Toughening Mechanisms in Quasi-Brittle Materials

Reporter's Summary: Session Three, Theoretical Fracture Mechanics Considerations

by

John W. Rudnicki

This session contained three invited talks: “Fracture Mechanics, Size Effect and Damage Localization in Brittle Heterogeneous Materials” by Zdenek P. Bazant (Northwestern University), “Micromechanics of Deformation in Rock” by Neville Cook (University of California, Berkeley) and “Asymptotic Analysis of Cohesive Cracks and Its Relation with Effective Elastic Cracks” by Jaime Flano (Ciudad Universitaria, Spain).

Following Professor Bazant’s talk, Professor Kobayashi noted that the fatigue law used by Bazant (equation 12) is known as Fomian’s Law in the material science literature. Bazant agreed with the comment by Karhaloo that the correction to the Paris law could be interpreted as appropriately defining the fracture toughness. In response to a question by Wu concerning the physical interpretation of the size effect law, Bazant noted that \( q \) is proportional to a characteristic length, with the proportionality factor depending on geometry; \( B \) is not related to the characteristic length but is a factor that depends on the specimen geometry.

Following Professor Cook’s talk, Horii commented that in analyzing the response of rectangular arrays of cracks, shear localization occurs only if the secondary cracks are not all the same length. Cook replied that his considerations of crack arrays pertained to prediction of the constitutive response; he views localization as a different phenomenon due to the highly heterogeneous stress field. Verrall asked whether the cases of overlapping and joining cracks had been examined since this is often observed in experiments. Cook replied that they had: In this case the simple analysis based on buckling of struts created by crack growth appears to be inappropriate; these configurations tend to fail by shear. Bazant noted that the strain softening constitutive relation predicted in the analysis will ultimately lead to localization and, hence, a localization limiter with coincident size effect will need to be introduced.

Following Professor Flano’s talk, Karhaloo asked a number of questions to clarify the range of validity of the analysis presented. In response, Flano noted that the analysis is not limited to large structures but that if the structure is large, the expansion can be truncated. The analysis does assume that the underlying cohesive zone model is a characteristic of the material and that the corresponding fracture energy is independent of size. In response to a question by Kobayashi about the need for a crack extension criterion, Flano replied that once the full softening curve is specified a crack extension criterion is not needed.
In addition to the invited talks, there was a short presentation by Professor Labuz (University of Minnesota) on the implications of the size effect for postpeak response, more specifically, whether the postpeak response is Class I or Class II (deterministic stress or force with increasing or decreasing displacement, respectively). Labuz noted that if the postpeak response is regarded as localized and a stress versus slip or opening cohesive zone relation is introduced, the subsequent behavior can be Class I or II depending on the size of the specimen.

The general discussion session opened with a question by Karitaloo about how geometric similarity of the bond slip relation was ensured in the experimental results described by Bazant. Bazant replied that it was difficult to ensure the geometric similarity of bond slip but that a comparison of tests with and without hooks suggested that this was not a problem.

There was considerable discussion of the solution for a crack emanating from a circular hole, to which reference was made in Cook's presentation. Cook noted that the solution for a circular hole (without a crack) in an infinite elastic body under biaxial compression predicts tensile stresses near the hole on a line parallel to the direction of maximum applied compression. Hence, the inhomogeneity caused by the hole does make it possible to generate opening cracks. Further discussion concerned whether the value of $K_1$ for a crack emanating from a circular hole increased for short crack lengths. Examination by this reporter of the results from Bowie ["Analysis of an Infinite Plate Containing Radial Cracks Originating at the Boundaries of an Internal Circular Hole", J. Mech. and Phys., 35, p. 60, 1956], based on a conformal mapping technique and summarized in Tada (The Stress Analysis of Cracks Handbook, H. Tada, P. Paris, and C. Irwin, Del Research Corporation, Hellertown, Pennsylvania, 1973), indicates that $K_1$ does indeed increase with crack length for small crack lengths. More specifically, the solution approaches that for an edge crack (suitably modified to account for the stress concentration due to the hole) for short crack lengths, as noted by Kobayashi in the discussion.

There was also some discussion of results of uniaxial compression tests. Cook described some tests in which the ends of the sample were constrained by steel wires and Shah described some results obtained in his laboratory. Uniaxial compression tests are notoriously susceptible to deviations from relatively homogeneous stress caused by the end constraints. The elimination of certain features of these tests with small amounts of confining stress suggests that the results are not indicative of material behavior and should be interpreted with considerable caution.

Atkinson questioned the need for the introduction of non-local theories to eliminate the crack-tip singularity and noted that there have been some problems identified with these theories when applied to crack problems. Bazant acknowledged that these theories were not required but argued that there is a rational basis for introducing them. Furthermore, he remarked that the non-local theory currently used by himself (and Flanagan) differs from that introduced by Eringen for which problems have been found.

Krajcicovic asked what was the origin of nonlinear (axial) deformation in cases for which all cracks are axial. Bazant noted that finite element simulations of uniaxial compression suggest that the amount of axial cracking (splitting) depends strongly on the amount of volume change allowed. Cook remarked that nonlinear axial deformation occurs when the axial cracks are driven by local tensile stresses due to inhomogeneities.

Francois commented that for perfectly plastic materials (no strain hardening) the strain singularity ahead of the crack tip is $r^{-1}$ for a stationary crack but decreases to $\ln r$ for a steadily moving crack. In response to his question about whether similar behavior is predicted for cohesive zone models, Bazant replied no.