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Physical Models of Earthquake Instability and Precursory Processes

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Abstract—This paper selectively reviews physical models of earthquake instability. In these models, instability arises as a result of interaction of a fault constitutive relation with deformation of the surrounding material that occurs in response to remote tectonic loading. In contrast to kinematic models in which the fault slip is imposed, it is calculated in physical models and, consequently, these models are essential for understanding precursory processes. Some kind of 'weakening' behavior for the fault constitutive relation is required to produce an instability analogous to an earthquake. Two commonly employed idealizations discussed here are rate-independent slip weakening and rate/state-dependent friction. When these constitutive models are employed on surfaces embedded in elastic half-spaces or layers, possibly coupled to a viscoelastic substrate, the results are capable of simulating realistically some aspects of earthquake occurrence. Common to all models is the prediction that earthquake instability is preceded by precursory slip which produces a departure of surface strain-rate from the background level. Near the epicenter of a moderate to large earthquake, the magnitude of this departure appears to be well within the range of current geodetic measurement accuracy, and its duration is of the order of months to years. However, details depend on a variety of factors, including the modelling of the constitutive relation near peak stress, coupling of elastic crust to the asthenosphere, and coupling of deformation with pore fluid diffusion.

Key words: Earthquakes, faulting, instability, prediction, friction.

Introduction

If data were more numerous and earthquakes more frequent, it is, perhaps, possible that earthquakes could be predicted purely on an empirical basis. This is, however, not the case: earthquakes that are large enough to possibly exhibit premonitory effects, and that occur in heavily instrumented areas, are infrequent. Consequently, purely empirical methods of earthquake prediction, no matter how promising, are viewed with skepticism, until there is a good understanding of the physical basis for the method. Physical models can help to understand the conditions that give rise to earthquake instability and the processes that precede them. Moreover, they are the only means of exploring the consequences of particular aspects of the earthquake process in isolation from the complexities of competing mechanisms occurring in the earth. Also, physical models can aid in the strategic deployment of

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instruments and in the interpretation of observations. Because there is no direct source of information about constitutive properties of materials *in situ*, information about material behavior mainly comes from laboratory observations at size and time scales that are vastly different from those in the field. Physical models provide the only sensible basis for extrapolating laboratory observation to field conditions.

Physical models are those that incorporate plausible constitutive relations consistent with laboratory observations and compatible with the laws of mechanics. In these models, an instability, which can be interpreted as an earthquake, arises because of the geometry of the fault or the nature of the constitutive relation. In quasi-static models, in which inertia is not included explicitly, the instability generally takes the form that the slip rate becomes unbounded on a portion of the fault. In models which do include inertia, the slip rate does not become unbounded but there is a transition from a situation in which inertia is negligible to one in which it is not. Physical models can be distinguished from kinematic models in which the distribution of subsurface slip is assumed or inferred, rather than obtained as part of the problem solution. Kinematic models are particularly useful for interpreting data and have been extensively used to infer distributions of subsurface slip that are consistent with surface deformation measurements. However, these models are not able to predict whether further slip will occur and thus, are limited in their capability to illuminate the physical mechanisms and geometric conditions that lead to instability.

Physical models have increased greatly in complexity and sophistication in the last decade. These advances have been motivated primarily by improvements in constitutive descriptions which are based on laboratory observations and in geodetic measurements. An indication of the progress in understanding is the current general agreement that some kind of weakening behavior is needed for instability; at a previous USGS workshop on fault mechanics (STUART *et al.*, 1977) this was a point of contention. Despite the progress, there is still much to be accomplished. Models are simplistic in terms of the geometric complexities of actual fault systems. There are still gaps in the parameter ranges explored by laboratory observations, and a general lack of understanding of how to extrapolate these observations to the field. Models have been limited, with a few exceptions, to two dimensions and straight, continuous faults. Even with these simplifications, investigation of relevant ranges of parameters has been limited by computational resources.

This review of physical models is neither comprehensive nor thorough. Instead, it concentrates on a few aspects of these models and, in parts, draws heavily on a recent more extensive review by RICE (1983). I begin by describing the constitutive relations that are typically employed in these models and the way in which they give rise to instabilities. Then, procedures for incorporating these relations into crustal scale fault models are described. After a brief outline of some of the results of these models, the paper concludes with a discussion of some of the limitations of these models, and suggestions for further research.

Constitutive Models

Most fault constitutive models relate the stress (traction) at a point on the fault surface to the relative displacement at a point on the fault surface. An implicit assumption in such models is that essentially all inelastic deformation is confined to the neighborhood of the fault surface. Although earthquake faults are generally localized features, there may be instances in which inelastic deformation is more broadly distributed prior to instability. In such cases, an inelastic constitutive relation for bulk deformation (e.g., RUDNICKI, 1977) is needed.

Because no ideal point measurements are possible, fault surface constitutive models are inferred from laboratory experiments in which slip occurs essentially simultaneously over the entire fault surface. As mentioned already, there is a consensus that some kind of weakening behavior is required for instability. Two common idealizations, to be discussed in more detail below, are the rate-independent slip weakening model and the rate- and state-dependent friction.

The slip weakening model is illustrated in Figure 1. The shear stress τ required for slip first increases to a peak τ_p, then decreases to a residual value τ_r after an amount of relative slip δ_0 (Figure 1b). Slip is generally accompanied by uplift or microcracking of the adjacent material (BARTON, 1976; TEUFEL, 1981; RALEIGH and MARONE, 1986) (Figure 1d). This dilatancy is typically small, corresponding to uplift equal to a few percent of the slip, but can be important if pore fluids are present (RUDNICKI and CHEN, 1986). The peak stress τ_p and the residual stress τ_r depend on the effective compressive stress on the fault $\sigma - p$ (where σ is the total compressive stress and p is the pore fluid pressure), the temperature, and, possibly, preparation of the surface, but are idealized as being independent of the rate of sliding (Figure 1c). The difference $\tau_p - \tau_r$ is thought to first increase with depth in the earth but then to decrease, reflecting a transition to more ductile conditions.

Slip weakening curves have been observed in laboratory shear tests (e.g., DIETERICH (1978), BARTON (1972, 1973)) and have been inferred from observations of postpeak deformation in axisymmetric compression tests on both initially intact and sawcut specimens (RICE, 1980; WONG, 1982). These models can be regarded as a generalization of the concept of static and kinetic friction: instead of an abrupt drop from the static to the kinetic value with the onset of slip, there is a transition reflected by the decrease of shear stress from peak to residual value. As will be discussed later, in some cases the area under the shear stress versus slip curve in excess of the residual stress τ_r can be identified as the energy that must be supplied to advance the fault edge. Note that this area vanishes in the limit of an abrupt drop from a static to a kinetic value of friction.

