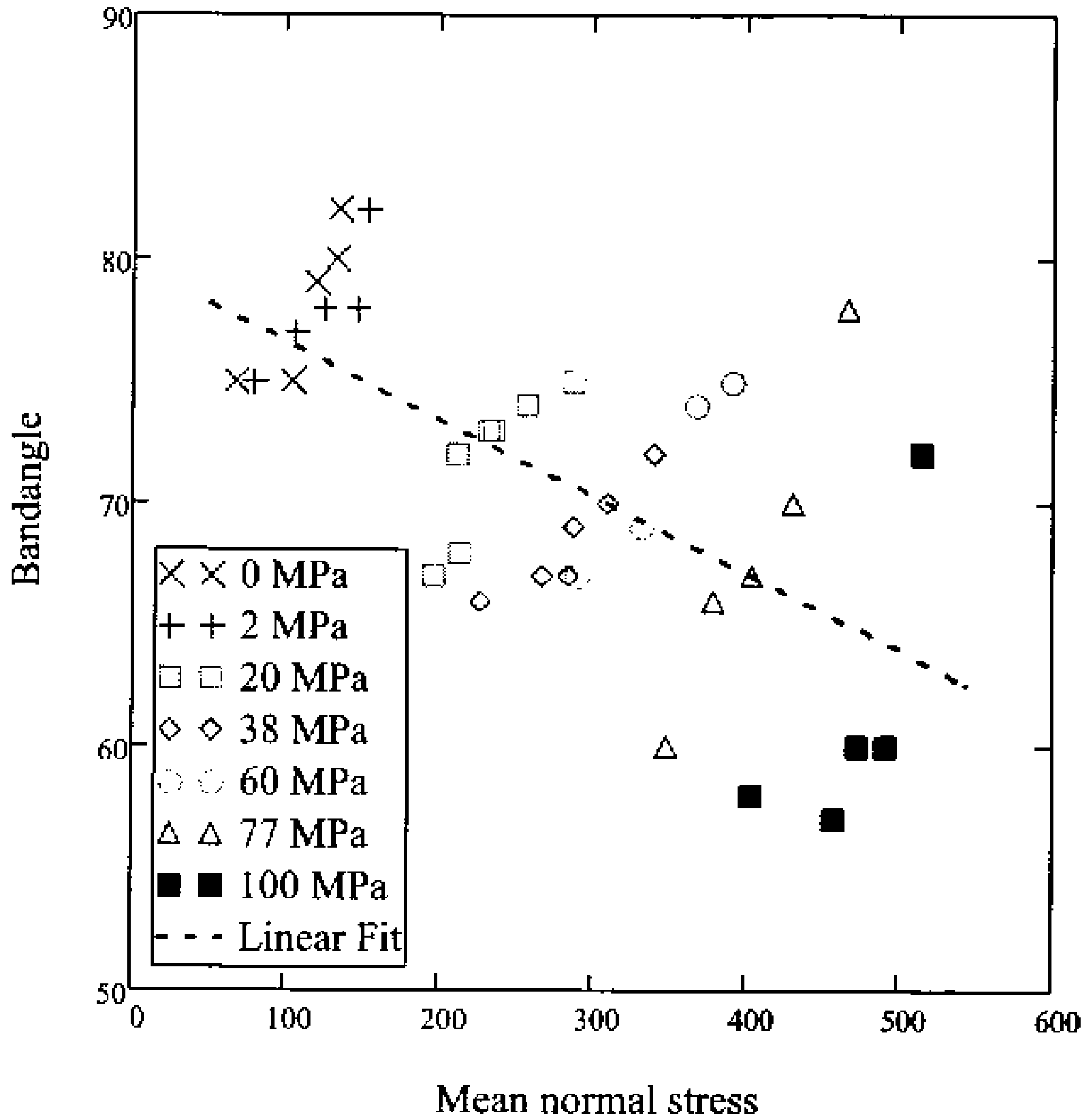

Figure 2. *Band angle vs. mean normal stress (MPa) for Westerly granite from true triaxial data from Haimson and Chang (00) and a linear fit through the data*

The invariants can be chosen in different ways but a convenient choice is the mean normal compressive (or octahedral) stress (1), the Mises equivalent stress

$$\tau = \sqrt{(1/6)\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right\}} \tag{3}$$

and the Lode angle

$$\theta = \frac{1}{3}\arcsin\left(-\frac{\sqrt{27}J_3}{2\tau^3}\right) \tag{4}$$

where

$$J_3 = \det(s_{ij}) \tag{5}$$

is the third invariant of the deviatoric stress $s_{ij} = \sigma_{ij} - (\sigma_{kk}/3)\delta_{ij}$. In the space of principal stresses, θ is the angle in planes that are normal to the hydrostat

$\sigma_1 = \sigma_2 = \sigma_3$ and for which $\sigma =$ constant. The angle (4) is zero for deviatoric pure shear ($\sigma_1 = -\sigma_3$), and varies between $-\pi/6\,(-30^\circ)$ for axisymmetric extension ($\sigma_2 = \sigma_3$) and $\pi/6\,(30^\circ)$ for axisymmetric compression ($\sigma_2 = \sigma_1$). For an isotropic material the remaining five sectors (of 60°) are given by symmetry. The Mises equivalent stress τ is equal to $\sqrt{3/2}$ the octahedral shear stress used by Mogi (67) and Haimson and Chang (00).

