Incorporating Complex Substitution Patterns and Variance Scaling in Long Distance Travel Choice Models

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ABSTRACT

This paper employs extensions to the MNL model to relax the known characteristic of MNL elasticities being a function of the size of variables which can cause under-sensitivity at low values and over-sensitivity at higher values and demonstrates the advantages of adopting the Generalized Nested Logit formulation in place of the more restrictive MNL and NL Generalized Extreme Value models. The resultant model, which includes distance scaling of distance related variables, the flexible covariance structure of the Generalized Nested Logit model and differential variances for first and second choice stated preference data. The analysis is undertaken in the context of intercity travel with multiple modes and multiple rail service classes. The resultant model strongly rejects more basic MNL and NL models and provides a highly intuitive interpretation of intercity choice behavior.

1. INTRODUCTION

Extensive market research and choice modeling of travel behavior has been undertaken in the last two decades focusing on rail travel in shorter corridor-oriented travel markets, such as the Northeast Corridor of the U.S., Montreal-Toronto corridor, Chicago hub corridors, the California corridors, and the Tampa-Orlando-Miami corridor in Florida (Charles River Associates, 1994; Peat Marwick, 1990; TEMS, 1999; KPMG et. al., 1996, KPMG, 1998). These studies provide a useful basis for evaluating the impacts of train pricing and service improvements in the respective corridor markets. In contrast, there has been limited travel behavior research in the long distance markets, where the choice of rail service class (coach or sleeper) is an added dimension to the travel choice decision.

This study addresses this need by estimation of stated preference based intercity mode and rail class choice models for long distance travel. Further, it demonstrates the advantage of adopting the more general, but still closed form, Generalized Nested Logit (GNL) choice model in place of both the MNL and NL models that have been widely used. Further, the model incorporates distance scaling of distance related variables to overcome the linear in variables elasticity property of the MNL and takes account of differential precision in first and second rank choices by inclusion of variance scaling. All the model components are estimated and demonstrate a substantial improvement in the goodness of fit to the data and the interpretation of long distance intercity choice mode and class choice behavior.

Our review of previous work identified only three studies which address the behavioral issues in choice of service class within a given mode of intercity travel (Koppelman, 1989; Proussaloglou and Koppelman, 1995; Hensher, 1998). The first two studies of these address choice of fare/service class within the air mode; the third study, which was aimed at understanding the determinants of rail service class choice, was limited to a single origin-destination pair.

The model system developed by Koppelman (1989) incorporates choice of service/fare class as part of a multi-dimensional choice process including trip frequency, destination, and mode choice. Nested logit models are developed to represent the interrelated choice structure of
these decisions, with fare/service class choice at the lowest level. However, the fare/service class models are estimated only for the air mode due to limited information on the range of service classes for the other modes in the National Travel Survey (1977) data used for the study. The estimated models show that both fare and departure frequency significantly influence fare/service class choice. In addition, travelers’ income and trip purpose are also important factors affecting the choice of fare/service class.

Proussaloglou and Koppelman (1995) developed a conceptual framework for analyzing air travel demand to measure the relative importance of factors that influence travelers’ choice of carrier, flight, and fare class for different purpose segments. Their analysis revealed that price levels, air travelers’ income, market presence and loyalty are the key determinants of fare class choice. However, this study addresses the issue of class choice in a single mode context and does not include explicit measures of service class attributes. Ignoring the interplay of competition among travel modes in the class choice behavior of travelers’ masks the complex substitution patterns that are likely to occur in real choice situations.

Hensher’s (1998) study of the Sydney-Canberra corridor in Australia, the only published study of service class choice in long distance passenger rail, reports an investigation of the demand for rail sleeper and auto train services between Sydney and Brisbane (12 to 14 hour trip) to quantify the demand elasticities with respect to price and service quality, using stated preference data. Nested logit models are developed with choice of mode at the upper nest and the rail service class alternatives in the lower nest. The key variables identified as important determinants of rail class choice include: fare levels of different service classes, travelers income, trip purpose, comfort and privacy provided by different classes, and whether or not the trip is overnight. The study provides valuable insights into class choice behavior of long distance rail travelers, but is limited in its generalizability to other contexts since the models are developed for only one city pair. Thus, behavioral differences of travelers arising from trip contexts (e.g., trip distance) are not taken into account.

The remainder of the paper is organized as follows. Section 2 identifies the key modeling issues that are addressed in this paper. Section 3 provides an overview description of the market research and stated preference survey approach adopted for this study. Section 4 describes the modeling framework and estimation methodology. The empirical results and modeling implications are described in Section 5. Section 6 provides the summary conclusions.

2. MODELING ISSUES

The above studies identify some of the key level-of-service, traveler, and trip related attributes that influence mode and rail service class choice decision of long distance travelers. However, the following modeling issues have not been adequately addressed in these studies.

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1 Frequency differs across alternatives since not all flights included all service classes.
2.1 Flexible Substitution Patterns

Effective representation of choices in models of intercity mode and service class choice can be improved through added flexibility in error correlation structure beyond that provided by Nested Logit (NL) models. Previous studies have considered service class choice as an extension of the mode choice decision using multinomial logit or nested logit structure as shown in Figure 2, where the choice of service class is conditional on the choice of travel mode (Koppelman, 1989; Hensher, 1998).