Although these models neglect rate-dependent effects to be discussed shortly and in the article by TULLIS (1988, this volume), they are consistent with more elaborate rate- and state-dependent models under conditions of uniformly accelerated motion (GU, 1984/85). The primary shortcoming of the rate-independent models is that they

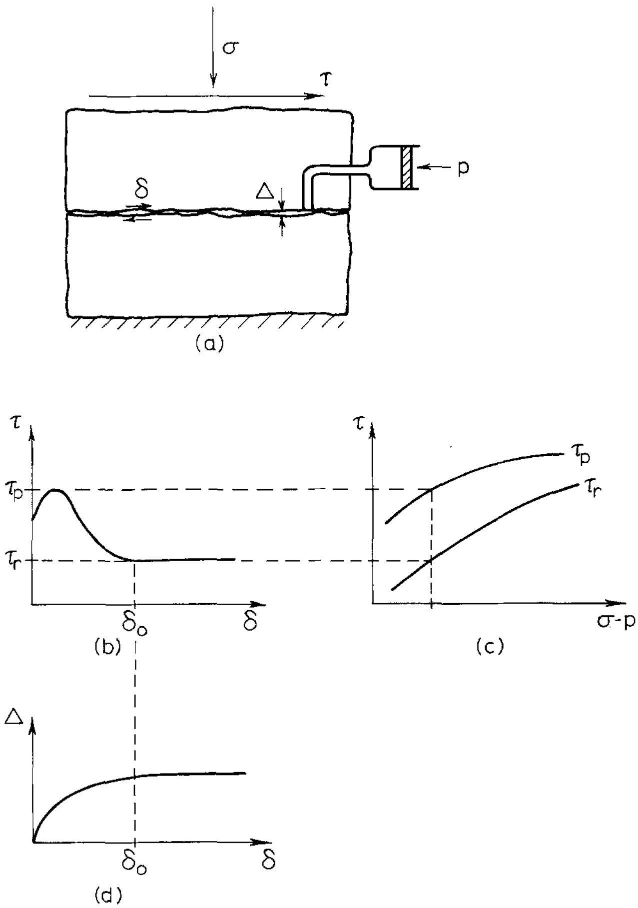

Figure 1

(a) Idealized experiment in which slip δ is assumed to occur simultaneously everywhere on the fault surface. τ is the shear stress, σ is the normal stress, Δ is the opening and p is the pore pressure. (b) Slip weakening friction relation. The shear stress increases with slip δ to a peak τ_p and then decreases to a residual value τ_r after an amount of slip δ_0. Sliding continues at $\tau = \tau_r$ for $\delta > \delta_0$. (c) Variation of τ_p and τ_r with effective normal stress $\sigma - p$. (d) Dilatancy Δ accompanying slip.

lack any mechanism for regaining strength after reduction to the residual level and, consequently, they are inadequate for simulating repeated events on the same fault segment. Nevertheless, the slip weakening models are a good approximation under many circumstances and, by comparison with the rate/state-dependent models, have an appealing simplicity.

Experiments initiated by DIETERICH (1978, 1979, 1981) (also, RUINA, 1983, 1984 and WEEKS and TULLIS, 1985) have demonstrated a rate-dependence of friction that is not modelled in the slip weakening idealizations. The canonical experiment is shown schematically in Figure 2. Sliding at constant normal stress at a velocity V_1

gives rise to a steady-state value of resisting shear stress $\tau_{ss}(V_1)$. The sliding velocity is now suddenly increased to a velocity V_2. The shear stress exhibits an instantaneous increase and then evolves to a new steady state value $\tau_{ss}(V_2)$ over a characteristic sliding distance, L. The new steady-state shear stress level may be greater or less (as shown in Figure 2) than $\tau_{ss}(V_1)$; that is, $\partial\tau_{ss}/\partial V$ may be positive or negative. Laboratory results at room temperature generally yield $\partial\tau_{ss}(V)/\partial V$ negative, but some results for adolescent surfaces and higher temperature yield positive values (e.g., see TSE and RICE, 1986 for a summary). As will be discussed shortly, the sign of $\partial\tau_{ss}(V)/\partial V$ is critical to the stability of small perturbations from steady-state sliding: instability is possible only if $\partial\tau_{ss}(V)\partial V < 0$ (velocity weakening).

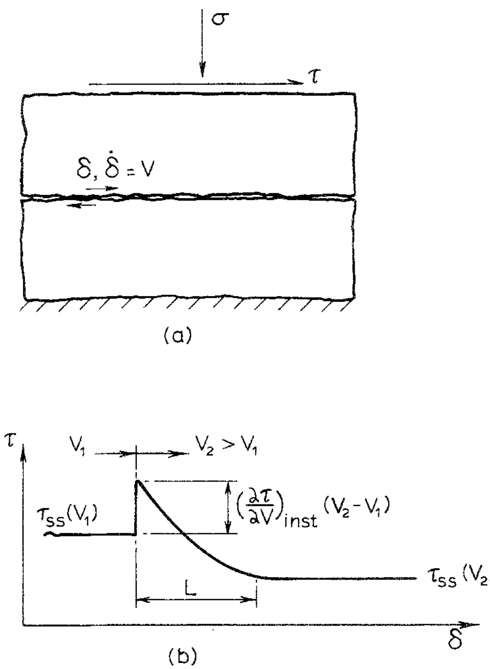

Figure 2
(a) Idealized experiment in which sliding occurs at a steady slip-rate $\dot{\delta} = V$. (b) Change in shear stress after a sudden increase in the sliding velocity from V_1 to V_2.

DIETERICH'S (1978, 1979) original modelling of these experimental observations has been reworked into a state variable description due to RUINA (1980, 1983):

$$\tau = F(V, \psi_i)$$

$$\frac{d\psi_i}{dt} = G(V, \psi_i) \qquad (i = 1, 2, \ldots, N) \tag{1}$$

where the ψ_i are a set of variables that describe the state of the surface. For steady sliding, the ψ_i are constant and the steady-state shear stress is

$$\tau_{ss} = F(V, \psi_i^{ss}). \tag{2}$$

The change in shear stress, in response to the sudden change of velocity illustrated in Figure 2, is assumed to occur at constant values of the state variables associated with steady sliding at velocity V_1. Thus, the evolution of the shear stress toward the new steady state level is associated with changes in the state variables. Equations (1) may depend additionally on the normal stress, and temperature or other environmental factors.

One state variable is usually adequate to describe experimental results, although sometimes two are needed. If one is sufficient RICE (1983) (also, RICE and TSE, 1986) has shown that the formulation can be recast into a form that eliminates specific reference to the state variable:

$$\frac{d\tau}{dt} = \left(\frac{\partial \tau}{\partial V}\right)_{\text{inst}} \frac{dV}{dt} - \frac{V}{L}[\tau - \tau_{ss}(V)] \tag{3}$$

where L is a length scale that is a measure of the slip distance needed to establish the new steady-state shear stress after a sudden change of velocity. Various minor modifications of this constitutive relation, and appropriate values of the parameters that enter it, have been widely discussed by a number of authors (also, see TULLIS, 1988).