3. True triaxial data

Figure 2 plots the band angle from the true triaxial data of Haimson and Chang (00 and personal communication, 2006) against the mean normal stress σ. Different symbols denote the constant values of the minimum compressive stress, $|\sigma_1|$, given in the legend. The band angle increases roughly linearly with σ for fixed values of $|\sigma_1|$, but decreases with increasing values of $|\sigma_1|$.

The data in Figure 2 are not, however, at fixed values of the Lode angle (4). Figure 3 plots the same data against $N = 2\sin\theta$. N, like θ, is a deviatoric stress state parameter. N varies between -1 and $+1$ and attains the limits in axisymmetric extension ($N = -1$) and compression ($N = +1$). (Note that N as defined here is $\sqrt{3}$ the parameter labelled as N by Rudnicki and Rice (75). Their $N = s_2/\tau$, the intermediate principal value of the deviatoric stress divided by the Mises equivalent stress). Since the minimum value of N in the data is 0.385, the tests span only about 20% of the possible range of deviatoric stress states and do not contain points near the axisymmetric extension data of Mogi (67). (Other tests by Chang and Haimson (00) on amphibolite and by Oku et al. (07) on a sandstone span a greater range of N). Also shown is the best fit line through all the data.

4. Predictions of failure angle

4.1 *Mohr-Coulomb*

The most common description of the failure angle is the Coulomb condition (Jaeger and Cook, 69)

$$\theta_C = 45^\circ + (1/2)\arctan\mu_C \tag{6}$$

where μ_C is the Coulomb friction coefficient, often written as the tangent of the friction angle ϕ_C. For a generalized Mohr-Coulomb failure envelope of the form (2) the Coulomb friction coefficient is the slope $\mu_C = F'_{MC}(p)$, where the prime denotes the derivative with respect to the argument p.

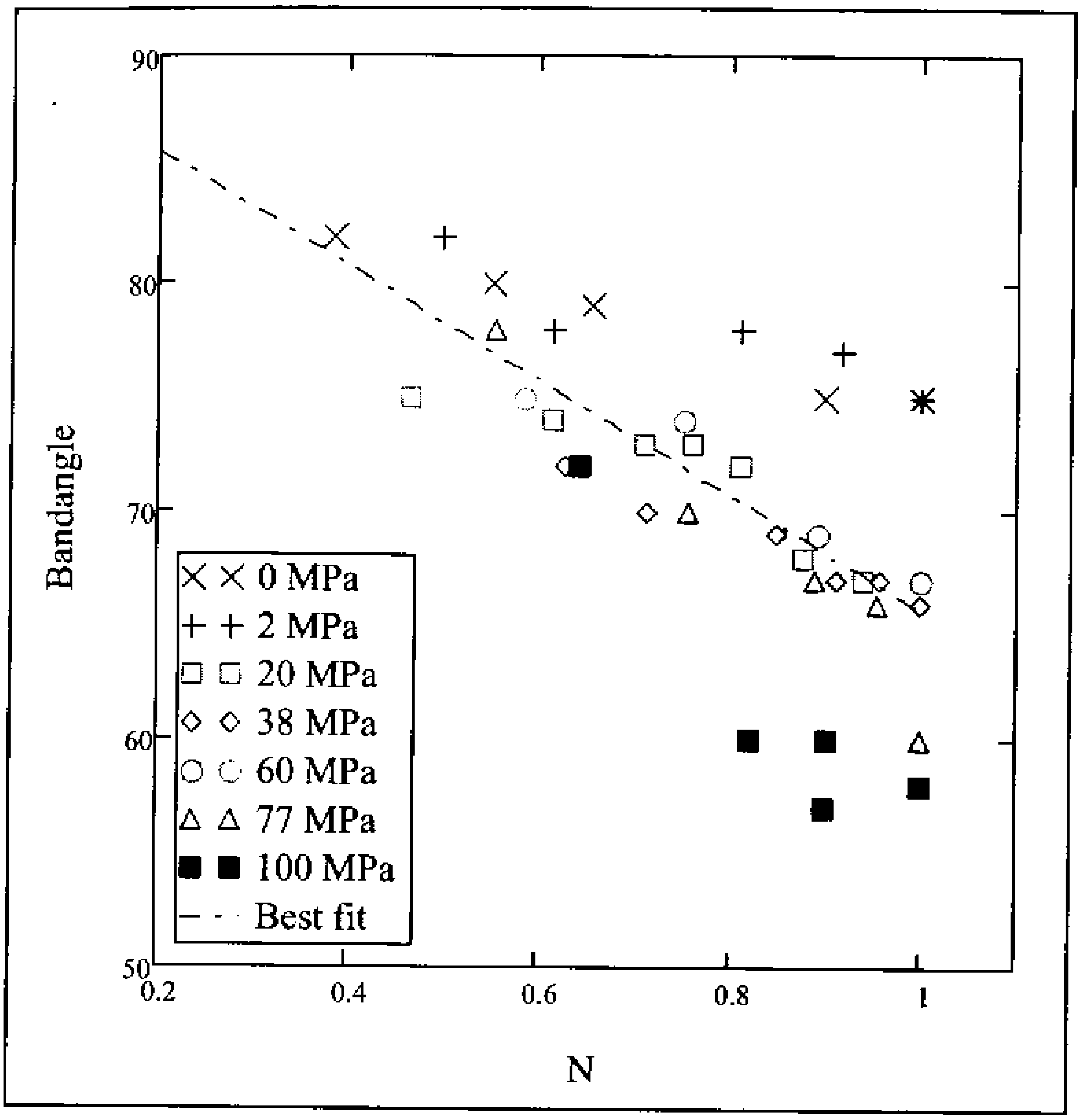

Figure 3. *Band angle vs.* $N = 2\sin\theta$ *for Westerly granite data from Haimson and Chang (2000)*