Figure 2: Hierarchical Structure for Modeling Mode and Class Choice Decision

The above structure implies a higher degree of substitution/similarity among alternatives in the nest (classes of rail service), but imposes the restriction of equal substitution between nested and non-nested alternatives (coach versus air/auto, and sleeper versus air/auto). However, the substitution/similarity relationships between alternative modes and rail service classes can be different as similarities may exist along a number of dimensions. We illustrate this idea in Table 1, which offers quality ratings along key observed and unobserved modal/service class attributes. These ratings indicate the possibility of complex similarity pattern in a choice situation with automobile, air, rail coach class, and rail sleeper class\(^2\). Although correlations of the random error terms are associated only with the unobserved attributes, relative quality ratings are provided for both observed and unobserved attributes, recognizing that certain observed attributes may not be available in specific studies.

\(^2\)The relative quality indices provided in the table are based on average trips in context of long distance intercity travel and may not necessarily be accurate for certain trip contexts.
### Table 1: Relative Quality Rating for Mode/Service Class Alternative for Observed and Unobserved Attributes

<table>
<thead>
<tr>
<th>Attribute Name</th>
<th>Attribute Type</th>
<th>Relative Quality Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Auto</td>
</tr>
<tr>
<td>Travel Time</td>
<td>Measurable/Observed</td>
<td>2</td>
</tr>
<tr>
<td>Travel Cost</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Service Frequency</td>
<td>N/A</td>
<td>1</td>
</tr>
<tr>
<td>Schedule Convenience</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Seat Comfort</td>
<td>Unmeasurable/Unobserved</td>
<td>2</td>
</tr>
<tr>
<td>Reliability</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>On-board Food Service</td>
<td>N/A</td>
<td>2</td>
</tr>
<tr>
<td>Privacy</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Safety</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The measurable/observed attribute dimensions shows that the rail service classes are more similar to each other than they are to other alternatives. However, the similarity relationship in the unmeasurable/unobserved attribute dimensions, which is the theoretical justification for nesting alternatives together, has different patterns as described in the following observations.

1. The rail sleeper class provides a distinctly higher level of seat comfort than air, automobile, and rail coach, which have similar levels of seating comfort.
2. Air and rail coach alternatives provide little or no privacy to travelers relative to rail sleeper class and automobile.
3. Rail coach and sleeper classes share a common level of service reliability, which is likely to be different from that provided by automobile and air alternatives.
4. The availability and/or quality of on-board food service differs across each of the travel alternatives.

Thus, unobserved similarities between alternatives may exist along different attribute dimensions and the overall similarity measure between alternatives (i.e., correlation among random error terms) may be complex, suggesting the need for a modeling framework that provides greater flexibility in accommodating differential substitution between pairs of alternatives.
2.2 Elasticity Scaling in Linear MNL Model

The linear utility specification in logit models results in elasticities that scale with the level of the variable \(X_k\) as shown in the direct elasticity expression for a MNL model:

\[
\eta^p_{X_{i,k}} = \beta_k X_{i,k} (1 - P_i)
\]  

where \(P_i\) is the probability of choosing alternative \(i\)
\(\beta_k\) is the parameter associated with the \(k^{th}\) attribute of alternative \(i\) \(X_{i,k}\).

The undesirable effects of this property are exacerbated in situations when the variable values vary over a very wide range, as is true in our empirical case where trip lengths vary from 250 miles to 2500 miles. In such cases, a simple linear representation of time and cost in modal utilities will result in an over-sensitivity to changes in these attributes for long trips and under-sensitivity for shorter trips. Stopher and Prashker (1976) addressed this problem by discounting costs, times, and distances by the “best” value of each variable across alternatives. However, they recognize that this approach is likely to produce counter-intuitive elasticities in situations where a change occurs in the “best” characteristic for the given observation. More recently, De La Barra (1994) postulated that decision makers perceive utility in relative rather than absolute terms and that models be structured to represent this. To reflect this behavior in the utility specification, De La Barra replaces the utility term in the logit model with the ratio of utility of each option to the utility of the best option in the choice set. This specification suffers from the same fundamental drawback of including cross-alternative attributes in the utility of each alternative (except, the alternative with the lowest utility) which can result in counter-intuitive elasticities under certain conditions (see, Ben-Akiva, 1974). An additional drawback of this formulation is that the lowest utility alternative varies across cases and is not known, \textit{a priori}, requiring iterative estimation for which convergence is not necessarily assured.

Another approach to this problem is the use of non-linear utility functions. Mandel \textit{et. al}. (1994) explored the implications of non-linear utility specifications on German high speed rail demand forecasts, using Box-Cox transformations (Gaudry and Wills, 1978). They report that non-linear specifications were statistically superior and yielded very different high speed rail market shares in comparison to linear utility specifications. They also demonstrate that the use of a Box-Cox specification greatly reduces the need for market segmentation by fare or distance classes. Other variations of non-linear specifications can also be found in the literature, the most common one being the use of natural logarithm of the variable\(^3\). This formulation results in a constant elasticity model (\textit{w.r.t.} to the log transformed variable) which may be overly restrictive in many situations.

We adopt an alternative approach to counter the undesirable effects of linear utility specification by scaling the relevant level-of-service variables (\textit{e.g.}, time and cost) by a

\(^3\text{The log transformation is a special case Box-Cox function}\)
distance transformation function (Equation 2), where $\gamma$ determines the shape of the distance scaling function$^4$.

$$Scale = (1 - \exp(-\gamma \times \text{Trip Distance}))$$  \hspace{1cm} (2)

The proposed function provides the flexibility of tempering the elasticities for longer distance trips, and simplifies the search process through the parameterization of the non-linear adjustment. It yields different elasticity profiles through appropriate specification of the scale parameter, $\gamma$, and results in a more reasonable elasticity range (across trip lengths) than both the linear and log specifications. The linear model produces rail fare and travel time elasticities that scale almost linearly with distance$^5$, whereas the log specification produces a constant elasticity. This property is illustrated in Figure 1 that shows the ratio of fare elasticities between a 2500-mile and a 250-mile trip for linear, log, and distance scaled specifications.