An important feature of both the slip weakening and the rate/state-dependent relations is the appearance of a characteristic sliding distance. In the slip weakening relation, this is δ_0, the slip needed to degrade the strength from peak to residual value. In the rate/state-dependent model, this is the distance L needed to establish new steady-state shear stress. In experiments, these distances are typically of the order of 10^{-6} to 10^{-4} m. These tend to increase with surface roughness, but the scaling to values appropriate to crustal faults is largely uncertain. Values considerably larger than observed in the laboratory, of the order of 10^{-2} to 10^{-1} m, are compatible with inferred energy release rates for elastic-crack fault models with slip weakening (see LI 1987 for an extensive discussion; also, RICE 1983).

The extrapolation of laboratory values for the critical distance to field situations would be aided by better understanding of the micromechanical processes that are responsible for the phenomenology. Presumably, the rate-dependent version results, as DIETERICH (1978, 1979) has argued, from competing processes of asperity deformation and remating of asperity contacts, but the details are not understood. Current efforts at scaling have focused on determining roughness values for faults in the field. However, there must be some question whether the micromechanical processes of sliding on well-defined surfaces or gouge zones in laboratory specimens are the same as those in the field, where overall macroscopic slip may be accomplished by more complex processes of slip on *en echelon* surfaces linked by regions of tensile fractures.

Instability

Both the slip weakening and rate/state-dependent models give rise to instabilities that can be interpreted as earthquakes. These instabilities can be explained most easily in terms of a single degree-of-freedom spring-slider system (Figure 3a). The single degree of freedom system is most appropriate as a model for laboratory experiments in which slip occurs essentially simultaneously on the entire sliding surface, but can also be used as a crude model of an isolated fault (Figure 3b). In the latter case, the friction law is regarded as relating the average stress and slip on the fault. The effective stiffness of the surroundings, corresponding to the spring constant k, depends on the elastic constants and is inversely proportional to the fault length 2a. This idealization neglects effects due to the inhomogeneity of slip on the fault and the stress concentrations at the ends. Although the single degree-of-freedom system is a crude model of a fault, most of the results to date for the rate/state-dependent friction relation, because of its complexity, are for this model.

The response of the slip weakening model is illustrated by the well-known graphical construction originated by RICE (1979) and depicted in Figure 3. In Figure 3c the solution for the stress and the slip of the slider is given by point B: the intersection of the slip weakening τ versus δ curve with the line of slope $-k$, representing the unloading stiffness of the spring. Figure 3d shows the evolution of the system as the load-point displacement δ_L is increased. In Figure 3d the spring is sufficiently stiff (large enough k), that the slider velocity remains bounded as the stress traverses the peak of the τ versus δ curve. Although the rate of slider slip increases, no instability occurs. If, however, the spring is sufficiently compliant (small enough k), then an instability occurs when the slope of the τ versus δ curve becomes equal to $-k$ (point I in Figure 3e). At this point, the slip rate of the block $\dot{\delta}_B$ becomes unbounded for a finite load-point displacement rate, i.e., finite $\dot{\delta}_L$. Thus, instability results from the interaction of the slip weakening constitutive relation with the unloading stiffness of the spring. Figure 3f illustrates a general precursory feature of models based on this constitutive relation: the ratio of the increments of block slip δ_B to imposed slip δ_L becomes larger as the instability point is approached.

The effects of certain types of time-dependent behavior on precursory processes can also be illustrated in terms of this graphical construction. Figure 4a shows the delay in instability that occurs when the short-time response of the material surrounding the fault is stiffer than the long-time response. As the instability point, predicted on the basis of the long-time or relaxed response, is approached (point I in

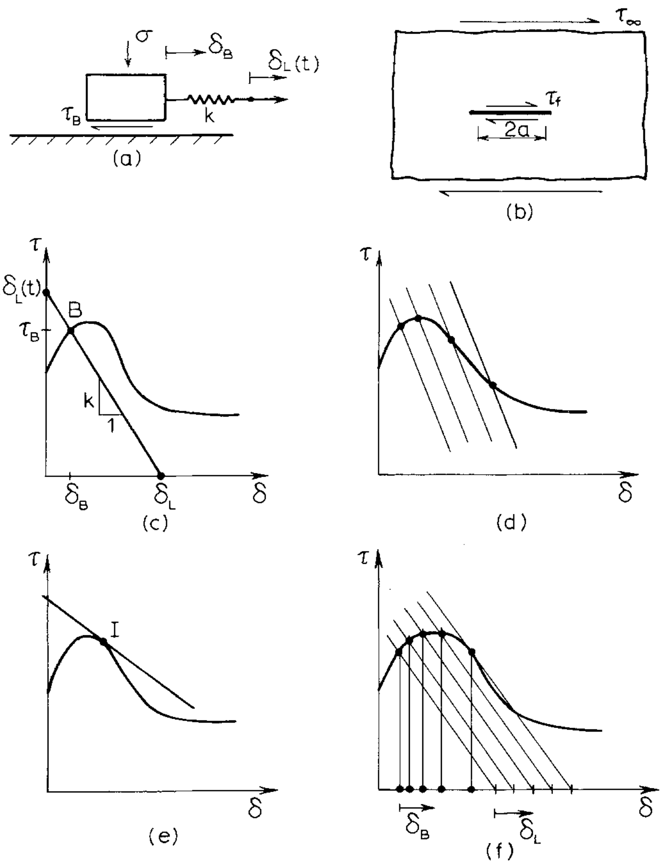

Figure 3

(a) One degree of freedom, spring-slider model for illustrating instability. The imposed displacement is $\delta_L(t)$, the spring stiffness is k, the displacement of the block is δ_B, the friction stress exerted on the block is τ_B and the constant normal stress is σ. (b) An isolated fault of length 2a, sustaining a uniform friction stress τ_f and loaded by a farfield stress τ_∞. (c) Graphical solution for the motion of the system in (a) with the slip weakening constitutive relation. Solution is given by point B. (d) Evolution of the system as the imposed displacement increases. The figure illustrates a case for which the spring is stiff enough that instability does not occur. (e) Instability corresponding to $d\delta_B/d\delta_L \rightarrow \infty$ occurs at point I when the stress versus slip curve becomes tangent to the spring unloading line. (f) Illustrates the acceleration of fault slip preceding instability. Figure shows increasing increments of δ_B corresponding to equal increments of δ_L.

Figure 4a), the acceleration of slip on the fault induces the stiffer short-time response. Thus, instability is delayed from I to I′ in Figure 4a. This type of time-dependent response can arise from mechanical coupling of deformation to pore fluid diffusion (Rice and Rudnicki, 1979; Rudnicki, 1979), from stress corrosion effects in the material surrounding the fault or from coupling of an elastic lithosphere to a viscoelastic substrate (Rice, 1980; Lehner, Li, and Rice, 1981; Li and Rice, 1983a,b). Of course, the time scales associated with these different mechanisms are different.