For typical values of μ_C, 0.6 to 0.75, θ_C varies from 60.5 to 81.9 degrees, which is comparable to the observed range. Because of its simplicity and rough consistency with data, the Coulomb condition is widely used and a reliable, albeit approximate, predictor of failure angle. But, as noted by Mogi (67) and others, the Coulomb condition does not account for the observed difference in the failure angles between axisymmetric compression and extension shown in Figure 1.

4.2 *Bifurcation Theory*

A prediction of failure angle also emerges from a theory of localized deformation as a bifurcation from homogeneous deformation (Rudnicki and Rice 75). In contrast to the Coulomb prediction, which depends on material properties only through μ_C, the predictions of bifurcation theory are sensitive to the description of material behaviour for homogenous deformation. Rudnicki and Rice (75) obtained results for a constitutive relation in which the strain increments are the sum of an elastic, isotropic portion and an inelastic portion. The yield surface and plastic potentials were of the form

$$\tau = f_{RR}(\sigma) \tag{7}$$

and

$$\tau = g_{RR}(\sigma) \tag{8}$$

Respectively. (Rudnicki and Rice (75) do not express the constitutive relation in terms of a yield surface and plastic potential but this interpretation is discussed by Issen and Rudnicki (01) and Rudnicki (04).) The yield surface is the boundary of stress states for which deformation is purely elastic and, in general, differs from the failure surface. Derivatives of the plastic potential with respect to stress components give the direction of inelastic strain increments. If f_{RR} and g_{RR} coincide, then the flow rule is said to be associated with the yield surface and the directions of inelastic strain increments are normal to the yield surface. Observations (and some theory) suggest that this is generally not the case of rocks, soils and other granular or frictional materials.

For the constitutive relation used by Rudnicki and Rice (75), Rudnicki and Olsson (98) expressed the angle between the band normal and the most compressive principal stress as

$$\theta_{RR} = (\pi / 4) + (1/2)\arcsin\alpha \tag{9}$$

where

$$\alpha = \frac{(2/3)(1+\nu)(\beta+\mu) - \sqrt{3}N(1-2\nu)}{\sqrt{4-N^2}} \tag{10}$$

In (10), ν is the isotropic elastic Poisson's ratio, $\mu = f'_{RR}(\sigma)$ is the slope of the yield surface, $\beta = g'_{RR}(\sigma)$ is a dilatancy factor, equal to the ratio of an inelastic increment of volume strain (positive for dilation) to an inelastic increment of shear strain. As already noted, the parameter N describes the deviatoric stress state and is related to the angle θ (4) by $N = 2\sin\theta$ (Recall that N here is equal to $\sqrt{3}$ times the value used by Rudnicki and Rice (75) and Rudnicki and Olsson (98)). Although the yield surface (7) and plastic potential (8) depend only on τ and σ, and not at all on J_3, dependence of the band angle on J_3, via N and θ, enters through the bifurcation analysis.

The expression (9) applies for $-1 \le \alpha \le 1$. For $\alpha > 1$, $\theta = 90°$ corresponding to a dilation band and for $\alpha < -1$, $\theta = 0°$, corresponding to a compaction band. Compaction bands are predicted to be possible for inelastically compacting materials, $\beta < 0$, or for stress states on the cap portion of a yield surface, $\mu < 0$ (Holcomb et al., 07). (Perrin and Leblond (93) and Ottosen and Runesson (91) have noted a correction to the range of applicability of the expression given by Rudnicki and Rice (75) that is equivalent to (9)).