![Figure 1: Ratio of Rail Fare Elasticities for 2500 by 250 Mile Trip for Alternative Utility Specifications](image)

$^4$ Alternative mathematical functions with similar properties are likely to produce similar results.

$^5$ This property is based on the assumption that rail fares and travel times increase linearly with distance and that rail market shares do not vary substantially over trip distance. Minor deviations from these assumptions do not change this property in any significant way.
2.3 Limited Representation of Service Quality Attributes

The influence of qualitative variables (e.g., service quality, seat comfort, reliability and convenience of schedules) in the choice decision is typically assumed to be absorbed in the random error terms of the primarily due to the lack of objective metric for measurement of these variables. We argue that qualitative attributes may assume greater importance in the decision process for long distance leisure trips, and to the extent possible, should be included in the systematic component of the utilities, to the extent possible. The explicit inclusion of these variables in the systematic utility can assist in policy evaluation, such as forecasting ridership impacts for improvements in service along these qualitative dimensions. Further, to the extent these variables are not included in the model, they should be considered in the formulation of structural elements which determine differential substitution patterns between pairs of alternatives.

2.4 Heteroscedasticity and Serial Correlation Across Observations

Heteroscedasticity (or differential error variances) between data sources (e.g., combining RP and SP data) has been explicitly accounted for in recent travel demand modeling work; however, heteroscedasticity within data sources has rarely been accommodated, especially outside the MNL model framework. Specifically, differential variances across SP observations may arise in rank-order data due to unequal reliability of data from different ranks. This expectation is based on the hypothesis that respondents are likely to be more thoughtful in indicating their most preferred alternative relative to less preferred alternatives. Therefore, lower error variances are expected for first choice observations relative to observations for lower ranks (Bradley and Daly, 1994). In addition, heteroscedasticity may exist due to differential complexity for a given choice situation in both RP and SP observations and/or fatigue effects that may occur from repeated choice made by a respondent in the SP experimental design (Swait and Adamowicz, 1996). Also, there is potential for correlation among observations obtained from repeated responses from a given individual in SP data which may produce biased parameter estimates and/or downwardly biased standard errors (Morikawa, 1994). The methods developed to correct for serial correlation are outside the framework of closed-form models and require the use of numerical integration techniques (Heckman, 1981).

3. MARKET RESEARCH AND PREFERENCE SURVEYS

A revealed and stated preference survey approach is used to investigate the attitudes and preferences of long distance intercity travelers including 1,000 current rail users and 500 non-users of passenger rail service. The survey is administered in two parts. The first part is a recruitment survey in which information regarding the qualifying trip is collected; and the second part is a follow-up telephone survey, tailored to the respondents’ trip, to collect attitudinal measures of services and responses to a series of stated preference experiments.

\[\text{Given the budget and calendar constraints on the study, priority was given to completing the user survey as they were deemed to be the most important part of the overall sample.}\]
The choice scenarios consisted of 8-9 future travel situations, where the respondent is ultimately asked to indicate his most preferred and second most preferred mode/class of travel among the alternatives presented to him based on travel cost, travel time (only for non-users), travel schedule, and other amenities available in each mode/class.

The stated choice experiments developed for the user sample required each respondent to indicate his most preferred and second most preferred choice from a set of three train service classes (each with and without auto train option), an alternative mode of travel7 (bus, auto, or air) and the option to not travel – a total of eight alternatives. The three train alternatives included both existing service classes (coach and sleeper), and one of two proposed service classes (premium coach or economy sleeper). The design for the non-users was simpler, requiring each respondent to choose from two train classes (each with and without auto train option), their existing mode of travel, and the option to not travel – a total of six alternatives. The two train alternatives presented to a respondent consisted of one of the following three combinations of coach and sleeper classes:

- Existing coach and existing sleeper
- Existing coach and economy sleeper
- Premium coach and existing sleeper

Each three combinations included one coach and one sleeper alternative, at least one of which was an existing rail class alternative. Table 2 summarizes the amenities offered in each class of train travel as described to the respondents.

### Table 2: Description of Service Classes for Train Travel

<table>
<thead>
<tr>
<th>Existing Coach</th>
<th>featuring big, comfortable seats with fold down trays and individual reading lights.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing Sleeper</td>
<td>providing a private room with upper and lower berths in a sleeping car with hotel style bedding for a comfortable overnight trip. This service includes complimentary meals, a bed-time sweet, and wake-up service with a newspaper, tea, coffee, and orange juice in the morning. Restroom facilities are not located within the room but are available nearby, as is a public shower.</td>
</tr>
<tr>
<td>Premium Coach</td>
<td>a proposed new class of coach service featuring deluxe seats with lots of leg room and greater reclining space for more comfortable overnight travel. Extra amenities include a big pillow and blanket, at seat audio and video, a public shower, and priority seating in the dining car where meals/beverages can be purchased. Complimentary newspaper, tea, coffee, and orange juice will be served in the morning.</td>
</tr>
<tr>
<td>Economy Sleeper</td>
<td>a proposed new class of sleeper service featuring small compact compartments with bunk style beds adequate but somewhat less spacious than the existing sleeper. Pillows and blankets will be provided on-board, and meals/beverages can be purchased by passengers in the dining car. Restroom facilities are not located within the compartment but are available nearby, as is a public shower.</td>
</tr>
</tbody>
</table>
The stated preference surveys are orthogonal by design consisting of five attributes with three or four levels each for users and four attributes with three levels each for non-users. Since a major aspect of the study is to determine revenue maximizing fare levels, four out of the five attributes varied in the user experimental design (and three out of four attributes in the non-user design) are the fares for the different classes of service. The remaining attribute, in the case of users, is a schedule convenience variable represented by alternative train schedules and, in the case of non-users, is line haul travel time.