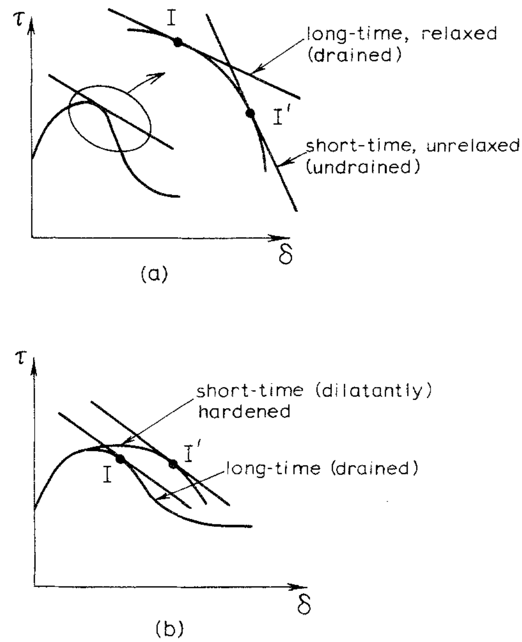

Figure 4

Delay of instability and enhancement of precursory slip in the slip-weakening model by time-dependent effects. (a) Elastically stiffer response of the surroundings to rapid straining delays instability from I to I′. Such time-dependence can arise from coupling of deformation with pore fluid diffusion, coupling of an elastic lithosphere with a viscoelastic asthenosphere or stress corrosion effects in the material surrounding the fault. (b) Hardening of frictional response to rapid slip can also delay instability from I to I′. Such hardening may arise from reduction of pore pressure (and increase of effective compressive stress) due to dilatancy accompanying rapid slip.

Time dependence may also enter via the response of the fault zone material. Figure 4b illustrates the stabilizing effect of dilatant hardening: if dilatancy (Figure 1d) accompanying slip occurs more rapidly than fluid mass can diffuse into the newly created void space, the local pore pressure decreases, increasing the effective compressive stress (total stress minus pore pressure) on the slip surface; this increase inhibits further frictional sliding. Thus the condition for instability is not met until the unloading stiffness becomes equal to the slope of the hardened response curve and instability is delayed from I to I′ in Figure 4b. RUDNICKI and CHEN (1986) have recently studied, in detail, dilatant hardening accompanying slip for a simple one-dimensional model intended to simulate experiments by MARTIN (1980). They find that very small amounts of dilatancy (only a few percent of the slip) localized on the slip surface can stabilize slip as long as the ambient pressure is high enough. If the ambient pressure is too low, the pressure reduction, caused by dilatancy accompanying rapid slip, can cause vapor to form or gases to come out of solution, reducing the pore fluid bulk modulus. This eliminates the stabilizing effect and causes an abrupt onset of instability.

Instability associated with the state/rate-dependent constitutive description also results from interaction with the spring stiffness, but the possibilities are more complex and varied than for the rate-independent relation. Instability occurs if a perturbation of the sliding velocity from a steady-state value tends to grow so that the net sliding velocity becomes unbounded. RICE and RUINA (1983) have shown that for a wide class of this type of constitutive relations, including (1), the response to small perturbations is unstable only if $d\tau_{ss}/dV < 0$ and, in addition, the spring stiffness is less than a critical value k_{cr}. Thus, as for the rate-independent slip weakening model, instability can be suppressed by a sufficiently stiff system. However, in contrast to the slip weakening model, this conclusion applies only for small perturbations: GU *et al.* (1984) have shown the one state variable relation can be unstable to large perturbations, even when $k > k_{cr}$. For the two-state variable model an even richer range of behavior is possible (GU *et al.*, 1984).

As already mentioned, laboratory results at room temperature generally indicate $d\tau_{ss}/dV < 0$, and thus steady-state sliding is potentially unstable. However, laboratory results at higher temperature, indicating $d\tau_{ss}/dV > 0$, suggest a transition to stable behavior. The implications of this transition for the depth cut-off seismicity, as explored by TSE and RICE (1986), will be discussed later.

Elastic Brittle Crack Model

The single degree-of-freedom, spring-slider system is most appropriate for modeling laboratory experiments in which slip which occurs essentially simultaneously on the entire surface. But for the rate-independent slip weakening model, the limit in which the slip distribution is strongly nonuniform is also easily understood. In this limit, most of the fault surface is either unslipped or has slipped an amount that exceeds δ_0 and hence is sliding at the residual value of shear stress τ_r. The transition from unslipped to slipped portions occurs over a distance ω behind the fault edge in which the shear stress is reduced from the peak value τ_p to τ_r (Figure 5). Advance of the slipping region is assumed to occur when the relative slip at the end of this transition, or so-called cohesive zone, achieves some critical magnitude (e.g., RUDNICKI, 1980). If the length ω of the transition zone is small compared to other relevant lengths in the problem, then this model is equivalent to an elastic brittle shear crack, in which the stress ahead of the fault edge is singular, as (distance)$^{-\frac{1}{2}}$ and the slipping portion of the fault sustains the residual value of the shear stress everywhere. The area under the shear stress versus slip curve in excess of the residual stress (Figure 5c) can now be identified with the critical energy release rate G_c (PALMER and RICE, 1973; RICE, 1980; RUDNICKI 1980). Advance of the fault is assumed to occur when the energy that would be released by a unit advance of the fault edge equals this critical value

$$G = G_c \tag{4}$$

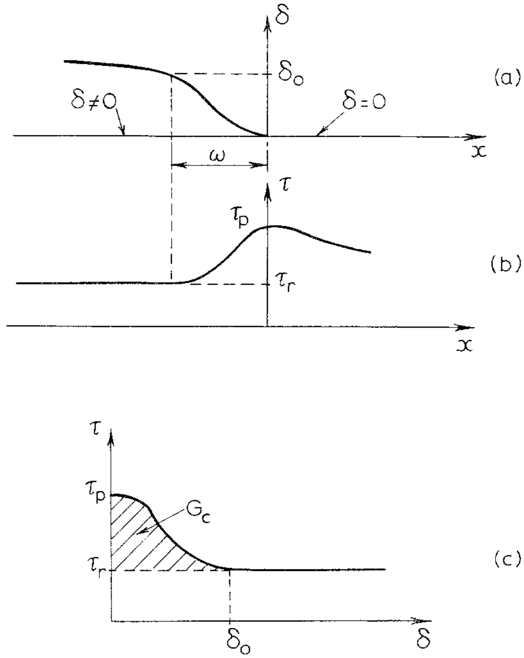

Figure 5
Cohesive zone fault model. (a) Variation of slip: for $x > 0$, $\delta = 0$ and for $x < \omega$, $\delta > \delta_0$. (b) Corresponding variation in stress. Note $\tau = \tau_r$ for $x < -\omega$. (c) Slip weakening relation between shear stress and slip. When the cohesive zone size ω is small compared to the overall fault length and other dimensions, the shaded area under the τ versus δ curve can be identified as the critical energy release rate G_c for a shear fault.

where the left-hand side is calculated from the solution for the cracked elastic body. For constant G_c, advance will be unstable if G increases with advance, but, in general, G_c will vary with position, reflecting the inhomogeneous strength of the fault. For a fault in a three-dimensional body (two-dimensional fault surface) G will also vary with position around the periphery of the fault edge. Although this variation is known for simple shapes such as circles or ellipses in infinite bodies, the effects of perturbations in the slip front and interactions with the free surface have not been explored. Some recent work has been done (RICE, 1985; GAO and RICE, 1986, 1987) on the stability of infinitesimal harmonic variations in edges of semi-infinite or circular cracks in infinite bodies, but more work is needed in this area.