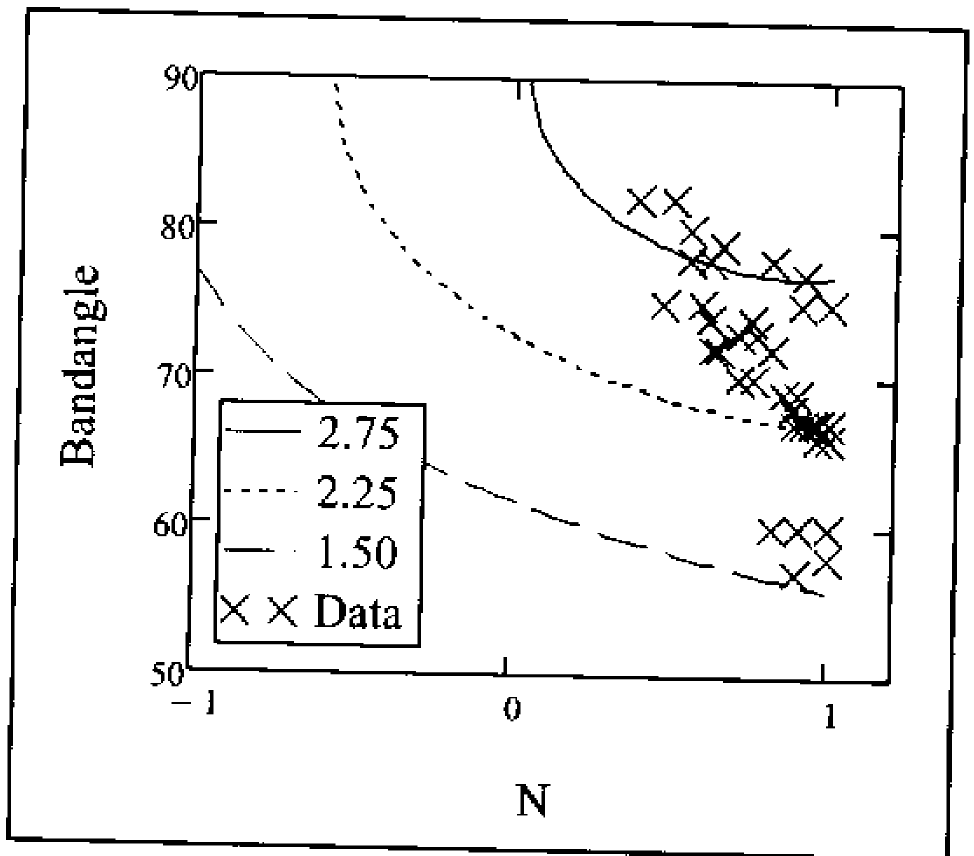

Figure 4. *Band angle vs.* $N = 2\sin\theta$ *from (9) for fixed values of* $\beta + \mu$, *1.50, 2.25 and 2.75,* $\nu = 0.1$ *and Westerly granite data of Haimson and Chang (00)*

5. Comparison with data

Figure 4 plots the band angle predicted by (9) for $\nu = 0.1$ and fixed values of $\beta + \mu$ that roughly span the data. The values of $\beta + \mu$ are on the large side but not implausible. The value of ν is less than typical for Westerly granite (0.2), but larger values yield curves that are flatter near the right end of the graph. Although the data are clustered at the right end of the graph, (9) predicts the occurrence of dilation bands for the larger two values of $\beta + \mu$ as the deviatoric stress state moves to the left, toward axisymmetric extension.

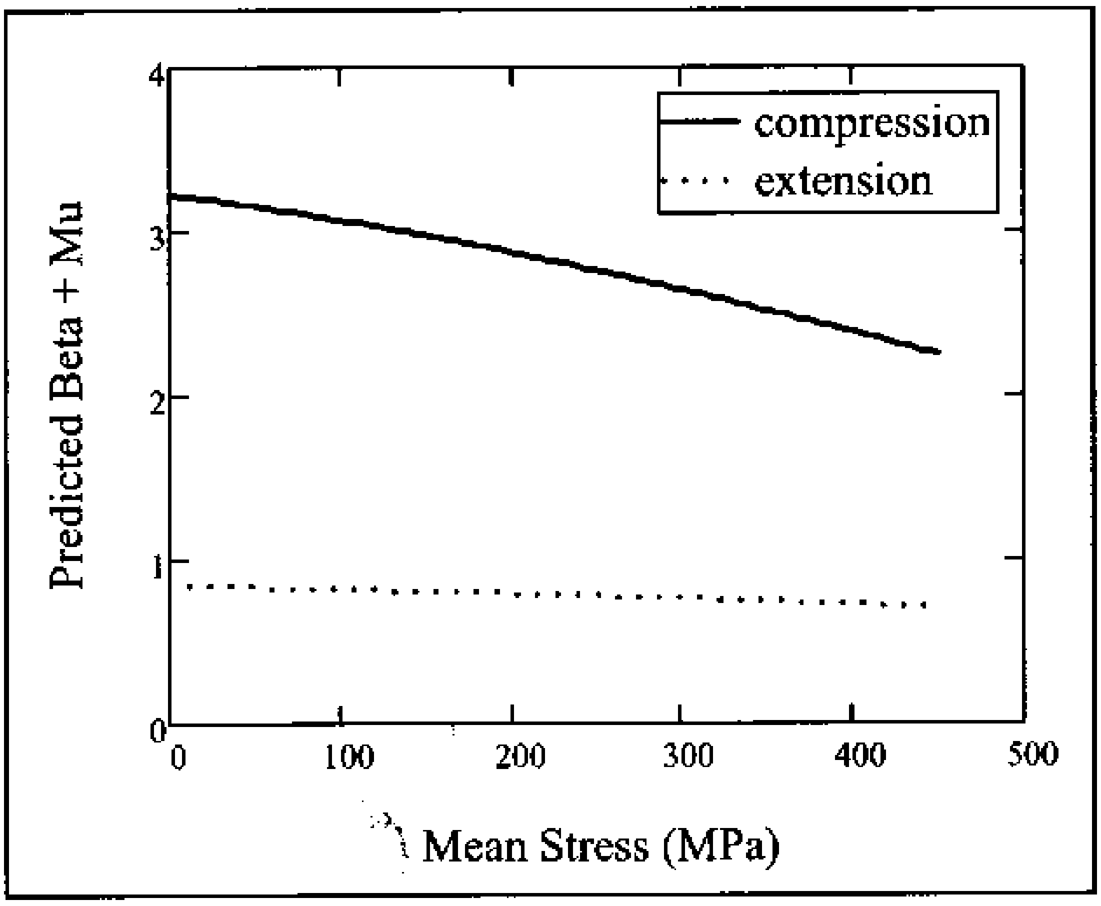

Figure 5. *Variation of $\beta + \mu$ predicted from (9) and linear fit to variation of band angle with mean stress in axisymmetric compression and extension shown in Figure 1. Poisson's ratio is taken as* $\nu = 0.2$