4. MODEL STRUCTURE AND ESTIMATION APPROACH

We estimate different Generalized Extreme Value (GEV) models that provide varying degree of flexibility in accommodating correlation in the random error terms of alternative utilities. The MNL is the most restrictive allowing no correlation among alternatives; the Nested Logit (NL) model allows differential correlation between alternatives, grouped in an hierarchical structure. The Generalized Nested Logit (GNL) provides the most flexibility in representing complex correlation/substitution patterns between each pairs of alternatives (Wen and Koppelman, 2000). We extend this model to allow heteroscedasticity in the variances of the random error terms due to different response precision obtained from 1st and 2nd preference responses in SP experiments using the variance scaling framework originally developed to accommodate differential error variances in combining revealed and stated preference data (Morikawa, 1989). The choice probability for the resulting model, the Heteroscedastic Generalized Nested Logit (HGNL) model, is:

\[
P_{i,t,n} = \sum_m P_{i,t,n/m} \times P_{t,m} = \frac{\alpha_{i,m}}{\sum_{j \in N_m} \alpha_{j,m} e^{\theta_m}} \times \left( \sum_{j \in N_m} \alpha_{j,m} e^{\theta_m} \right)^{-\theta_m} \sum_m \left( \sum_{j \in N_m} \alpha_{j,m} e^{\theta_m} \right)^{-\theta_m}
\]

where \( V_{i,n} \) is the systematic component of the utility for alternative i, individual n
\( N_m \) is the set of all alternatives included in nest m,
\( \theta_m \) is the similarity parameter for nest m that satisfies the condition, \( 0 < \theta_m \leq 1 \),
\( \alpha_{i,m} \) is the allocation parameter which characterizes the portion of alternative i assigned to nest m and must satisfy the following conditions:

i) \( \sum_m \alpha_{i,m} = 1 \), \( \forall i \)

ii) \( \alpha_{i,m} > 0 \)

\( \mu_t \) is the variance scale parameter for observation type t; fixed to 1.0 for 1st choice observations and estimated for 2nd choice observations.

Calibration of the HGNL model requires simultaneous estimation of the utility, similarity,
allocation, and scale parameters, taking account of restrictions implied in the model formulation to ensure consistency with random utility maximization. The maximization of the constrained log-likelihood function is mathematically represented as:

Maximize \[ LL = \sum_{n} \sum_{i=1}^{2} \sum_{t=1}^{I} \delta_{i,t,n} \log P_{i,t,n} \]  

s.t.

\[ 0 < \theta_m \leq 1, \forall m \]
\[ \sum_{m} \alpha_{i,m} = 1, \forall i, m \]
\[ 0 \leq \alpha_{i,m} \leq 1, \forall i, m \]
\[ \mu_t > 0, \forall t \]

where \( \delta_{i,t,n} \) is 1 if individual \( n \) chooses alternative \( i \) for choice observation \( t \), and 0 otherwise, and \( P_{i,t,n} \) is the estimated probability that individual \( n \) chooses alternative \( i \) for choice observation \( t \). \( \mu_t \) is the variance scale parameter for observation type \( t \); fixed to 1.0 for 1st choice observations and estimated for 2nd choice observations.

The model estimation is implemented in GAUSS using its constrained maximum likelihood module (Aptech, 1995). This framework accommodates all the modeling issues raised in Section 3 except for serial correlation across SP responses for each respondent.

5. EMPirical Analysis

We focus the model development in this paper on the SP data. The extension to including RP data requires an additional scale parameter to account for the differential error variances between RP and SP data sources.

The data includes over 13,000 SP observations obtained from approximately 1,500 survey respondents. The choice set for each observation consists of up to six of the following seven alternatives:

- Rail Coach
- Rail Sleeper
- Rail Premium Coach (PC)
- Rail Economy Sleeper (ES)
- Auto Train
- Bus
- Other

\(^8\) Constrained Maximum Likelihood framework was adopted to simplify the GAUSS code. However, it is possible to estimate the GNL model in a standard Maximum Likelihood framework by incorporating the appropriate constraints within the code.

\(^9\) Auto Train and Bus were excluded from the analysis due to the low choice frequency of these alternatives.
- Air
- Automobile
- Not Travel

The first step in model development is selection of a preferred utility specification for the MNL model. The estimation results of four MNL models are reported in Table 3 (T-statistics in parenthesis) which includes a base model (Model 1), a model which takes account of distance scaling of cost variables (Model 2), a model that incorporates differential error variances between most preferred choice and second most preferred choice observations (Model 3), and both (Model 4). The variables included in the utility specification of the base MNL model can be grouped into the following categories:

1. **Inertia variables** which represent the resistance to make an alternative change obtain positive signs, as expected. The large parameters for air and sleeper travelers imply that they are most “attached” to their chosen alternative.