Critical values of the energy release rate, estimated from field studies, are of the order of 10^6–10^7 J/m^2 (IDA, 1973; RUDNICKI, 1980; LI, 1987). These values are about 10^3 larger than those inferred from laboratory experiments (WONG, 1982; RICE, 1980). As remarked by RICE (1983), this discrepancy might result because slip *in situ* does not occur on a single surface, but on a series of surfaces that are offset. Consequently, the larger values of G_c reflect the significant amounts of inelastic deformation that are required to accommodate slip.

As noted by RICE (1983), there is not yet a corresponding model for the rate/state-dependent friction relation. Presumably, such a model would yield a velocity dependent critical fracture energy. In this case, the conditions for stability of advance will also be more complicated than for the rate-independent model.

The applicability of the elastic-brittle crack model depends on the endzone size ω. Because in this model ω scales with δ_0, the critical sliding distance (Figure 1a), uncertainties in the appropriate values of δ_0 *in situ* lead to a similar uncertainty in ω. Intermediate cases, in which the slip is neither so uniform that the single degree-of-freedom model is appropriate, nor so nonuniform that the elastic brittle crack model is appropriate, are more complex and must be investigated numerically. The constitutive relation for the fault surface must be combined with modeling of the deformation of the surrounding material. When inelastic deformation is confined to a narrow neighborhood of the fault surface, the deformation of the surrounding material can be sensibly idealized as linear elastic. In this case, the shear stress τ at a point P of the fault surface must be related to the slip (assumed to be unidirectional) $\delta(P', t)$ at points P' as follows (e.g., RICE, 1983):

$$\tau(P, t) = \tau_0(P, t) - \int_S K(P, P')\delta(P', t)\, dS(P') \tag{5}$$

where $\tau_0(P, t)$ is the shear stress that would be induced on the fault surface, in the absence of slip and $K(P, P')$, is the elastostatic Green's function. The stress $\tau_0(P, t)$ is assumed to be known and increasing at a slow rate due to large scale plate motions. If the surrounding material is modeled as linearly viscoelastic or the elastic crust is coupled to a viscoelastic asthenosphere, the Green's function will depend additionally on time, and a further integration over past history is required. Implicit in (5) is the assumption that the fault surface is straight, so that changes in normal and shear stress are not coupled via the slip.

As noted by RICE (1983), when (5) is used with either slip weakening or the rate/state-dependent constitutive relations, instabilities analogous to those described for the single degree-of-freedom system can occur. More specifically, $d\delta/dt$ may become unbounded for some set of points for which $d\tau_0/dt$ is finite, and this is interpreted as an earthquake instability. In the next section, some applications of this type of modeling to crustal faults are reviewed.

Applications to Crustal Faulting

A complete description of slip progression on a planar surface, in response to farfield loads employing either the rate-independent or rate/state-dependent constitutive relations discussed earlier and, possibly, including coupling of the elastic crust to the viscoelastic asthenosphere, is a formidable three-dimensional nonlinear problem. Consequently, most modeling efforts have considered strike-slip geometries and focused on particular aspects of the problem, for example, two-dimensional models of slip progression from depth or along strike.

Seminal work in studying the progression of slip from depth on a long (infinite) strike-slip fault was done by STUART (1979a) and STUART and MAVKO (1979). They used a slip weakening model in which shear stress τ at any point on the fault was related to the slip δ by

$$\tau = \tau_p \exp\{-(\delta - \delta_0)^2/a^2\} \tag{6}$$

where δ_p is the slip at peak stress τ_p and a is constant that measures the width of the peak. The peak stresses were assumed to have a Gaussian distribution with depth z:

$$\tau_p(z) = \tau_p^{\max} \exp[-(z - z_0)^2/b^2] \tag{7}$$

where the maximum peak stress $\tau_p^{\max}$ occurs at a depth z_0 and b is a constant governing the width of the peak. This variation is consistent with conceptual models (e.g., SIBSON (1982, 1983)) in which strength initially increases with depth due to increasing pressure, but then reaches a maximum and decreases due to increasing temperature. This distribution is also consistent with the supposed behavior of $\tau_p - \tau_r$, discussed in relation to the slip weakening model. However, the detailed form of (7) is chosen primarily for convenience. Depending on the parameters entering their model, STUART (1979a) and STUART and MAVKO (1979) find that the response can be stable or unstable, and that unstable response is preceded by a rapid straining of the ground surface compared with that due to the background rate of tectonic loading. These effects are similar to those discussed for the one degree-of-freedom model. However, an additional feature that does not appear in that model is the progression of slip from depth, as points closer to the surface are driven over the peak of the τ versus δ relation. STUART (1979b) has also used a similar two-dimensional model, modified for a thrust fault geometry to study deformation preceding the San Fernando earthquake. He finds that the predictions of the model can be made consistent with uplift, observed before and after the earthquake.

LI and RICE (1983a,b) extend the model of STUART (1979a) and STUART and MAVKO (1979) by including coupling of the elastic crust to a viscoelastic asthenosphere and the effects of a finite rupture length along the strike of the fault. The modeling of LI and RICE (1983a,b) is based upon the 'line spring' approximation originated by RICE and LEVY (1972) for the analysis of part-through surface cracks. As illustrated in Figure 6, this procedure approximates solution of the three-dimensional problem (Figure 6a) by two compatible two-dimensional problems (Figure 6b, c). Rupture propagation along strike is described by a relation between stress and slip averaged through the thickness of a thin plate (Figure 6b) overlying a viscoelastic substrate. This generalized Elsasser model (ELSASSER, 1969; RICE, 1980; LEHNER *et al.*, 1981) treats the lithosphere as a linear Maxwell foundation mechanically coupled to the elastic lithosphere. At each position along strike, the relation between thickness averaged stress and slip is obtained by solving the anti-plane problem shown in Figure 6c.