For a fixed deviatoric stress state (N) in (9), the band angle depends only on $\beta+\mu$ (assuming ν is also constant). This sum is, in general, not constant but may depend on the mean normal stress σ though there are few precise measurements of this dependence. Nevertheless, the dependence of $\beta+\mu$ on σ can be inferred by using (9) and (10) with the roughly linear dependence of the band angle on σ for axisymmetric compression and extension (Figure 1). Since the slopes of the lines are different for extension and compression, the actual dependence on N and σ must be more complex, but a simple variation with σ that may be acceptable for data in a limited range deviatoric stress states near axisymmetric compression. Figure 5 shows the variation of $\beta+\mu$, predicted from (9) with $\nu = 0.2$ using the linear fits to the variation of band angle with mean stress in axisymmetric compression and extension shown in Figure 1. The decrease of β and μ with increasing mean compressive stress in Figure 5 is consistent with observation. The values inferred from axisymmetric extension are much less than those for compression and decrease less rapidly with mean stress.

Figure 6 shows the band angles predicted by using the dependence of $\beta+\mu$ on mean normal stress for axisymmetric compression shown in Figure 5. Predictions are shown for the same values of N and mean normal stresses as in the tests of Haimson and Chang (00). Predictions based on the values $\beta+\mu$ inferred from the axisymmetric extension data (Figure 5) fall well below the observed values and, consequently, are not shown. Although the predictions generally lie slightly below the data, they do capture both the general downward trend of the band angles with increasing mean stress and the roughly linear increase of band angle for fixed values of the least compressive stress.

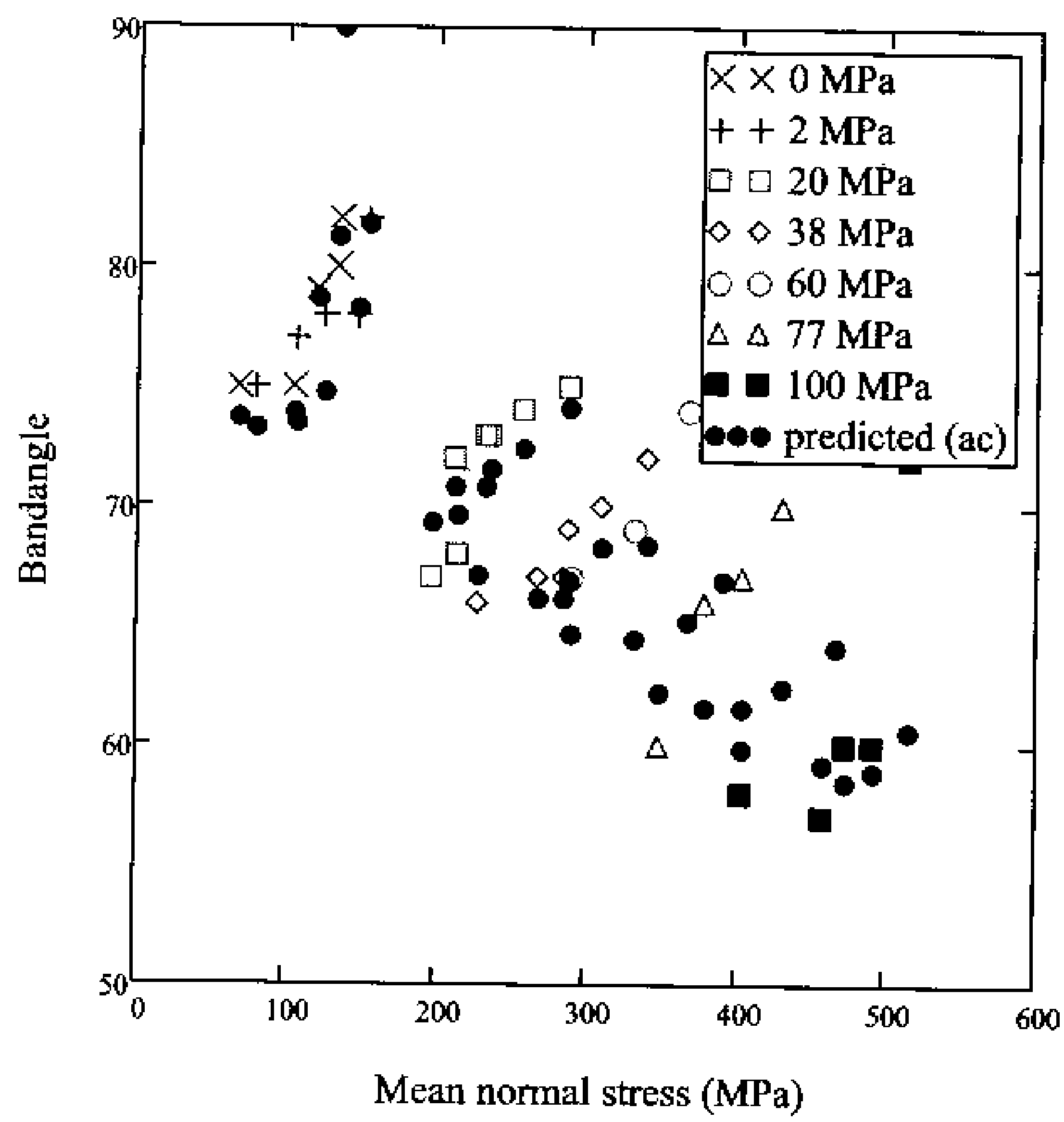

Figure 6. *Comparison of observed band angles reported by Haimson and Chang (2000) with those predicted (solid circles) for the same mean normal stresses and* N *values using the mean stress dependence of* $\beta + \mu$ *inferred from the axisymmetric compression data and shown in Figure 5*

Figure 7 plots the same data and predictions against N. Although the predictions for the band angles generally lie below the data, they do capture the roughly linear decrease of band angle with N. The slopes of the best fit lines to the data and predictions are -25 and -22.4, respectively.