2. **Rail cost** which is divided between the basic transportation cost for travel by coach class and the upgrade cost for premium coach, economy sleeper, and sleeper (with respect to rail coach). Differential sensitivities to upgrade cost with respect to household income are incorporated in the model by including distinct upgrade cost variables for travelers with annual household income less than or greater than $50,000 per year. Both income groups are more sensitive to upgrade cost than to transportation cost and low income travelers are more sensitive to upgrade cost relative to high income travelers.

3. **Schedule convenience** represented by bad departure hour and bad arrival hour dummy variables, specific to all rail alternatives and to the not travel alternative. The variables take a value of one if the train departs/arrives between midnight and 4:00 A.M., and zero otherwise. As expected, the negative signs on the rail specific bad departure/arrival time variables confirms the undesirability of such a schedule for rail travel. Not only does this reduce the utility of the rail alternatives, but also increases the utility of the not travel alternative. This suggests that rail travelers who face inconvenient travel schedule will shift to no travel in greater proportion relative to other alternatives. Further, the larger magnitude of the “bad arrival time” parameters suggests that an inconvenient arrival time is more undesirable than an inconvenient departure time.

4. **Overnight rail trip** dummy variables specific to sleeper and air all have positive parameters associated with them, implying an increased preference for these alternatives relative to rail coach and automobile options for trips requiring overnight train travel.

5. **Quality of service** perception variables include an unreasonable delay dummy variable indicating that the delay experienced by rail user was more than reasonable (as defined by the rail user) and a quality rating score on a scale of 1.0 to 10.0. The negative sign on the delay dummy variable confirms that expected reliability of service and perceived quality of rail service have a positive influence on rail choice.
6. **Group size** dummy variable parameters (for group size 1 and 2) specific to sleeper and automobile suggest, as expected, that the larger the size of the travel party, the more likely it is that they will choose to travel by sleeper and auto relative to other alternatives.

7. **Trip distance** and **household income** specific to sleeper, air and automobile alternatives indicates that increasing income favors air the most, followed by sleeper and automobile (relative to the other three rail and not travel alternatives). Increasing trip distance favors sleeper and discourages travel by automobile. Surprisingly, the air specific distance variable is close to zero and statistically not significant, possibly due to high correlation with the overnight dummy variable.

8. **Not travel** variables reflect the propensity of travelers to forgo travel (*i.e.*, choose to not travel) if their currently chosen alternative becomes unavailable. The positive sign on these variables implies a greater increase in the probability of the Not Travel alternative (relative to other alternatives) if the currently chosen alternative is unavailable. Further, the larger magnitude for the non-rail specific variable suggests that automobile and air travelers are more likely than rail users to forgo travel if their current mode of travel is unavailable.

9. **Alternative specific constants** with rail coach as the base alternative. The interpretation of these parameters is somewhat difficult due to the presence of other alternative specific variables (*e.g.*, income, distance). For example, the large negative constant for the sleeper alternative can be offset by the positive utility contribution from the distance variable for longer trips and the income variable for higher income travelers. However, the negative value of the sleeper constant is counter-intuitive for even short-trips and low-income travelers, *ceteris paribus.*
### Table 3: Estimation Results for MNL Model Specifications

<table>
<thead>
<tr>
<th>Variable Names</th>
<th>Model 1 Base MNL</th>
<th>Model 2 Distance Scaled MNL</th>
<th>Model 3 Variance Scaled MNL</th>
<th>Model 4 Distance and Variance Scaled MNL</th>
</tr>
</thead>
<tbody>
<tr>
<td>inertia_coach</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inertia_sleeper</td>
<td>1.1513 (20.9)</td>
<td>1.1446 (20.7)</td>
<td>1.2890 (19.6)</td>
<td>1.2702 (19.6)</td>
</tr>
<tr>
<td>inertia_air</td>
<td>1.3933 (17.0)</td>
<td>1.6406 (20.3)</td>
<td>1.5875 (16.7)</td>
<td>1.8613 (19.8)</td>
</tr>
<tr>
<td>inertia_auto</td>
<td>2.4696 (16.8)</td>
<td>2.3765 (16.9)</td>
<td>2.7154 (16.7)</td>
<td>2.5991 (16.3)</td>
</tr>
<tr>
<td>Scale Parameter for Second Choice Observations</td>
<td>0.9015 (-0.9)</td>
<td>0.8024 (-1.8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log likelihood at zero</td>
<td>-17883.6</td>
<td>-17883.6</td>
<td>17883.6</td>
<td>17883.6</td>
</tr>
<tr>
<td>Log likelihood at convergence</td>
<td>-14348.9</td>
<td>-14225.7</td>
<td>14258.1</td>
<td>14147.9</td>
</tr>
</tbody>
</table>

* Scaled Rail Transportation Cost = Rail Transportation Cost / (100*(1-exp(-0.001*Distance)))
** Scaled Rail Upgrade Cost = (Rail Cost – Rail Transportation Cost) / (100*(1-exp(-0.0005*Distance)))
The unavailability of level-of-service data for non-rail modes precluded the inclusion of \textit{travel time} as an explanatory variable in the utility specification. However, we speculate that the potential biases due to the omission of travel time variable may not be significant due to the similar travel times for rail and automobile modes combined with the limited variability in air travel times across observations.

