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Development of Localization in Undrained Deformation

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In recent experiments, *Finno et al.* [1] have used stereophotogrammetry to examine in detail the development of shear localization in undrained plane strain compression on saturated loose sand. In some experiments, zones of localized deformation initiate, then stabilize and disappear, to be followed by the formation of persistent bands of localization in different orientations. Although the tests are globally undrained, the nonhomogenous deformation indicates that there is internal fluid flow driven by compaction and/or dilation accompanying shear. This suggests the possibility that the evolution of the band structure is controlled by the effects of pore pressure changes on the strength via the effective stress principle.

Preliminary investigation of the effects of pore fluid diffusion on the development of localized deformation is explored in terms of a simple model of a body subjected to simple shear (at fixed total normal stress). Constitutive relations for the drained (constant pore pressure) response are assumed in the form of shear stress (τ) and porosity (ϕ) as a functions of shear strain (γ). For drained (constant pore pressure) deformation the shear stress is assumed to be a function of the shear strain as follows:

$$\tau(\gamma) = \tau_0 + (\tau_p - \tau_0)g(\gamma) \tag{1}$$

where $g(\gamma)$ is chosen so that τ rises from the initial yield stress τ_0 at $\gamma_0 = \tau_0/G$, where G is the elastic shear modulus, to the peak stress (at the ambient value of normal stress) τ_p at $\gamma = \gamma_p$. If the effective normal stress changes, then a term is appended to (1) to yield

$$\tau(\gamma) = \tau_0 + (\tau_p - \tau_0)g(\gamma) + m_0(\sigma - p) \tag{2}$$

where m_0 is a friction coefficient; both σ, the total normal stress (positive in

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compression), and p, the pore pressure, are measured from their ambient values at $\gamma = \gamma_0$.

The material is assumed to exhibit behavior typical of loose sand: compaction upon initial shearing in the inelastic regime, followed by dilation. Thus, the porosity ϕ is represented as follows:

$$\phi(\gamma) = \phi_0 - \frac{(\sigma - p)}{M} - (\phi_0 - \phi_{\min}) f(\gamma) \tag{3}$$

where M is an elastic modulus and $f(\gamma)$ is chosen so that ϕ decreases from an initial value ϕ_0 to a minimum value $\phi_{\min}$ and then increases to a residual value ϕ_r. (The form of the elastic response, the second term in (3), assumes incompressible solid and fluid constituents, an appropriate approximation for soils.)

For undrained response, there is no change in fluid mass and, for incompressible constituents, there is no change in the porosity. Consequently, the change in effective normal stress can be obtained from (3):

$$(\sigma - p)_{und} = -M(\phi_0 - \phi_{\min}) f(\gamma) \tag{4}$$

Substituting (4) into (2) yields the undrained shear stress versus shear strain relation:

$$\tau_{und}(\gamma) = \tau_0 + (\tau_p - \tau_0) g(\gamma) - m_0 M (\phi_0 - \phi_{\min}) f(\gamma) \tag{5}$$

Note that compaction (porosity decrease) causes the effective compressive stress (4) and the undrained shear stress to decrease (5). For dilation (porosity increase) the effective compressive stress and undrained shear stress increase. Figure 1 shows an example of the drained response, porosity variation and undrained response.

The stability of homogeneous, undrained behavior can be examined by introducing a small non-uniformity in the form of a layer of thickness h with properties slightly different than those of the adjacent material. The fluid mass in the layer is $\rho h \phi$, where ρ is the fluid mass density. The rate of change of fluid mass within the layer is assumed to be proportional to the difference between the pore pressure in the layer p and that in the surrounding material p_∞:

$$\frac{d}{dt}(\rho h \phi) = -\rho h \kappa (p - p_\infty) \tag{6}$$

where κ is a phenomenological coefficient with dimensions (stress x time)$^{-1}$, and ρ and h have been inserted for convenience on the right hand side. Carrying out the differentiation (assuming incompressible pore fluid, ρ = constant) and substituting (3), yields

$$\dot{p} - M(\phi_0 - \phi_{\min}) f'(\gamma) \dot{\gamma} = -(p - p_\infty)/t_D \tag{7}$$

where $t_D = \kappa M$ is the time scale of fluid mass transfer, the superposed dot denotes the time derivative and the prime denotes the derivative of the function

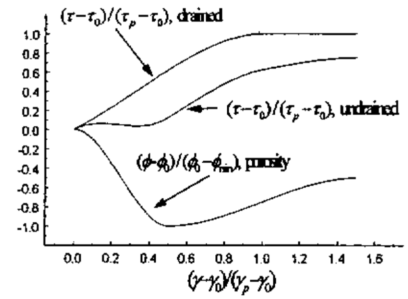

Figure 1:

with respect to its argument. The normal stress σ has been assumed to be constant and, hence, can be removed from the problem.