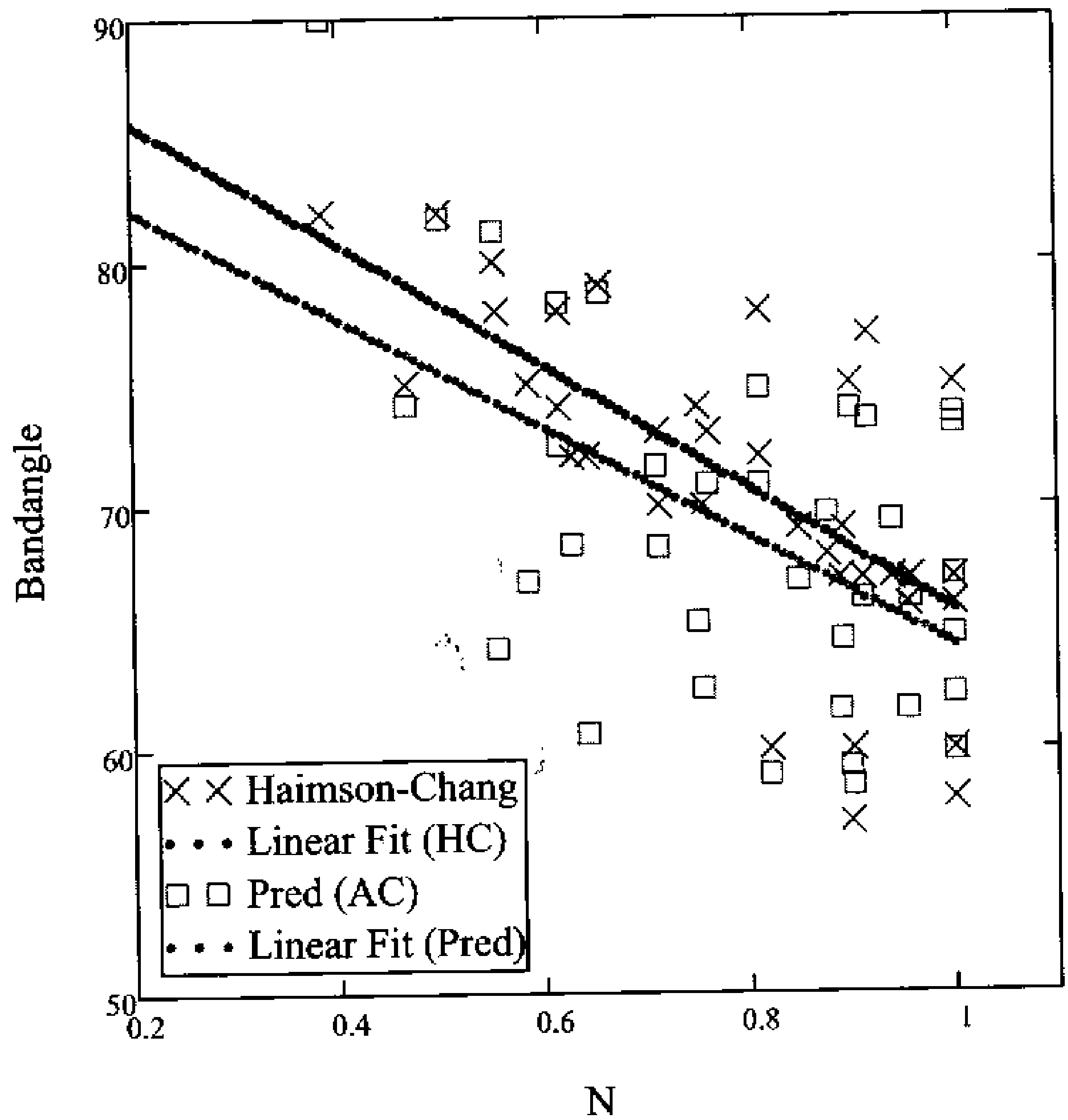

Figure 7. *As with Figure 6 but plotted against the deviatoric stress state parameter N (data values at the same minimum compressive stress are not distinguished). Also shown are best fit lines to the data and predictions*

Figure 8 compares the data with predicted curves of the band angle against N for constant values of the mean normal stress (100, 200, 300 and 400 MPa) that span those in the tests. These curves correspond to values of $\beta+\mu$ of 3.1, 2.9, 2.6 and 2.4. Although the curves lie within the spread of the data for the tested range of N, a few of the observed values (at the highest values of least compressive stress) lie below the curve for 400 MPa and quite a few of the observed values lie above the curve for 100 MPa. The band angles for these latter points are nearly as large as those for axisymmetric extension although they are for N values greater than about 0.38. The curves rise steeply to the left of the data (N values closer to extension) but have trends similar to the data (even though the data are not for fixed mean normal stress). All four of the curves predict dilation bands (band angle of 90°) as the values of N become negative and approach axisymmetric extension. Mogi (67) reports shear bands at angles lower than this for tests in which the mean stresses (at failure) ranged from about 200 to 400 MPa.