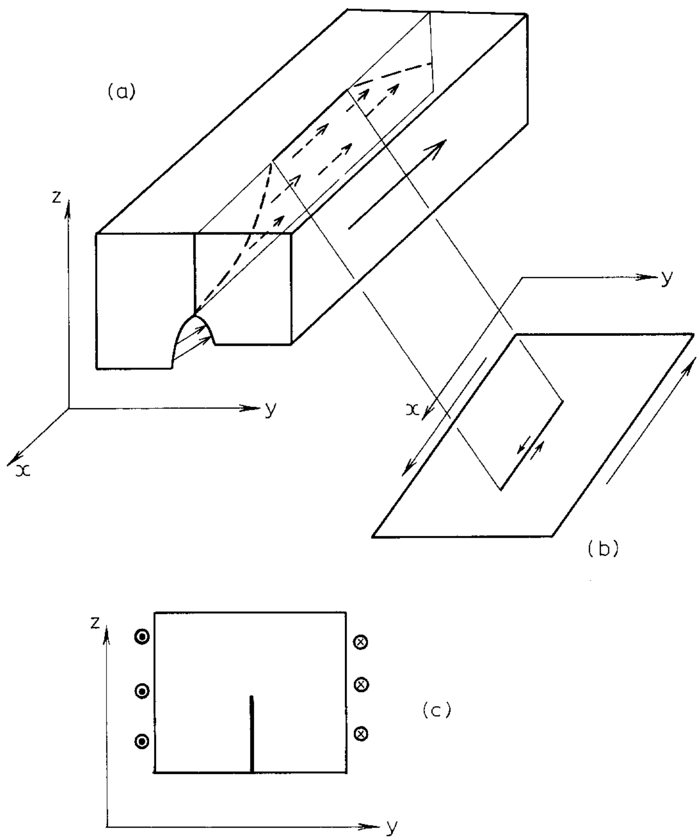

Figure 6
Illustration of the line-spring approximation. (a) A long strike-slip fault. The depth of slip below the surface varies along the strike of the fault (x direction). This three-dimensional problem is approximated by the two-dimensional problems shown in (b) and (c). (b) Thin plate approximation is used to relate variations of thickness-averaged stress and slip along the strike (x direction). The thickness-averaged relation between stress and slip at each section (each x) is obtained by solution of the anti-plane problem in (c).

In employing the line spring procedure with the elastic brittle crack idealization, LI and RICE (1983a) use a Gaussian distribution of the critical energy release rate to model the upward progression of slip. The maximum of the distribution is assumed to be 7 to 10 km below the surface, coinciding approximately with the hypocenters of large earthquakes. They also assume conditions are uniform along strike and, as a consequence, the model reduces to a one degree-of-freedom system for which the response is governed by the relation of a single curve of thickness-averaged stress versus thickness averaged slip to the effective unloading stiffness of the surroundings. This stiffness is time-dependent, because of coupling to the asthenosphere. LI and RICE (1983a) find that this coupling is an important factor in modeling the response immediately prior to instability. The stiffer short-time response delays the onset of instability and gives rise to a period of self-driven precursory deformation, as discussed earlier in relation to Figure 4a.

LI and RICE (1983b) discuss the implications of their results (LI and RICE, 1983a) for precursory surface deformation preceding earthquakes. They predict that the shear strain increases rapidly near the fault but decreases at moderate distances from the plate boundary. The decrease, also noted by STUART (1981), results from a decrease in the thickness averaged stress. Near the fault this decrease in strain is outweighed by a larger increase due to the approach of upward propagating slip to the ground surface. Unfortunately, the magnitude of the strain decrease is so small that it is unlikely to be detected by current geodetic methods. However, the shear strain increase near the fault is an order of magnitude larger than current geodetic detection levels, and the shear strain rate exceeds twice the background rate for time periods ranging from a few months to 5 years, depending on the tectonic loading rate. LI and RICE (1983a,b) also note that the details of their results are dependent on the modeling of the stress versus slip relation in the vicinity of peak stress, but are not sensitive to other features of the constitutive relation. This comment also applies to the work of STUART (1979a,b) and STUART and MAVKO (1979).

The work just described considers the upward progression of slip in a vertical section, but does not consider variations along strike. Hence, these models are appropriate for analyzing the response on a segment of a long strike-slip fault. TSE *et al.* (1985) use the line spring approximation to consider effects of variations along strike, but they neglect the coupling to the asthenosphere, and do not consider the upward progression of slip. In one calculation, they consider the effects of perturbations in the depth of slip. They find that positions along strike where the slip front is deeper (shallower), the energy release rate (left hand side of (4)) is higher (lower). This suggests a tendency for the deeper portions of the slip front to catch-up to adjacent shallower portions, possibly resulting in minor seismicity. In another calculation, they examine the loading of slip resistant, locked patches by adjacent creeping sections of the fault. As expected, they find that stresses and energy release rates are highest at the edges of the locked patches.

TSE *et al.* (1985) also use this model with slip rate data for the creeping portion of the San Andreas fault and seismicity patterns near Parkfield, to infer the nature of an effectively locked patch near Parkfield. Although, as they note, the inherent inaccuracy of the line spring procedure for describing short wavelength variations prevents detailed resolution of the patch geometry, their results are consistent with work of STUART *et al.* (1985), to be discussed below. TSE *et al.* (1985) also infer a remote stressing rate of 0.3×10^{-6} μ/year where μ is the shear modulus and a lithospheric thickness of 30 to 40 km.

STUART *et al.* (1985) also study the geometry of the slip resistant patch near Parkfield by means of a three-dimensional model, and use their results to forecast the creep and strain changes before a future Parkfield earthquake. They idealize the San Andreas fault near Parkfield as a vertical fault in an elastic slab. The slab extends to a depth of 54 km, chosen somewhat arbitrarily, below which no slip occurs. They assume that each point on the fault is either freely slipping or locked, except for a circular patch near Parkfield. The stress and slip on this patch are related by the slip weakening law (6), and the peak stresses are assumed to have a radial Gaussian distribution. The parameters of the model are adjusted so that the predicted surface deformation and creep are consistent with measured values. The model predicts a departure of these data from their background rates as the time of the next Parkfield earthquake approaches. The predicted departure is most notable in the creep records at stations nearest to the inferred location of the resistant patch. Predicted deviations from the background rate are less obvious in the length changes of trilateration lines; the deviation is, however, clearly visible in a line passing just northwest of the patch location, but the modeling of present data is poorest for this line (see Figure 5 of STUART *et al.*, 1985). Extrapolation of the results is unable to improve prediction of the time of the next earthquake, based on the recurrence interval, but refinement of the prediction may, in principle, be possible with time. The creep and trilateration data cited in the paper extend only to 1982, and no departure of the strain and creep changes from steady accumulation was evident. However, the simple model used by STUART *et al.* (1985) appears to be inadequate to fit current data; evidently, it would be necessary to include the interaction effects of slip on other nearby faults (W. D. Stuart, personal communication, 1986).