Model 2 differs from the base model in the distance scaling of the cost variables as described in Section 2\textsuperscript{10}. This specification yields a substantially improved statistical fit and rejects the base model at a very high level of significance. More importantly, Model 2 provides a more intuitive interpretation of the effect of mode cost as illustrated in Figure 3. First, the Figure shows that cost sensitivity declines as trip length increases which effectively tempers the scaling of elasticities with trip length. Second, travelers are more sensitive to upgrade cost relative to the transportation cost at shorter distances, but this difference in sensitivity diminishes with increasing trip length. Finally, the use of distance scaling corrects, to a large extent, the problem of the negative constant for the sleeper alternative for short distance trips\textsuperscript{11}.

![Figure 3: Rail Transportation and Upgrade Cost Coefficients as a Function of Distance](image)

\textsuperscript{10}The scale parameters, gamma1 and gamma2, were estimated by a grid search in the MNL model and fixed at these values for all further models. Otherwise, all model parameters are estimated simultaneously.

\textsuperscript{11}Even when not used for sleeping, the sleeper class offers an increased level of comfort relative to coach travel alternative. Therefore, intuitively we expect a positive bias constant for sleeper class for all trip lengths.
Model 3 retains the utility specification of the reference model but allows for variance scaling between 1st and 2nd preference observations. The estimated scale factor is not significantly different than 1.0. However, the scale factor becomes marginally significant in model 4, which incorporates both distance and variance scaling in its specification. Therefore, we retain the scale parameter for testing in advanced model specifications given that its value (less than one) is consistent with our expectation that 2nd choice observations have higher error variances; that is, people are less careful in decisions about their second choice than about their first choice. Model 4 rejects both models 2 and 3 at a confidence level of 0.001. We therefore adopt model 4 as the preferred utility specification as the basis to explore model structures that allow for differential substitution among alternatives.

Figure 4 shows the tree structures of four Nested Logit (NL) models that yielded logsum parameters significantly different from 1.0 and bounded by zero and one. The values of the respective logsum parameters (t-statistics) in each of the models and the log-likelihood (LL) values at convergence are also shown in the figure. The parameters for other variables in the utility specification, which were similar to those in the MNL model, are not reported here in the interest of brevity.

The auto-not travel nest is common to and significant in all four nesting structures suggesting a strong substitution between automobile and not travel alternatives. This nest has an intuitive interpretation suggesting that automobile travelers are more likely to give up their leisure trip than to shift to any other travel alternative.

The first three NL models (5, 6, and 7) have two-level nesting with similar goodness-of-fit statistics. All three reject the corresponding MNL model but lead to different behavioral interpretations. Model 5 implies higher substitution amongst the rail alternatives relative to non-rail modes. Model 6 implies higher substitution between coach and premium coach and Model 7 implies higher substitution among coach, premium coach, and air. A statistical comparison of these three models using the non-nested hypothesis test (Ben-Akiva and Lerman, 1985) leads to the rejection of the hypothesis that Model 7 is the correct model. However, neither of the remaining two models (5 and 6) can reject each other at any reasonable level of confidence.

Model 8 provides increased flexibility in representing differential substitution patterns amongst alternatives by including an additional level of nesting. In this three-level NL structure, the lower level includes three distinct nests for coach-premium coach, sleeper-economy sleeper, and auto-not travel. The upper level groups all four rail alternatives in a common nest to reflect similarities amongst these alternatives. The model outperforms all the two-level NL models rejecting them at a significance greater than 0.001. In addition to strong goodness-of-fit results, this nesting structure has the most intuitive behavioral interpretation. It implies an intermediate level of error correlation similarity among all rail alternatives, and a higher level of error correlation between coach and premium coach and between sleeper and economy sleeper. Among the NL models, we prefer model 8 due to its statistical superiority and intuitive appeal. However, even this three-level nesting structure imposes a number of restrictions on the similarity relationships between pairs of alternatives that may be unrealistic in this empirical setting. For example, it does not allow similarity in
unobserved attributes of rail alternatives and air, which is contrary to the relationships implied by the statistically significant coach-premium coach-air nest in Model 7. Therefore, despite its statistical superiority over the other NL structures, Model 8 fails to accommodate all the substitution patterns that are implied by the earlier estimation results. These results support our earlier hypothesis that correlations in unobserved component of the utility exists along a number of dimensions, which may lead to acceptance of a number of different nesting structures, and the preferred nesting structure in not clear, \textit{a priori}.

\textit{Figure 4: Tree Structures for Estimated Nested Logit Models}
The above limitations of the NL model are rooted in its restriction that each alternative can be included in one and only one nest. The GNL model addresses this problem through the use of allocation parameters, $\alpha$'s, which allow proportions of alternatives to be allocated to different nests. We illustrate this by estimating three GNL models (9, 10, and 11), the tree structure and estimation results for which are shown in Figure 5 and Table 4, respectively. Each of these GNL models have two distinct types of nests: 1) **similarity nest(s)** which include portions of alternatives grouped together to represent different similarity relationships, and 2) a **dis-similarity nest** which includes portions of all alternatives that are not allocated to any of the similarity nest(s). The logsum parameter for the dis-similarity nest is restricted to 1.0 and in essence provides MNL type relationship for the portions of alternatives included in this nest.

The utility specification used for these GNL models is identical to the MNL specification in Model 4. An important observation to be made in comparing the GNL utility parameters to the MNL model is that scale parameter for the 2nd choice observations becomes highly significant. This result is consistent with the findings of Bradley and Daly (1994) in which they observe declining precision in parameter estimates with increasing rank.

GNL Model 9 is similar to NL Model 5 with a rail nest and an auto-not travel nest, but allows different proportions of each rail alternative to be allocated to that nest producing different pairwise similarities amongst rail alternatives. are accommodated through differential allocation parameters, "$\alpha$'s. The GNL Model 9 rejects the NL Model 5 at a high level of significance, a reflection of the fact that all allocation parameters (except for Not Travel alternative in the Auto-Not Travel nest) obtain values that are significantly lower than 1.0.