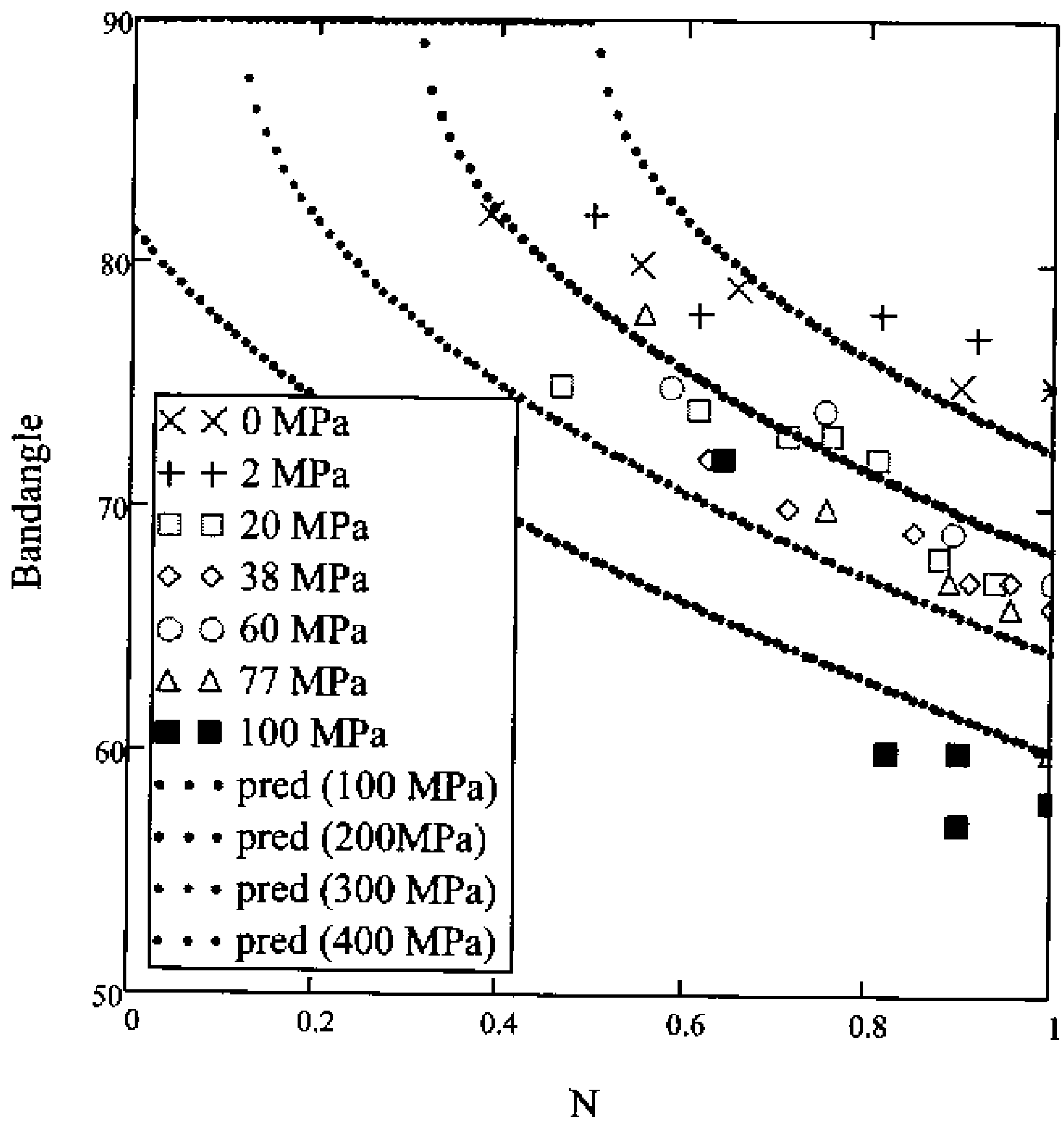

Figure 8. *Comparison of band angles of Haimson and Chang (2000) with predictions of the variation of band angle with N at four constant values of the mean normal stress in MPa: 100, 200, 300 and 400. Predictions assume that the variation of $\beta + \mu$ with mean normal stress is determined by the nearly linear decrease of the band angle in axisymmetric compression (Figure 1)*

6. Discussion

Although the Mohr-Coulomb condition (6) gives a reasonable and simple prediction of failure angle, it is at odds with the different band angles observed in axisymmetric extension and compression (even at the same values of σ or p). More generally, it predicts that the band angle does not depend at all on σ_2. The result from Rudnicki and Rice (75) does predict a decrease of band angle, at least for constant $\beta + \mu$, the sum of a friction coefficient and a dilatancy factor, as the deviatoric stress state varies from axisymmetric extension to axisymmetric compression. This is consistent with the axisymmetric compression and extension observations. Using a variation of $\beta + \mu$ with mean normal stress inferred from observations of band angle in axisymmetric compression, the prediction of Rudnicki and Rice (75) also appears to be consistent with band angles observed in true triaxial tests of Haimson and Chang (00). The predictions tend to underestimate the band angle but are consistent with the observed trends with both mean normal stress and deviatoric stress state. In particular, the predictions capture the increase in band angle with mean stress for fixed values of the least compressive stress and the decrease with mean stress for increasing values of the least compressive stress.