Because of the complexities of the rate/state-dependent constitutive relation, results have been limited primarily to single degree-of-freedom, spring-slider models. In addition to the frequently referenced but unpublished work of Mavko, (see MAVKO, 1980 for a summary) and the work of Stuart described in this volume, I am aware of only two calculations employing the rate/state-dependent constitutive relation for faults embedded in an elastic continuum, those of TSE and RICE (1986) and of HOROWITZ and RUINA (1986). Both of these calculations are two-dimensional, and use the one state variable version of the constitutive relation.

Following the earlier unpublished work of MAVKO, TSE and RICE (1986) use the rate/state-dependent constitutive relation on a long vertical strike-slip fault. Although Mavko assumed a plausible depth dependence for the parameters of the constitutive relation, TSE and RICE (1986) infer this depth dependence from observed temperature dependence of laboratory specimens. Laboratory data at room temperature indicates that $d\tau_{ss}/dV < 0$. Although there is apparently no data between room temperature and 300°C, data from STESKY (1975, 1978) at higher temperatures indicate $d\tau_{ss}/dV > 0$. TSE and RICE (1986) convert this temperature dependence to depth dependence by using the LACHENBRUCH and SASS (1973) geotherm for the San Andreas. The result is that $d\tau_{ss}/dV$ is negative (velocity weakening) above 11 km and is positive below. As discussed earlier instability is possible when $d\tau_{ss}/dV < 0$, but not when $d\tau_{ss}/dV > 0$. Thus, the inferred depth dependence provides an explanation of the confinement of earthquakes to shallow depths. As TSE and RICE (1986) discuss, this explanation is a refinement of conceptual models (e.g., SIBSON, 1982) in which the depth cut-off is explained in terms of a transition from brittle to ductile behavior, but suggests that this transition can be described by the depth dependence of constitutive parameters. A noteworthy feature of this explanation is that it does not require a transition from deformation that is localized on the fault at shallow depths, to deformation that is more broadly distributed below.

TSE and RICE (1986) use their inferred depth variation on a long vertical strike-slip fault in an elastic plate. Conditions are assumed to be uniform along the strike, but the length enters via the line spring approximation. The plate is subjected to a stress increase compatible with a uniform farfield plate velocity. The resulting fault motion is remarkably evocative of many featues observed for the occurrence of large earthquakes on the San Andreas fault system. Deep portions of the fault slip continuously at the plate velocity, but shallow portions are effectively locked during most of the earthquake cycle. Driven by continued slip at depth, the slipping zone gradually penetrates upward (as in the previously discussed slip weakening models). Eventually a small region in the locked zone unloads and the spread of this unloading zone initiates an instability. The depth at which instability initiates, ranges unpredictably from 3 to 8 km. The instability is followed by rapid afterslip with depth. The cycle then repeats itself with periods ranging from 50 to 160 years, for parameters chosen by TSE and RICE (1986).

The most uncertain feature of the calculation is the value of the parameter L in the constitutive relation (3) to which the results are sensitive. For values of L larger than 160 mm, the entire fault slips stably at the plate velocity. For L between 80 and 160 mm, the shallower portions of the fault do not exhibit earthquake-like instabilities, but instead keep up with the deep slip by episodic creep events. Most simulations of TSE and RICE (1986) use values of L in the 10 to 40 mm range. These values are 2 to 3 orders of magnitude larger than values inferred from laboratory tests, but may be appropriate in reflecting the larger scale irregularities in the field. TSE and RICE (1986) include one simulation for $L = 5$ mm, but, because the required mesh size decreases with L, the prohibitive cost of simulations prevents a full exploration of results for small values of L.

A result of the calculations, particularly relevant to earthquake prediction efforts, is the rapid surface slip that precedes instability. However, the magnitude of this slip increases with increasing L (and with decreasing $Vd\tau_{ss}/dV$) and, hence, reflects the uncertainties in the magnitude of L. These results, and their implications for field measurements, are discussed in more detail by TULLIS (1988).

HOROWITZ and RUINA (1986) also use a one-state variable model similar to that used by TSE and RICE (1986), but differing slightly in the way it models high velocity slip. The constitutive relation is used on an infinite planar fault embedded in an elastic slab, loaded by a remote constant velocity. In contrast to the depth dependence incorporated in the calculations of TSE and RICE (1986), HOROWITZ and RUINA (1986) assume that each point on the fault behaves identically. Nonuniformity enters only through the initial conditions. The resulting solutions exhibit a remarkable variety of responses, depending on parameter values. Solutions may be periodic, quasi-periodic or aperiodic in time; spatial variations include the possibility of simultaneous occurrence of local creep events, rapid (seismic) slip and steady sliding. The variety of solutions illustrates the richness of fault models based on the rate/state-dependent friction relations, but at the same time demonstrates the limited understanding of the models employing these relations for slip between elastic continua. The calculation also suggests at least the possibility that such models may be capable of simulating features of earthquake occurrence that are often attributed to random heterogeneities, e.g., the frequency distribution of different size events.

Although the results of these simulations, using rate/state-dependent friction, are intriguing and encouraging, the computational resources required for investigation are discouraging. Both the TSE and RICE (1986) and HOROWITZ and RUINA (1986) models are two-dimensional and, although complex in comparison to efforts based on rate-independent constitutive relations, are simplistic in terms of the geometric complexity posed by real faults. In both studies the full exploration of geophysically relevant parameter ranges was inhibited by computational resources (even though the TSE and RICE (1986) calculations were done on a supercomputer). Furthermore, both calculations used the one-state variable constitutive model, although two state variables are needed to describe some experimental results. This is a sobering prospect for future research along these lines and suggests that an essential part of this research will be the development of asymptotic methods that will be helpful in improving the computational efficiency.

Discussion and Suggestions for Research

Current physical models, examples of which have been discussed in this review, are capable of realistically simulating many aspects of crustal earthquakes. Although these models oversimplify the actual geometry of faults, and employ constitutive behavior having many uncertain aspects, they produce results that are consistent with (or can be made consistent with by adjusting a few parameters) geodetic data. However, current geodetic data are not precise enough to place many constraints on these models or to distinguish among them. More generally, the interpretation of observations is complicated by the likely presence of a variety of competing effects. The modeling efforts have, quite sensibly, focused on particular aspects, for example, upward progression of slip or coupling of the crust to the asthenosphere. Yet, all of these effects, as well as those due to coupling between deformation and pore fluid diffusion, are likely to contribute to precursory processes. Thus, although physical models have increased greatly in complexity and sophistication, there is still much to accomplish.

Better three-dimensional modeling is needed not only to provide better comparison with field observations, but also to investigate geometric effects that are not evident in two-dimensional models. As reviewed here, most fault models have been two-dimensional. Some approximate three-dimensional modeling has been done, using the line spring procedure. The great advantage of this approach is its relative simplicity, compared with the fully three-dimensional modeling. However, in most cases in which the line spring approach has been implemented, variations in one of the dimensions has been simplified, for example, by assuming that conditions are uniform along strike. When variations over the entire fault plane are considered, the advantageous simplicity of the line spring method may be eroded. Another limitation of the line spring method is, as noted by TSE *et al.* (1985), its inherent inaccuracy for short wavelength variations. Often, however, abrupt transitions between slipping and effectively locked portions of the fault are of great interest as, for example, at Parkfield. Some of these drawbacks may be overcome by hybrid approaches, for example, combining the line spring procedure with a boundary element method (LI and FARES, 1986), but recourse to direct numerical methods should be considered as an alternative.