GNL Model 10 adds a coach-air nest to Model 9 to allow for stronger substitution between coach and air alternatives – an insight gained from the estimation results of NL Model 6. As expected, the coach-air nest has logsum parameter and allocation parameters significantly different from 1.0 and 0.0 respectively. The log-likelihood ratio test rejects Model 9 at a high level of significance. Finally, Model 11 adds an additional nest for coach-premium coach to allow additional flexibility in representing a stronger substitution between coach and premium coach alternatives than perhaps can be accommodated in either GNL Model 9 or GNL Model 10, as implied by the three level NL model (Model 8). GNL Model 11 rejects NL Model 5 but does not reject Model 10 as the correct model. This suggests that the flexibility provided by the allocation parameters in terms of accommodating differential substitutions for the same logsum parameter reduces the need for searching multiple levels of nests. Therefore, we prefer the GNL Model 10 as our preferred specification based on both goodness of fit and interpretation.
Figure 5: Tree Structures for Estimated GNL Models
Table 4: Estimation Results for Selected GNL Models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model 0, Rail, Auto, Not Travel Nests</th>
<th>Model 10, Rail, Auto, Not Travel, Coach-Air, Nests</th>
<th>Model 11, Rail, Auto, Not Travel, Coach-Air, Coach-PC, Nests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omitted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inertia_Chock</td>
<td>1.195 (11.8)</td>
<td>0.915 (19.5)</td>
<td>1.132 (13.0)</td>
</tr>
<tr>
<td>Inertia_Sleeper</td>
<td>1.013 (12.6)</td>
<td>1.106 (13.6)</td>
<td>0.962 (12.3)</td>
</tr>
<tr>
<td>Inertia_Air</td>
<td>2.783 (7.6)</td>
<td>2.534 (7.2)</td>
<td>2.734 (8.6)</td>
</tr>
<tr>
<td>Inertia_Auto</td>
<td>1.277 (13.1)</td>
<td>1.180 (13.1)</td>
<td>1.140 (13.8)</td>
</tr>
<tr>
<td>Scaled Rail Transportation Cost (in $1,000)</td>
<td>-0.127 (-10.5)</td>
<td>-0.103 (8.0)</td>
<td>-0.120 (-10.4)</td>
</tr>
<tr>
<td>Scaled Rail Upgrade Cost_Low Income (in $1,000)</td>
<td>-0.016 (-11.2)</td>
<td>-0.012 (-11.5)</td>
<td>-0.016 (-11.9)</td>
</tr>
<tr>
<td>Scaled Rail Upgrade Cost_High Income (in $1,000)</td>
<td>-0.051 (-12.2)</td>
<td>-0.051 (-11.6)</td>
<td>-0.051 (-11.6)</td>
</tr>
<tr>
<td>Rail Departure Hour_Rail</td>
<td>-0.369 (-5.7)</td>
<td>-0.369 (-5.7)</td>
<td>-0.372 (-5.5)</td>
</tr>
<tr>
<td>Rail Departure Hour_Rail_Not Travel</td>
<td>-0.368 (-5.6)</td>
<td>-0.369 (-5.3)</td>
<td>-0.372 (-5.5)</td>
</tr>
<tr>
<td>Rail Departure Hour_Not Travel</td>
<td>0.623 (4.9)</td>
<td>0.623 (4.9)</td>
<td>0.623 (4.9)</td>
</tr>
<tr>
<td>Rail Departure Hour_Not Travel</td>
<td>0.743 (4.4)</td>
<td>0.633 (4.4)</td>
<td>0.623 (4.9)</td>
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<tr>
<td>Schedule_Demand_Dummy_Rail</td>
<td>0.501 (9.1)</td>
<td>0.501 (9.1)</td>
<td>0.501 (9.1)</td>
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<tr>
<td>Schedule_Demand_Dummy_Rail_Not Travel</td>
<td>0.501 (9.1)</td>
<td>0.699 (7.9)</td>
<td>0.701 (7.9)</td>
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<tr>
<td>Quality_Day_Dummy_Rail</td>
<td>-0.234 (-4.1)</td>
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<td>-0.245 (-4.3)</td>
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<tr>
<td>Quality_Rating_Rail</td>
<td>0.104 (13.8)</td>
<td>0.104 (13.8)</td>
<td>0.105 (13.8)</td>
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<tr>
<td>Group Size 1_Auto</td>
<td>-0.411 (-4.5)</td>
<td>-0.423 (-4.5)</td>
<td>-0.422 (-4.4)</td>
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<td>Group Size 2_Auto</td>
<td>-0.475 (-4.7)</td>
<td>-0.428 (-4.9)</td>
<td>-0.414 (-10.1)</td>
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<td>Group Size 1_Sleeper &amp; Economy Sleeper</td>
<td>-0.131 (-1.8)</td>
<td>-0.248 (-1.8)</td>
<td>-0.246 (-1.9)</td>
</tr>
<tr>
<td>Group Size 1_Sleeper &amp; Economy Sleeper</td>
<td>-0.131 (-1.8)</td>
<td>-0.248 (-1.8)</td>
<td>-0.246 (-1.9)</td>
</tr>
<tr>
<td>Income_Sleeper (in $1,000)</td>
<td>0.038 (7.6)</td>
<td>0.036 (7.6)</td>
<td>0.035 (7.6)</td>
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<tr>
<td>Income_Air (in $1,000)</td>
<td>0.087 (7.9)</td>
<td>0.087 (7.9)</td>
<td>0.087 (7.9)</td>
</tr>
<tr>
<td>Income_Auto (in $1,000)</td>
<td>0.030 (3.7)</td>
<td>0.030 (3.8)</td>
<td>0.031 (2.5)</td>
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<tr>
<td>Distance_Sleeper</td>
<td>0.052 (1.0)</td>
<td>0.051 (1.0)</td>
<td>0.052 (1.0)</td>
</tr>
<tr>
<td>Distance_Air</td>
<td>0.001 (1.3)</td>
<td>0.001 (1.1)</td>
<td>0.001 (1.1)</td>
</tr>
<tr>
<td>Distance_Rail</td>
<td>-0.004 (-1.1)</td>
<td>-0.029 (-5.0)</td>
<td>-0.028 (-5.1)</td>
</tr>
<tr>
<td>Alternative Specific Covariates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternative Shortest Distance</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Non-User Mode Not Available Not Travel</td>
<td>1.387 (12.3)</td>
<td>1.406 (12.0)</td>
<td>1.447 (11.4)</td>
</tr>
<tr>
<td>User Alternative Not Available Not Travel</td>
<td>0.036 (0.7)</td>
<td>0.172 (2.2)</td>
<td>0.189 (1.5)</td>
</tr>
<tr>
<td>Constant_Sleeper</td>
<td>-0.033 (-1.0)</td>
<td>-0.283 (-4.5)</td>
<td>-0.129 (-1.9)</td>
</tr>
<tr>
<td>Constant_Premium Coach</td>
<td>0.239 (6.7)</td>
<td>0.143 (3.9)</td>
<td>0.157 (4.8)</td>
</tr>
<tr>
<td>Constant_Economy Sleeper</td>
<td>0.343 (6.0)</td>
<td>0.343 (2.3)</td>
<td>0.239 (6.5)</td>
</tr>
<tr>
<td>Constant_Air</td>
<td>-0.780 (-4.7)</td>
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<td>-0.731 (-4.4)</td>
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<tr>
<td>Constant_Not Travel</td>
<td>-0.756 (-7.9)</td>
<td>-0.827 (-7.8)</td>
<td>-0.831 (-5.6)</td>
</tr>
<tr>
<td>Auto-Not Travel Nests_logsum</td>
<td>0.101 (49.1)</td>
<td>0.101 (49.1)</td>
<td>0.101 (49.1)</td>
</tr>
<tr>
<td>Rail Not Logsum</td>
<td>0.018 (422.5)</td>
<td>0.011 (382.0)</td>
<td>0.013 (758.6)</td>
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<tr>
<td>Coach-Air Not Logsum</td>
<td>0.017 (2.7)</td>
<td>0.017 (2.7)</td>
<td>0.017 (2.7)</td>
</tr>
<tr>
<td>Coach-Premium Coach Coach Nest Logsum (All alternatives)</td>
<td>0.024 (7.7)</td>
<td>0.024 (7.7)</td>
<td>0.024 (7.7)</td>
</tr>
<tr>
<td>Loglikelihood at zero</td>
<td>-17883.6</td>
<td>-17883.6</td>
<td>-17883.6</td>
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<tr>
<td>Loglikelihood at convergence</td>
<td>-14010.3</td>
<td>-13596.7</td>
<td>-13594.7</td>
</tr>
</tbody>
</table>