The comparison between observations and predictions is complicated by the limited range of deviatoric stress states spanned by the data. Whether the predictions would be consistent with observations for the wider range of deviatoric stress states, as tested in Chang and Haimson (00) and Oku et al. (07), is not clear. The predictions do indicate a more rapid increase of band angle as the deviatoric stress state moves toward axisymmetric extension. For the range of mean normal stress spanned by the data, and the inferred values of $\beta+\mu$, dilation bands (band angles of 90^0) are predicted for axisymmetric extension. Mogi (67) reported, however, band angles ranging from 78 to 83 degrees, similar to the largest values reported by Haimson and Chang (00) at deviatoric stress states much closer to axisymmetric compression (values of N greater than 0.38, compared with $N=-1$ for axisymmetric extension). Further work is needed to determine whether this discrepancy is an anomaly of the extension test, a consequence of the overly idealized constitutive relation used by Rudnicki and Rice (75) or other factors.

The variation of $\beta+\mu$ with mean normal stress is based on the decrease in band angle with mean normal stress observed in axisymmetric compression (for the combined data of Mogi (67) and Haimson and Chang (00)). This dependence is acceptably idealized as linear, but the values and trend differ significantly for extension and compression. The variation inferred from the axisymmetric extension data significantly underpredicts band angles for the true triaxial observations. Whether this is, again, a peculiarity of the extension test or an indication of a dependence of $\beta+\mu$ on θ is not clear. Such a dependence is not, however, consistent with the constitutive formulation of Rudnicki and Rice (75).

As noted previously, the bifurcation approach strongly depends on details of the constitutive relation used to describe homogeneous deformation. In particular, Rudnicki and Rice (75) assume a form that depends on stress only through τ and σ. Molenkamp (85) compared the bifurcation predictions based on various combinations of plastic potential and yield surfaces with the Mises, Mohr-Coulomb and Lade and Duncan (75) shapes in deviatoric planes. He found significant differences between predictions for the onset of localization and the orientation of the failure plane. A variety of more elaborate constitutive models have been proposed for frictional materials (see, e.g., Borja et al., (03); Jiang and Pietruszczak (88)) but there appears to be little systematic study, such as that of Molenkamp (85), of the differences between them with respect to onset of localization and orientation of the failure plane.

The prediction for the band angle, (9) with (10), can be generalized for a plastic potential and yield surface depending on all three invariants: The yield surface is given by

$$F(\tau,\sigma,\theta)=0 \tag{11}$$

and the plastic potential by

$$G(\tau, \sigma, \theta) = 0 \tag{12}$$

Normality (or associated flow) in the deviatoric plane requires $F_\tau = G_\tau$ and $F_\theta = G_\theta$, where the subscript indicates partial differentiation with respect to that argument. Although data for frictional materials indicate that normality is not the case in planes containing the hydrostatic axis, there is little reason not to assume normality in the deviatoric plane. A consequence is that F and G can differ only by a function of the mean stress σ (a condition that is not met by all the combinations considered by Molenkamp (85)). Then the band angle is given again by (9) with α replaced by

$$\alpha = \left\{(1+\nu)\cos\psi\left(F_\sigma + G_\sigma\right)/G_\tau - (1-2\nu)\sin(\psi+\theta)/\sqrt{3}\right\}/\cos(\psi+\theta) \tag{13}$$

where

$$\tan\psi = (G_\theta/\tau)/G_\tau \tag{14}$$

Ottosen and Runesson (91) have derived an equivalent expression. They also show that when the yield surface and plastic potential coincide and have a Mohr-Columb form, then $\alpha = \sin\phi_C$ and (9) reduces to (6) with $\mu_C = \tan\phi_C$ Thus, although the Mohr-Coulomb and bifurcation predictions have been contrasted here, they coincide for certain cases. In addition, the type of comparison reported here could be done for predictions employing plastic potentials and yield surfaces that depend on all three stress invariants.

7. Conclusions

The prediction of Rudnicki and Rice (75) for the band angle at failure is consistent with true triaxial observations on Westerly granite from Haimson and Chang (00). It is, however, necessary to include variation of a constitutive parameters (sum of friction coefficient and dilatancy factor, $\beta + \mu$) with mean normal stress inferred from axisymmetric compression tests. Using the variation inferred from axisymmetric extension yields predictions for band angle that are significantly lower than observations. Although there is reasonable agreement between predictions and observations, the latter span only about one-third of possible range in deviatoric stress states. Additional true triaxial data are essential for obtaining better understanding of appropriate constitutive relations and more precise predictions of the variation of failure angle (band angle) with mean stress and deviatoric stress state. At least from the theoretical point-of-view, it would be advantageous to collect data for several values of N at different fixed values of the mean stresses or for different values of mean stress at fixed values of N rather than at fixed values of the least compressive stress. Although the Rudnicki and Rice (75) prediction can be generalized for more complex constitutive relations depending on all three stress invariants, it remains unclear whether using these would significantly improve the agreement of predictions with data or whether the remaining discrepancies are due to other factors, such as anisotropy.

8. Acknowledgments

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