Equilibrium requires that the shear stress within the layer and in the surrounding material must be equal:

$$\tau_0^\infty + (\tau_p^\infty - \tau_0^\infty) g_\infty(\gamma_\infty) - m_0 p_\infty = \tau_0 + (\tau_p - \tau_0) g(\gamma) - m_0 p \tag{8}$$

where the sub- or super-script "∞" refers to the material outside the layer. Because the thickness of the outer material is assumed to be much larger than h, the thickness of the layer, the response of the outer material is undrained. Consequently, the pore pressure there is given by (4). A single ordinary differential equation for the evolution of the strain in the nonuniform layer can be obtained by using (8) to eliminate the pore pressure from (7). The result is

$$\varepsilon \{ g'(\gamma) - \beta f'(\gamma) \} \frac{d\gamma}{dT} = \varepsilon \Delta \{ g'_\infty(\gamma_\infty) - \beta_\infty f'_\infty(\gamma_\infty) \} + \Delta g_\infty(\gamma_\infty) - g(\gamma) \tag{9}$$

where, for simplicity, $\tau_0 = \tau_0^\infty$ and the following nondimensional quantities have been introduced: $T = \dot{\gamma}_\infty t$ (where $\dot{\gamma}_\infty$ = constant), $\varepsilon = \dot{\gamma}_\infty t_D$, $\Delta = (\tau_p^\infty - \tau_0^\infty)/(\tau_p - \tau_0)$, $\beta = m_0 M (\phi_0 - \phi_{\min})/(\tau_p - \tau_0)$, and β_∞ is the corresponding quantity in the outer material. The parameter ε is the product of the imposed strain-rate and the time scale of fluid mass diffusion. A rough estimate of ε for the experiments of *Finno et al.*[1] is about 10^{-8}[2]. Although the presence of this small parameter requires that some care be taken in the numerical solution of (9), it does make possible an asymptotic analysis.

In the limit $\varepsilon \to 0$, the deformation occurs so slowly that the pore pressure in the non-uniform layer is equal to that in the surrounding material. The equation obtained by setting $\varepsilon = 0$ in (9) is equivalent to the solution for drained response obtained by eliminating the pore pressure from (8). In this limit, the response becomes unstable, in the sense that the ratio of the strain-rate in the layer to that imposed $\dot{\gamma}_\infty$ becomes unbounded only when the *drained* response in the layer has a peak, i.e., $g'(\gamma) = 0$. If, however, the undrained response in the outer region has a peak prior to this, an inertial instability will occur under load control.

In contrast to the result just cited for $\varepsilon = 0$, the coefficient on the left hand side of (9) vanishes when the *undrained* response in the layer has a peak allowing for the possibility that $d\gamma/dT$ becomes unbounded at this point. For a compacting material, a peak in the undrained response will occur prior to a peak in the drained response. For small values of ε, the value of the imposed strain γ_∞ at which $\dot{\gamma}$ becomes unbounded can be estimated accurately by using the drained solution ($\varepsilon = 0$) to the determine the value of γ_∞ at which the coefficient of $d\gamma/dT$ in (9) becomes unbounded. Consider an example in which the layer response and the surrounding material have identical drained response: hardening to a peak stress at γ_p except that the peak stress of the layer is somewhat smaller so that $\Delta = 1.05$. The porosity response is also assumed to be identical and similar to that shown in Figure 1, except that $\beta_\infty = 0.4$ and $\beta = 0.5$. For these values, the undrained response in the surrounding material does not have a peak but that in the layer has a peak at $\hat{\gamma} = 0.143$, where $\hat{\gamma} = (\gamma - \gamma_0)/(\gamma_p - \gamma_0)$ is the normalized strain. Predicted on the basis of the drained reponse, the coefficient of $d\gamma/dT$ in (9) vanishes when $\hat{\gamma}_\infty =$ 0.137. Limited experimentation suggests that for small values of ε this is an accurate prediction of when the numerical solution of (9) becomes unstable. For comparsion, the purely drained solution does not become unstable until $\hat{\gamma}_\infty = \hat{\gamma} = 1.0$; in contrast, the purely undrained response (corresponding to the limit $\varepsilon \to \infty$) becomes unstable at $\hat{\gamma}_\infty = 0.08$.

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