* T-statistics for tests of logsum parameters ($\theta_m$) are reported against the null hypothesis: $2_m = 1$.  
** Degenerate nest with logsum parameter constrained to 1.00.  
*** Portions of alternatives not allocated to any similarity nest, i.e., $1 - \sum M \alpha_{i,m}$, are assigned to the dis-similarity nest.  
**** T-statistics are not reported for parameters at boundary values in Gauss's CML Module.
6. SUMMARY AND CONCLUSIONS

This paper identifies and addresses some limitations in extant long distance travel choice models, both in terms of utility specification and model structure. We adopt the Generalized Nested Logit model to allow the representation of complex patterns of substitution among alternatives and add variance scaling to account for differential precision in responses obtained from SP ranking experiments. Further, a scaling adjustment is incorporated in portions of the systematic utility function to temper the elasticity scaling property of linear logit models. Superior estimation results support the benefit of combining these different advances to develop a preferred model for long distance intercity travel.

This research demonstrates the theoretical and empirical superiority of the GNL model structure over MNL and Nested Logit models in accommodating complex substitution patterns among alternatives. Further, it demonstrates the ability to estimate GNL models for a relatively large number of elemental alternatives. The GNL model structure allows complete flexibility in representing differential similarities between pairs of alternatives within the closed-form GEV framework. However, it imposes a greater responsibility on the analyst to explore and select among many alternative structures - a problem it shares (and potentially exacerbates) with the search required to select a preferred NL model. Our perspective is that testing an exhaustive set of GNL structures is neither advisable nor necessary. Rather, we prescribe using behavioral understanding and theory, and empirical observations from NL and/or Paired Combinatorial Logit (Chu, 1989; Koppelman and Wen, 2000) modeling results to develop hypotheses regarding similarity relationships among alternatives to be tested in a GNL framework.

Finally, the empirical insights gained from this research can provide a basis for evaluating the ridership and revenue potential for the proposed new classes of rail service for long distance intercity travel.
ACKNOWLEDGEMENTS

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REFERENCES