Recent three-dimensional modeling has been done by STUART *et al.* (1985). Their procedure divides the fault plane into a series of cells in which the displacement is uniform. Hence, the displacement is discontinuous and the stresses at the cell boundaries are unbounded. Because of the strong stress singularity introduced by the uniform displacement solution, the average stress in each cell is also unbounded. Nevertheless, comparison of the results with known solutions indicates that this method yields satisfactory results.

Although three-dimensional calculations using the rate/state-dependent friction model do not appear to be feasible at present, there is much that can be done with simpler models, particularly the elastic brittle crack model. For the crack models, efficient and accurate procedures for three-dimensional calculations of fracture progression are available (LEE and KEER, 1987; LEE *et al.*, 1987). Conceptual models of earthquakes, based primarily on seismic observations, attribute considerable significance to slip resistant portions of the fault plane, usually called barriers or asperities. The procedures just mentioned could be used to explore the progression of slip fronts through or around slip resistant patches on the fault plane. More generally, there has been little exploration of effects of geometric perturbations in an advancing slip front, perhaps caused by encountering slip resistant zones. TSE *et al.*

(1985) have made some calculations for the variations in energy release rate due to along-strike depth variations in a stationary slip front. Also, RICE (1985) and GAO and RICE (1986) have made progress in this direction with analytical calculations for the stability of infinitesimal harmonic variations in the straight front of a crack in an unbounded body. However, they find that the critical wavelength above which perturbations are unstable is sufficiently large that the idealization used by them, of a half-plane crack in an infinite body, is unsuitable. Consequently, there is a need for further work in this area, including the effects of the free surface. An important goal is to comprehend conditions that distinguish large and small events.

Current physical models simulate realistically large events, but due to the prohibitive costs of using very fine grid sizes, it has not been possible to model small scale instabilities that could be interpreted as precursory seismicity. This shortcoming is unfortunate, because minor seismicity is often routinely and reliably monitored. At present there is no clear path around this difficulty. Possibilities include use of more efficient computational algorithms, asymptotic multiscaling techniques, or more supercomputer time, but it seems unlikely that large computations, involving disparate size scales, will be feasible in the near future. Some progress may, however, be possible again by using crack models, if increases in seismicity can be identified with the increase of stresses due to an advancing slip front. Such models can also be used to predict surface deformation and, thus, can be used to relate surface deformation to time and space variations in minor seismicity, an objective repeatedly emphasized by RICE (1984, 1985).

Because of the difficulties of including minor instabilities, representative of minor seismicity, in continuum calculations, there has been much work on multi degree-of-freedom spring-block systems. (See CAO and AKI, 1986 or NUSSBAUM and RUINA, 1987 for summaries). The conclusions of these studies have been varied and seem difficult to reconcile with one another, suggesting a strong sensitivity to choice of parameters. A persistent question has been the physical basis of observed seismicity patterns and, more specifically, the magnitude frequency relation. To what extent are these attributable to fault surface heterogeneity, fault surface constitutive relation, and/or some as yet unidentified characteristic of a dynamical system model of the Earth? Recent simulations by Cao and Aki, using a slip weakening (CAO and AKI, 1984/85) and rate/state-dependent (CAO and AKI, 1986) friction relations, have produced reasonably realistic seismicity patterns. The simulations using the slip weakening law were not, however, capable of continually repeatable, production of small earthquakes. Because the rate/state-dependent friction relation does include restrengthening of fault segments that are sliding at low velocities, a heterogeneous distribution of strength can be continually regenerated on the fault. Consequently, on the basis of their simulations using the rate/state-dependent friction relation, CAO and AKI (1986) conclude that this relation provides a physical mechanism for the roughening process that is necessary to explain the observed stationary magnitude—frequency relation. However, NUSSBAUM and RUINA (1987) have shown that a simple model with two identical degrees-of-freedom, and employing a static/dynamic friction, repeatedly tends toward an inhomogeneous state, after many slip events. Moreover, one class of solutions is found to be structurally unstable, in the sense that the results are strongly sensitive to perturbation. Although the simplicity of their model makes direct application to the earth difficult, their results suggest the possibility that spatial seismicity patterns may be due, at least in part, to nonlinear interactions in a complex dynamical system.

The further development of physical models is inhibited by uncertainties in how to extrapolate laboratory measurements to the crust, in particular measurements of the critical distance parameters entering the friction relations. This difficulty is compounded because the appropriate discretization in crustal scale models is tied to the size of this length scale, and computational costs increase as the size scale diminishes. This effort would be aided by a better understanding of the micro-mechanical mechanisms that give rise to the observed phenomenology. Current efforts on scaling have concentrated on the effects of surface roughness, but variations in frictional behavior among different rock types suggests that the behavior may also be related to constituent minerals, or whole rock deformation at pressures approximating those at asperity contacts. A possibility that cannot be ignored is that slip on faults in the earth is quite different from that on a well-defined surface or gouge layer in the laboratory. RICE (1983) has pointed out that the discrepancy between values of the critical energy, inferred from laboratory and field studies, may result because macroscopic fault slip *in situ* involves slip on offset segments, requiring considerable inelastic deformation between them. Such processes may also be relevant to the larger values of the critical sliding distance inferred from crustal fault models.

Although models have, for the most part, considered straight faults, bends are important because the normal stress induced across the fault can inhibit slip. STUART (1986) does approximate the curved San Andreas by a series of straight line segments, but does not account for induced variations in normal stress. The effects of normal stress on the slip weakening constitutive relation are reasonably well understood (WONG, 1986), but there is, yet, only preliminary information on the effects of normal stress changes in the rate/state-dependent model (OLSSON, 1985; HOBBS and BRADY, 1985; LINKER and DIETERICH, 1986).

In all the models discussed here, the inelasticity is confined to the fault surface. This would seem to rule out modeling precursory phenomena associated with widespread cracking or dilatancy, such as resistivity changes, radon transport or travel time changes. Models based on embedded volumes of inelastically deforming material have been suggested (RUDNICKI, 1977, RICE, and RUDNICKI, 1979). However, there has been no mechanism convincingly demonstrated for causing inelasticity to spread from a narrow zone, although dilatancy hardening is a possibility (RICE, 1975; RUDNICKI, 1984; RUDNICKI and CHEN, 1986). It may be that widespread inelasticity is induced in the vicinity of bends and offsets in the fault trace. These have been suggested as places where ruptures start and stop (SIBSON, 1986), and the mechanics of these regions warrants more study.

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